School of Mathematics

Course 415 — Topics in Analysis (Fourier Analysis and Wavelets) 2000-01
(JS & SS Mathematics)

Lecturer: Prof. R. M. Aron & Dr. R. M. Timoney

Requirements/prerequisites: 221

Duration: 21 weeks

Number of lectures per week: 3 including tutorials

Assessment: No continuous assessment.

End-of-year Examination: 3-hour end of year examination.

Description:

1. **Basics of Functional Analysis**, including
   - Baire category theorem and consequences: Uniform Boundedness principle, closed graph theorem, open mapping theorem, et. al.
   - Schauder basis, Haar system for $L_2[0,1]$, Schauder system for $C[0,1]$ (with proofs). Comparison with Hamel basis.
2. **Hilbert Spaces**
   - revision of basis properties of inner product spaces
   - L_p, conjugate indices, Hölder's inequality, completeness of L_p
   - Convexity and Hilbert spaces
   - Every subspace of a Hilbert space is complemented
   - Orthonormal bases and examples.
   - Haar system for $L_2(\mathbb{R})$
3. **Fourier Analysis**
   - Fourier transform, basic definitions of $f * g$ and $\hat{f}$, basic properties, Fourier inversion theorem, Plancherel theorem (without proof).
   - Riemann-Lebesgue lemma, Paley-Wiener theorem, uncertainty principle, Nyquist sampling rate.
   - Discrete Fourier transform
4. **Continuous Wavelet Transform** on $\mathbb{R}$.
5. **Orthonormal Wavelet bases**, dilation equations, multiresolution analysis.
6. **Discrete Wavelet transform**

Textbooks: The following may be used as references, but no one book is being followed closely.

1. Berberian, S. K., Introduction to Hilbert Space, Oxford (1961).

2. Burke Hubbard, B., The world according to Wavelets: The story of a Mathematical Technique in the Making (2nd ed.), A. K. Peters, Natick, Massachusetts (1998).

3. Chui, C., Wavelets: a mathematical tool for signal processing, SIAM, Philadelphia, PA, 1997.

4. Daubechies, Ingrid, Ten Lectures on Wavelets, CBMS-NSF Regional Conference Series in Applied Mathematics volume 61, Society for Industrial and Applied Mathematics (1992).

5. Firth, Jean M., Discrete Transforms, Chapman & Hall (1992).

6. Rudin, W., Real and Complex Analysis, McGraw-Hill (3rd ed., 1987).

7. G. Strang, *Wavelet transforms versus Fourier transforms*, Bull. Amer. Math. Soc. **28** (1993) 288–305.

8. Wojtaszczyk, P., A mathematical introduction to wavelets. London Mathematical Society Student Texts, 37, Cambridge University Press, Cambridge, 1997.

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