On the Absence of Cross–Confinement for Dynamically Generated Multi–Chern–Simons Theories

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Abstract

We show that when the induced parity breaking part of the effective action for the low–momentum region of $U(1) \times \ldots \times U(1)$ Maxwell gauge field theory with massive fermions in 3 dimensions is coupled to a ϕ^4 scalar field theory, it is not possible to eliminate the screening of the long-range Coulomb interactions and get external charges confined in the broken Higgs phase. This result is valid for non-zero temperature as well.

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1 Introduction

In a recent paper Cornalba et al. [1] have proposed a novel topological way of confining charged particles. The method uses the special properties of $U(1) \times U(1)$ Chern–Simons gauge theory, interacting with external sources in two spatial dimensions, with a scalar Higgs field providing condensates. The idea of the approach is to note that, when charge/flux constraints of a certain type are not satisfied, the fall off of the Higgs fields at infinity will not be fast enough and will lead to configurations with infinite energy; hence, such configurations are confined. The analysis is based on number–theoretic properties of the couplings and charges and shows the intriguing possibility for confinement even for integral charge particles. The confinement mechanism is topological in origin.

A Chern–Simons term of the form considered in [1] can be dynamically generated as the parity–breaking part of the low–momentum region of the effective action of a three–dimensional $U(1) \times \ldots \times U(1)$ Maxwell gauge filed theory with fermions, after integrating out the fermionic degrees of freedom [2], [3]. Indeed, we carry out this procedure for the system at non-zero temperature. The effective action, obtained by us, following the approach of [2] has the correct temperature dependence for the multiple $U(1)$ Chern–Simons term and yields in its zero-temperature limit a multiple Chern–Simons term of the form considered in [1], [4], [5].

Such multiple $U(1)$ gauge theories have been considered before, for example: in the study of spontaneously broken abelian Chern–Simons theories [4], [5]; in the study of two–dimensional superconductivity without parity violation [6].

Our original motivation was to investigate if the mechanism for cross–confinement, proposed in [1], continues to hold for the system with a temperature slightly deviated from zero and if confinement is lost for high temperature with the system still in the Higgs phase.

Surprisingly, with this dynamically generated parity–breaking term, the arguments of Cornalba et al. [1] do not hold, namely, the proposed scheme of confinement is not possible. This result is valid, as we show, for zero and non–zero temperatures. In this model it is not possible to eliminate the screening of the long–range Coulomb interactions. We claim that, if confinement occurs, it happens when the broken $U(1) \times \ldots \times U(1)$ gauge symmetry is restored in at least one of the directions of the gauge group.

By the standard Higgs mechanism, the gauge group is spontaneously broken down to a product of the cyclic groups $Z_1 \times \ldots \times Z_N$. This residual symmetry represents the non-trivial holonomy of the Goldstone boson. The photon fields $A_\mu^{(i)}$, $i = 1, ..., N$ now acquire masses by their coupling to the Goldstone bosons. In the broken Higgs phase the Higgs currents are proportional in magnitude to the massive vector fields and screen the Coulomb interaction and we are left with purely quantum Aharonov–Bohm interactions [4], [5]. In this phase, at temperature well below the critical, all conserved charges can reside in the zero–momentum mode due to the bosonic character of the particles. When the temperature increases, some of the charges get excited out of the condensate and at sufficiently high temperature the condensate becomes thermally disordered and the symmetry is restored. When this happens the charges introduced by the matter currents will not be screened and the energy of the Coulomb field will logarithmically diverge with distance (in two spatial dimensions) and this will lead to confinement.

2 The Model

We will determine first the parity–breaking part of the effective action for $U(1)\times\ldots\times U(1)$ Maxwell gauge field theory coupled to massive fermions and ϕ^4 scalar field theory in 3 dimensions at finite temperature. Contact with the multiple Chern–Simons term, considered by [1], [4], [5] is made by taking the zero-temperature limit. The effective action for the low–momentum region of the theory is:

$$e^{-\Gamma(A^{(k)},M_k)} = \int\prod_{k=1}^{N}\mathcal{D}\psi_k\mathcal{D}\bar{\psi}_k\mathcal{D}\phi\ exp\Big\{-\int_0^\beta d\tau\int d^2x\Big\{\sum_{k=1}^{N}\Big(\bar{\psi}_k\not{D}^f_k\psi_k + j^{(k)}A^{(k)}\Big) + \\ + (D_\alpha\phi)(D^\alpha\phi)^* - m^2\phi\phi^* - \lambda(\phi\phi^*)^2\Big\}\Big\} \quad (1)$$

where $\not{D}^f_k = \not{\partial} + iQ_{kj}\not{A}^{(j)} + M_k$ are the fermionic covariant derivatives with $Q_{kj},\ k,j = 1,\ldots,N$ being the matrix of the fermionic charges with respect to the N gauge groups, $D_\alpha = \partial_\alpha + iq_kA^{(k)}_\alpha, k = 1,\ldots,N$ are the covariant derivatives for the scalar field with q_i being the charge of the scalar field with respect to the i^{th} gauge group. In this action $\beta = \frac{1}{T}$ is the inverse temperature and Dirac matrices are in the representation $\gamma_\mu = \sigma_\mu$. We have also introduced external currents coupled to the gauge fields.
We shall consider first the parity-breaking part of the fermionic part of the action and at this stage the scalar field is only a spectator.
For this purpose we will follow the approach of Fosco et al. [2].
The fermionic fields obey antiperiodic boundary conditions, while the gauge fields are periodic. The considered class of configurations for the gauge fields is:

$$A^{(k)}_3 = A^{(k)}_3(\tau), \quad A^{(k)}_{1,2} = A^{(k)}_{1,2}(x), \quad k = 1,\ldots,N \qquad (2)$$

There is a family of gauge transformation parameters, which allow us to gauge the time-components $A_3^{(i)}(\tau)$ to the constants $a^{(i)}$ [2]. This makes the Dirac operator invariant under translations in the time coordinate (as the dependence on τ comes solely from the $A_3^{(i)}$ fields) and therefore we could Fourier–expand ψ_i and $\bar{\psi}_i$ over the Matsubara modes. Following steps, similar to those in [2], one finds that the parity-odd bit of the fermion part of the effective action is given by:

$$\Gamma_{odd} = \frac{i}{2\pi} \sum_{k,j=1}^{N} \sum_{n=-\infty}^{+\infty} \phi_n^{(k)} \int \epsilon_{lm} Q_{kj} \partial_l A_m^{(j)} d^2x \tag{3}$$

where $\phi_n^{(k)} = arctg\left(\frac{\omega_n + Q_{kj} a^{(j)}}{M_k}\right)$ and $\omega_n = (2n+1)\frac{\pi}{\beta}$ is the Matsubara frequency for fermions.

Performing the summation we get that for $U(1) \times \ldots \times U(1)$ gauge group the parity-odd part of the action is:

$$\begin{aligned} e^{-\Gamma_{odd}} = \int \mathcal{D}\phi \; exp\Bigg\{ -\frac{i}{2\pi} \sum_{k,j,n=1}^{N} arctg\left[th\left(\frac{\beta M_k}{2}\right) \, tg\left(\frac{1}{2} \int_0^\beta Q_{kj} A_3^{(j)}(\tau) d\tau \right) \right] \\ \times \int \epsilon_{lm} Q_{kn} \partial_l A_m^{(n)} d^2x \\ + \int_0^\beta d\tau \int \left[\, (D_\alpha \phi)(D^\alpha \phi)^* - m^2 \phi\phi^* - \lambda(\phi\phi^*)^2 + j^{(k)} A^{(k)} \right] d^2x \Bigg\} \end{aligned} \tag{4}$$

As the temperature T approaches 0 (that is $\beta \to \infty$) this reduces to $U(1) \times \ldots \times U(1)$ Chern–Simons gauge theory.

We will use now the effective parity–odd temperature dependent action (with the induced $U(1) \times \ldots \times U(1)$ parity breaking term) to re-examine the confinement argument of Cornalba et al. [1]. First of all, let us perform the integration (using Stokes' theorem) of the gauge fields over the spatial coordinates. This gives the relation with the magnetic fluxes Φ_l:

$$\begin{aligned} e^{-\Gamma_{odd}} = \int \mathcal{D}\phi \; exp\Bigg\{ -\frac{i}{2\pi} \sum_{k,j,n=1}^{N} arctg\left[th\left(\frac{\beta M_k}{2}\right) \, tg\left(\frac{1}{2} \int_0^\beta Q_{kj} A_3^{(j)}(\tau) d\tau \right) \right] Q_{kn} \Phi_n \\ + \int_0^\beta d\tau \int \left[(D_\alpha \phi)(D^\alpha \phi)^* - m^2 \phi\phi^* - \lambda(\phi\phi^*)^2 + j^{(k)} A^{(k)} \right] d^2x \Bigg\} \end{aligned} \tag{5}$$

The equations of motion, obtained by varying the action with respect to the magnetic fields, are:

$$-\frac{i}{4\pi} \sum_{k,m,n=1}^{N} \frac{th\left(\frac{\beta M_k}{2}\right) Q_{kl} Q_{kn} \Phi_n}{cos^2\left(\frac{1}{2} \int_0^\beta Q_{km} A_3^{(m)} d\tau \right) + th^2\left(\frac{\beta M_k}{2}\right) sin^2\left(\frac{1}{2} \int_0^\beta Q_{km} A_3^{(m)} d\tau \right)}$$

$$-\, q_l \int (\phi D_3\phi^* - \phi^* D_3\phi) d^2x \; + \; \int \rho^{(l)} d^2x = \int \partial_j F^{(l)\, j3}\, d^2x \qquad (6)$$

where $\rho^{(l)} = j_3^{(l)}$ are the charge densities. Here we have included explicitely the contribution of the Maxwell term $F^{(i)}_{\mu\nu} F^{(i)\,\mu\nu}$. We ignore temperature dependent terms which come from $O(A^4)$ terms in the effective action. These are of higher order $\left(O(Q^4)\right)$ in the fermionic chagres. The Coulomb charges on the r.h.s. vanish because all $U(1)$ fields are massive.
Denote by u the integral over the third component of the conserved Nöther current: $u = \int (\phi D_3 \phi^* - \phi^* D_3 \phi) d^2x$ and by $C^{(l)} = \int \rho^{(l)} d^2x$ the total external charge. So we have:

$$\mu \Phi = C - uq \qquad (7)$$

where $\Phi = \begin{pmatrix} \Phi_1 \\ \vdots \\ \Phi_N \end{pmatrix}$, $\quad q = \begin{pmatrix} q_1 \\ \vdots \\ q_N \end{pmatrix}$, $\quad C = \begin{pmatrix} C^{(1)} \\ \vdots \\ C^{(N)} \end{pmatrix}$, $\quad$ and:

$$\mu_{ln} = \frac{i}{4\pi} \sum_{k,m=1}^{N} \frac{th\left(\frac{\beta M_k}{2}\right) Q_{kl} Q_{kn}}{cos^2\left(\frac{1}{2}\int_0^\beta Q_{km} A_3^{(m)} d\tau\right) + th^2\left(\frac{\beta M_k}{2}\right) sin^2\left(\frac{1}{2}\int_0^\beta Q_{km} A_3^{(m)} d\tau\right)} \qquad (8)$$

As in [1] there is another condition which must be satisfied by the magnetic fluxes. The Higgs field ϕ should be completely condensed, i.e. $\phi(x) = ve^{i\sigma(x)}$, where $\sigma(x)$ is the Goldstone boson field (the mass and the coupling constants of the scalar field are temperature–dependent). In order that this holds we have to require that the covariant derivative of the scalar field vanishes. After integration we get:

$$2\pi l = q_1 \Phi_1 + \ldots + q_N \Phi_N = \; {}^t q \; \Phi \qquad (9)$$

where $2\pi l$ is the non-trivial holonomy of the Goldstone boson (reflecting a topological property of the Higgs field). Combining the two conditions (7) and (9) for the fluxes we get:

$$\begin{aligned} \mu \Phi &= C - uq \\ 2\pi l &= {}^t q \; \Phi \end{aligned} \qquad (10)$$

Following the analysis of [1] we identify u as a continuous parameter, representing the ability of the condensate to screen the electric charge.
The matrix μ can be written as $\mu = \; {}^t Q F(\beta) Q$, where $F(\beta)$ is a diagonal matrix with entries:

$$F_{kj}(\beta) = \frac{i}{4\pi} \frac{th\left(\frac{\beta M_k}{2}\right)}{cos^2\left(\frac{1}{2}\int_0^\beta Q_{km} A_3^{(m)} d\tau\right) + th^2\left(\frac{\beta M_k}{2}\right) sin^2\left(\frac{1}{2}\int_0^\beta Q_{km} A_3^{(m)} d\tau\right)} \delta_{kj} \qquad (11)$$

As $F(\beta)$ is diagonal we can always write μ in the form:

$$\mu = {}^tQ(T)\ Q(T) \tag{12}$$

where $Q_{mn}(T) = F^{1/2}_{mj}(\beta)Q_{jn}$.
Let us now try to eliminate the screening in (10) by inverting the matrix μ. We get that if the determinant of μ is not zero and if

$${}^tq\ \mu^{-1}q = 0 \tag{13}$$

then the screening would be eliminated (the condition ${}^tq\ \mu^{-1}q = 0$ is the condition for confinement, proposed by Cornalba et al. [1]. According to their analysis, if the determinant of μ vanishes, then μ^{-1} should be interpreted as the transposed matrix of co-factors).
Assuming that the determinant of μ is not zero, we can re-write this as:

$${}^t\Big(\tilde{Q}(T)q\Big)\tilde{Q}(T)q = 0 \tag{14}$$

where $\tilde{Q}(T)$ is the matrix of co-factors. This equation shows that the vector $\tilde{Q}(T)q$ is orthogonal to itself ("orthogonal" with respect to the matrix multiplication of column vectors) and, therefore, this is the null vector:

$$\tilde{Q}(T)q = 0 \tag{15}$$

This is an equation for the values of the boson field charges, which would eliminate the screening mechanism. As we see, we can have a non–trivial solution if, and only if, $det\ Q = 0$, which contradicts to our initial assumption ($det\ \mu \neq 0$). Therefore, we cannot eliminate the screening. Otherwise, this theory would be inconsistent with the induced parity–breaking term. This argument is valid for all values of the temperature.
Intuitively, one can expect that if condition (9) is violated, after elimination of screening, there would be currents which would not fall off faster than $1/r$ at infinity and the resulting long–range forces will lead to diverging energies. We argue that condition (9) can never be violated — this condition represents the fact that we are left with a residual symmetry after the spontaneous symmetry breakdown. If this condition does not hold, it would mean that the symmetry is restored. This, on its turn, will lead to diverging energy straight away, but not in the broken Higgs phase.
We conclude that confinement is not possible in the Higgs phase in the presence of the dynamically generated parity–breaking term (which coincides with Chern–Simons term in zero–temperature limit). If there are configurations with infinte energy, they must necessarily be outside the broken Higgs phase — where the gauge symmetry is restored.

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