School of Mathematics

Course 381 — Mathematical Economics 2000–01
(J.S./S.S. Mathematics, J.S./S.S. TSM)

Lecturer: Dr P Waldron

Requirements/prerequisites: 211 is an essential prerequisite, 221 and 251 are strongly recommended and 212 would be useful, but not essential.
This course and 412 (Measure theory and probability) are complementary and students with an interest in finance should take both, in any order.

Duration: Michaelmas, Hilary and Trinity

Number of lectures per week: 2lectures + 1 tutorial per week.

Assessment: Problem sets will be distributed regularly throughout the course and students are expected to attempt them in advance of the tutorials in which they are discussed. Collaboration on the more difficult problems is encouraged. The course mark will be based entirely on the annual examination, but this examination will be based to a significant extent on the problem sets. The examination paper will comprise nine questions and each student is expected to attempt six of these.

End-of-year Examination: 3-hour end of year exam.

Description: Mathematics course 381 is a course in mathematical economics (including a substantial component on mathematical finance) taught by Dr. Patrick Waldron (Economics Department). It is a compulsory mathematics course for Junior Sophister TSM students in mathematics and economics and may also be chosen by the following:

- Sophister students in single honor mathematics
- Senior Sophister TSM students in mathematics whose other subject is not economics

Objective The objective of this course is to introduce students of mathematics to a few of the countless applications of mathematics in modern economics and finance. Much of the mathematics will be familiar, and the emphasis will be on applying it in economics.

Course outline

Following a brief introduction to economics, the remainder of the course will be broken down into the following topics:

1. Convexity and Optimisation:
   (a) Vector calculus prerequisites
   (b) Convexity and concavity
   (c) Unconstrained optimisation
   (d) Equality constrained optimisation: The Lagrange Multiplier Theorems
   (e) Inequality constrained optimisation: The Kuhn-Tucker Theorems

   (f) Quadratic programming
   (g) Duality

2. Single-Period Choice Under Certainty:
   (a) Definitions
   (b) Axioms
   (c) Marshallian and Hicksian demand
   (d) Indirect utility and expenditure
   (e) Further results in demand theory
   (f) General equilibrium theory
   (g) The Welfare Theorems
3. Multi-Period Choice Under Certainty
4. Single-Period Choice Under Uncertainty:
   (a) Review of basic probability
   (b) Taylor's theorem: Stochastic version
   (c) Pricing state-contingent claims
   (d) The expected utility paradigm
   (e) Jensen's inequality and Siegel's paradox
   (f) Risk aversion
   (g) The mean-variance paradigm
   (h) Other non-expected utility approaches
5. Multi-Period Choice Under Uncertainty
6. Portfolio Theory:
   (a) Notation and preliminaries
   (b) The single-period portfolio choice problem
   (c) Mathematics of the portfolio frontier
   (d) Market equilibrium and the Capital Asset Pricing Model

All of the above topics will not necessarily be covered in every year.

Bibliography

Further details about the course are available at the web site `http://pwaldron.bess.tcd.ie/teaching/ma381/2000-1/`.

The accompanying bibliography contains a variety of items, ranging from classics such as Debreu (1959) and von Neumann and Morgenstern (1953), through popular works on the use of mathematics in financial markets such as Bernstein (1992) and Bass (1999) to original journal

articles such as Black (1972) and Merton (1972). Students are of course encouraged to read widely, and appropriate references to the literature will be provided in lectures. As far as course textbooks are concerned, for the first section, the primary text, insofar as there is one, is Takayama (1994). For the second section, it is Varian (1992) and for the remaining sections, it is Huang and Litzenberger (1988).

October 10, 2000