School of Mathematics

Course 211 — Linear algebra & Differential Forms 2001-02
(SF Mathematics, SF Theoretical Physics, SF Two-subject Moderatorship with Economics & JS Two-subject Moderatorship)

Lecturer: Prof. D.J. Simms

Requirements/prerequisites: 111, 131

Duration: 24 weeks

Number of lectures per week: 3

Assessment: Some assignments, which do not contribute to the final grade

End-of-year Examination: One 3-hour end of year examination

Description: Vector space over a field, coordinate functions with respect to a basis, dual space, M and M^* mutually dual, homogeneous linear equations, linear operators, matrix with respect to a basis, Einstein index notation, conjugacy classes, trace, characteristic polynomial, rank, direct sums, invariant subspaces, eigenspaces, diagonalisability, simultaneous diagonalising, Hamilton-Cayley, primary decomposition theorem, Jordan form, minimum polynomial, zeros of minimum polynomial, diagonalizability iff minimum polynomial is a product of distinct linear factors.

Bilinear forms, symmetric, skew-symmetric, hermitian, non-singular scalar product space, covariant and contravariant components, raising and lowering indices, $M = N \oplus N^{\perp}$ if N finite dim non-singular, classification of scalar products: complex symmetric, real symmetric, hermitian, diagonalisation, quadratic forms, Sylvester law, diagonalisation using determinants (Jacobi), condition for positive definite, Schwarz and triangle inequalities, adjoint operator, self-adjoint, isometry, normal operator, spectral theorem, orthogonal, unitary, Lorentz transformations, simultaneous reduction of quadratic forms.

Tensor addition, multiplication, contraction, construction of invariants, wedge product of skew-symmetric tensors, volume element, Hodge star operator.

Tensor fields, metric tensor and line elements, gradient vector, tangent and normal, orientation, volume and area elements.

n-dim vector analysis with wedge product and Hodge star operator, Poincare lemma, Gaussian curvature, Stokes's theorem and applications.

October 10, 2001