

Ramond-Ramond Couplings on Brane-Antibrane Systems

Conall Kennedy* and Andy Wilkins†

*Department of Mathematics
Trinity College
Dublin 2
Ireland*

(May 1999)

Couplings between a closed string RR field and open strings are calculated in a system of coincident branes and antibranes of type II theory. The result can be written cleanly using the curvature of the superconnection.

*E-mail: conall@maths.tcd.ie

†E-mail: andyw@maths.tcd.ie

1 Introduction

Unstable D-branes can decay and produce branes of lower dimension. The most simple decay is that of a Dp -brane annihilating with a Dp -antibrane to yield a $D(p-2)$ -brane. In this scenario the lightest open-string mode stretching between the brane-antibrane pair is tachyonic and it can condense into a configuration with non-trivial winding. Then, in order to have finite energy, the gauge field living on the branes must have non-zero first Chern class which implies the existence of a non-zero $(p-2)$ -brane charge. The happy state of affairs is that the negative energy density of the condensed tachyon field exactly cancels with the positive energy density of the brane-antibrane pair asymptotically, leaving a $D(p-2)$ -brane with finite tension [1].

One can also start with non-supersymmetric Dp -branes in type II theory (p is odd for IIA and even for IIB). Here too there is a tachyon in the spectrum of the open string connecting the brane to itself. It can condense into a kink and the unstable non-supersymmetric brane will decay to yield a stable $D(p-1)$ brane. This type of brane production was studied in detail in [2] where it was argued that all supersymmetric D-branes in IIA can be constructed as bound states of a number of D9-branes. Further examples of such decay processes are studied in [2–8] and can be understood within the context of K-theory [9, 10]. The unstable and non-BPS branes have also been used in testing various duality conjectures [1, 8, 11, 12], indeed it was in this context that they made their first major appearance.

The configurations that are studied in this note are a straightforward generalisation of the scenario described in the first paragraph to N Dp -branes coincident with N Dp -antibranes. Without the antibranes the coupling of the $U(N)$ massless world-volume vectors (denoted by A) to the closed-string RR fields (denoted by C) is given by the ‘Wess-Zumino’ action [13, 14]

$$S = \int_{\Sigma_{(p+1)}} C \wedge \text{Tr} e^F, \quad (1)$$

where $\Sigma_{(p+1)}$ is the world volume, Tr is over the Chan-Paton factors and F is the field strength for A . Numerical factors such as α' have been suppressed, as have the contributions from the antisymmetric 2-form and the A-roof genus [15–17]. When the antibranes are included, the light degrees of freedom are two $U(N)$ gauge fields — one living on the branes (A^+) and the other on the antibranes (A^-) — and a tachyon (T) and antitachyon (\bar{T}) living in the (N, \bar{N}) and (\bar{N}, N) representations respectively. We would like to know the generalisation of the Wess-Zumino action. To this end we perform a tree-level string calculation of the effective action to low orders in the tachyon field.

We find a non-zero contact interaction of the form

$$\lambda \int_{\Sigma_{(p+1)}} C_{(p-1)} \wedge d \text{Tr} \left(T \wedge \overline{DT} \right), \quad (2)$$

where the covariant derivatives are

$$DT = dT + A^+ T - T A^- \quad \text{and} \quad \overline{DT} = d\bar{T} - \bar{T} A^+ + A^- \bar{T}.$$

In this note we will not calculate the overall normalisation of such terms (except to say their coefficients are not zero) since they are unimportant for our purposes. We also consider the brane-antibrane pairs to have indistinguishable world volumes.¹ The result Eq. (2) means the total charge of the $(p-2)$ -brane is measured by $\int \text{Tr} \left(F^+ - F^- + \lambda d(T\bar{D}T) \right)$ which contains the first Chern class of the gauge configuration on the brane, the antibrane, and the winding number of the tachyon configuration. However, this latter term does not add unwanted charge to the $(p-2)$ -brane because the covariant derivative, DT , must vanish at infinity in order for the solitonic configuration to have finite energy.

The coupling to the RR $(p-1)$ -form given by Eqs. (1) and (2) can be rewritten to read

$$\int_{\Sigma_{(p+1)}} C_{(p-1)} \wedge \text{Tr} \left(F^+ - F^- - \frac{1}{2} \{F^+, T\bar{T}\} + \frac{1}{2} \{F^-, \bar{T}T\} + DT \wedge \bar{D}T \right), \quad (3)$$

Interestingly, this can be written in a more compact form by employing the superconnection [19–21]

$$\mathcal{A} = \begin{pmatrix} d + A^+ & T \\ \bar{T} & d + A^- \end{pmatrix},$$

which transforms under the $U(N) \times U(N)$ symmetry as

$$\mathcal{A} \rightarrow \mathcal{G} \mathcal{A} \mathcal{G}^{-1} \quad \text{where} \quad \mathcal{G} = \begin{pmatrix} g & 0 \\ 0 & \bar{g} \end{pmatrix}.$$

The curvature of this is

$$\mathcal{F} = \begin{pmatrix} F^+ - T\bar{T} & DT \\ \bar{D}T & F^- - \bar{T}T \end{pmatrix},$$

and a “supertrace” is defined by

$$\text{STr} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \text{Tr} a - \text{Tr} d.$$

Then the terms in Eq. (3) are just those in an expansion of

$$S = \int_{\Sigma_{(p+1)}} C \wedge \text{STr} e^{\mathcal{F}}. \quad (4)$$

We propose that this generalises the usual Wess-Zumino action Eq. (1).

Before moving on to the string calculation, let us show that our proposal gives the correct charges for all decay products. This can be seen by using the ‘transgression formula’ [20, p. 47] which reads

$$\text{STr} \exp \mathcal{F}_1 - \text{STr} \exp \mathcal{F}_0 = d \int_0^1 dt \text{STr} \frac{d\mathcal{A}_t}{dt} \exp \mathcal{F}_t,$$

¹Further comments on this point are made in [18] in which the tachyon potential was shown to assume a Mexican hat shape for weak fields

where \mathcal{A}_t is a superconnection depending on a continuous parameter t , and \mathcal{F}_t is its curvature. Let us choose

$$\mathcal{A}_t = \begin{pmatrix} d + A^+ & tT \\ t\bar{T} & d + A^- \end{pmatrix} .$$

When the transgression formula is integrated over the $2k$ directions perpendicular to the $(p - 2k)$ -brane's world volume we can set $DT = 0 = \overline{DT}$ on the RHS. The RHS then integrates to zero because it contains only *odd* dimensional forms. This implies

$$\int_{2k} \text{STr} \exp \mathcal{F} = \int_{2k} \text{Tr} \exp F^+ - \int_{2k} \text{Tr} \exp F^- ,$$

as required.

2 The calculations

The process of interest is one where a RR boson annihilates onto the brane-antibrane world volume to create some open strings. By string duality this is just an open string graph with insertions on the boundary (the brane's world volume) and one RR insertion in the interior. Similar calculations have been performed in [22–27] and we will follow their notations and conventions:

Map the disc to the upper-half plane so the boundary of the disc becomes the real axis. On this axis the worldsheet bosons (X^μ) and fermions (ψ^μ) obey the following boundary conditions

$$\text{Neumann: } \begin{cases} X^a(z) = \tilde{X}^a(\bar{z}) \\ \psi^\mu(z) = \tilde{\psi}^\mu(\bar{z}) \end{cases} \quad \text{and} \quad \text{Dirichlet: } \begin{cases} X^i(z) = -\tilde{X}^i(\bar{z}) \\ \psi^i(z) = -\tilde{\psi}^i(\bar{z}) \end{cases} ,$$

while the ghosts (c) and superghosts (ϕ) obey trivial boundary conditions. Indices $\mu = 0, \dots, 9$, while an index a lives on the world volume ($a = 0, \dots, p$) and i fills the transverse space ($i = (p + 1), \dots, 9$). Because of these boundary conditions the correlators mix;

$$\begin{aligned} \text{Neumann: } \langle X^a(z) X^b(w) \rangle &= -\eta^{ab} \log(z - w) & \text{and} & & \langle X^a(z) \tilde{X}^b(\bar{w}) \rangle &= -\eta^{ab} \log(z - \bar{w}) , \\ \text{Dirichlet: } \langle X^i(z) X^j(w) \rangle &= -\eta^{ij} \log(z - w) & \text{and} & & \langle X^i(z) \tilde{X}^j(\bar{w}) \rangle &= +\eta^{ij} \log(z - \bar{w}) . \end{aligned} \tag{5}$$

and similarly for the fermions (we use the conventions $\alpha' = 2$). Now use the “doubling trick” in which the fields $X(z)$ and $\psi(z)$ are extended to the lower-half plane by defining

$$X(z) = \begin{cases} \tilde{X}(z) & \text{Neumann} \\ -\tilde{X}(z) & \text{Dirichlet} \end{cases} \quad (z \in \text{LHP}) .$$

Then if we think of \bar{w} being in the LHP the correlation function of this extended holomorphic field

$$\langle X^\mu(z) X^\nu(w) \rangle = -\eta^{\mu\nu} \log(z - w) ,$$

correctly reproduces all of Eq. (5). Thus, when considering scattering off p -branes, the rule is to replace

$$\tilde{X}^\mu(\bar{z}) \rightarrow D_\nu^\mu X^\nu(\bar{z}) , \quad \tilde{\psi}^\mu(\bar{z}) \rightarrow D_\nu^\mu \psi^\nu(\bar{z}) , \quad \tilde{\phi}(\bar{z}) \rightarrow \phi(\bar{z}) \quad \text{and} \quad \tilde{c}(\bar{z}) \rightarrow c(\bar{z}) ,$$

where

$$D = \begin{pmatrix} \mathbf{1}_{p+1} & 0 \\ 0 & -\mathbf{1}_{9-p} \end{pmatrix} ,$$

and then use the usual correlators

$$\begin{aligned} \langle X^\mu(z) X^\nu(w) \rangle &= -\eta^{\mu\nu} \log(z-w) , \\ \langle \psi^\mu(z) \psi^\nu(w) \rangle &= -\eta^{\mu\nu} (z-w)^{-1} , \\ \langle c(z) c(w) \rangle &= (z-w) , \\ \langle \phi(z) \phi(w) \rangle &= -\log(z-w) . \end{aligned}$$

The vertex operators for the tachyon are

$$V_T^{(0)}(x) = k \cdot \psi e^{ik \cdot X}(x) \quad \text{and} \quad V_T^{(-1)}(x) = e^{-\phi} e^{ik \cdot X}(x) ,$$

where the superscripts label the superghost number. The momentum k is constrained to lie in the world volume; $k^\mu = (k^a, 0)$ with $k^2 = 1/4$ in our conventions ($\alpha' = 2$). In the coincident brane-antibrane system the vertex operators for the tachyon and the antitachyon look the same — in order to distinguish them a Chan-Paton factor must be understood. After doubling, the vertex operators become [28]

$$\begin{aligned} V_T^{(0)}(z) &= k \cdot \psi e^{2ik \cdot X}(z) , \\ V_T^{(-1)}(z) &= e^{-\phi} e^{2ik \cdot X}(z) . \end{aligned}$$

The RR vertex operators are

$$V_{RR}^{(-1)}(w, \bar{w}) = (P_- \mathbf{H}_{(m)})^{\alpha\beta} : e^{-\phi/2} S_\alpha e^{ip \cdot X}(w) :: e^{-\tilde{\phi}/2} \tilde{S}_\beta e^{ip \cdot \tilde{X}}(\bar{w}) :$$

with the projector $P_- = \frac{1}{2}(1 - \gamma^{11})$ assuring that we are using the correct chirality and

$$\mathbf{H}_{(m)} = \frac{1}{m!} H_{\mu_1 \dots \mu_m} \gamma^{\mu_1} \dots \gamma^{\mu_m} ,$$

where $m = 2, 4$ for type IIA and $m = 1, 3, 5$ for type IIB. The spinorial indices are raised with the charge conjugation matrix, eg $(P_- \mathbf{H}_{(m)})^{\alpha\beta} = C^{\alpha\delta} (P_- \mathbf{H}_{(m)})_{\delta\beta}$ (further conventions and notations for spinors can be found in appendix B of [23]). The RR bosons are massless so $p^2 = 0$. The spin fields can also be extended to the entire complex plane. In calculations we replace

$$\tilde{S}_\alpha(\bar{w}) \rightarrow M_\alpha^\beta S_\beta(\bar{w}) ,$$

where

$$M = \frac{1}{p!} \gamma^{\alpha_0} \gamma^{\alpha_1} \dots \gamma^{\alpha_p} \epsilon_{\alpha_0 \dots \alpha_p} .$$

Finally, a couple of correlators containing two spin fields are

$$\begin{aligned}\langle S_\alpha(w)S_\beta \rangle &= w^{-5/4}C_{\alpha\beta}^{-1}, \\ \langle \psi^\mu(z)S_\alpha(w)S_\beta \rangle &= (z-w)^{-1/2}z^{-1/2}w^{-3/4}\gamma_{\alpha\beta}^\mu.\end{aligned}$$

where $S_\beta = S_\beta(0)$ and $C_{\alpha\beta}$ is the charge conjugation matrix.

2.1 The two-point amplitude

The amplitude containing one RR field and one tachyon (or antitachyon) is

$$\begin{aligned}\mathcal{A}^{T,RR} &= \int \frac{dzdwd\bar{w}}{V_{\text{CKG}}} \langle V_T^{(-1)}(z)V_{RR}^{(-1)}(w,\bar{w}) \rangle, \\ &= \int \frac{dzdwd\bar{w}}{V_{\text{CKG}}} \langle e^{-\phi(z)}e^{2ik\cdot X(z)}e^{-\phi(w)/2}S_\alpha(w)e^{ip\cdot X(w)}(P_-H_{(m)}M)^{\alpha\beta} \\ &\quad \times e^{-\phi(\bar{w})/2}S_\beta(\bar{w})e^{ip\cdot D\cdot X(\bar{w})} \rangle.\end{aligned}$$

We have chosen the vertex operators according to the rule that the total superghost number must be -2 . Including the ghost contribution, $\langle c(z)c(w)c(\bar{w}) \rangle = (z-w)(z-\bar{w})(w-\bar{w})$, and using the kinematic constraint, $k^\alpha + p^\alpha = 0$, the volume of the conformal Killing group can be canceled by fixing the three insertion points. The amplitude then reads

$$\mathcal{A}^{T,RR} = \text{Tr} \left(P_- H_{(m)} M \right).$$

Repeated use of $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ yields

$$\text{Tr} \left(\gamma^{[\mu_1} \dots \gamma^{\mu_m]} \gamma^{[a_0} \dots \gamma^{a_p]} \right) = \frac{1}{2} 32(p+1)! \delta_{m,p+1} \eta^{\mu_m a_0} \eta^{\mu_{m-1} a_1} \dots \eta^{\mu_1 a_p},$$

implying that the amplitude vanishes since there is no $H_{(p+1)}$ in the type II string. On the other hand, in the case of a non-BPS brane (as studied recently in [29]) this amplitude implies the effective action contains the term $\int C \wedge dT$.

2.2 The three-point amplitude

The three-point amplitude between one RR field, and two tachyonic particles is

$$\begin{aligned}\mathcal{A}^{T,T,RR} &= \int \frac{dzdz'dwd\bar{w}}{V_{\text{CKG}}} \langle V_T^{(0)}(z)V_T^{(-1)}(z')V_{RR}^{(-1)}(w,\bar{w}) \rangle, \\ &= \int \frac{dzdz'dwd\bar{w}}{V_{\text{CKG}}} \langle k\cdot\psi(z)e^{2ik\cdot X(z)}e^{-\phi(z')}e^{2ik'\cdot X(z')}e^{-\phi(w)/2}S_\alpha(w)e^{ip\cdot X(w)}(P_-H_{(m)}M)^{\alpha\beta} \\ &\quad \times e^{-\phi(\bar{w})/2}S_\beta(\bar{w})e^{ip\cdot D\cdot X(\bar{w})} \rangle.\end{aligned}$$

It is convenient to fix the points $(z, z', w, \bar{w}) = (x, -x, i, -i)$ to cancel the volume V_{CKG} by inserting the ghost contribution

$$(c(z')c(w)c(\bar{w})) = (z' - w)(z' - \bar{w})(w - \bar{w}) .$$

Introduce the Mandelstam variable $t = -(k + k')^2$. Then using various kinematic constraints such as

$$k^a + k'^a + p^a = 0 \quad \text{and} \quad p \cdot D \cdot p = -2t = -2p \cdot (k + k') ,$$

the amplitude reduces to

$$\begin{aligned} \mathcal{A}^{T,T,RR} &= \int_{-\infty}^{\infty} dx \left(\frac{(1+x^2)^2}{16x^2} \right)^{\frac{1}{2}+it} \frac{1}{1+x^2} \text{Tr} (P_- \mathbb{H}_{(m)} M \gamma^a) k_a , \\ &= \pi \frac{\Gamma[-2t]}{\Gamma[\frac{1}{2}-t]^2} \text{Tr} (P_- \mathbb{H}_{(m)} M \gamma^a) k_a . \end{aligned} \quad (6)$$

Next the trace must be evaluated;

$$\text{Tr} (\gamma^{\mu_1} \dots \gamma^{\mu_m} \gamma^{a_0} \dots \gamma^{a_p} \gamma^a) H_{\mu_1 \dots \mu_m} \epsilon_{a_0 \dots a_p} = \frac{1}{2} 32(p+1)! \delta_{m,p} (-1)^{p(p+1)/2} H_{a_0 \dots a_{p-1}} \epsilon^{a_1 \dots a_{p-1} a} .$$

The trace containing the factor of γ^{11} ensures the following results also hold for $p > 3$ with $H_{(m)} \equiv \star H_{(10-m)}$ for $m \geq 5$.

The prefactor of Eq. (6) has the interesting property that at non-negative integer values of t there is a pole corresponding to an open-string resonance with mass-squared $m^2 = t = -p_a^2$. On the other hand, at positive half-integer values of t it vanishes, implying that strings with half-integer mass-squared do not propagate in this channel. In [24] it was shown that these are also properties of the amplitude for one NS-NS string to decay into two massless open strings stuck to a D-brane.

The low-energy effective Lagrangian of massless and tachyonic particles will contain the terms

$$\mathcal{L}_{\text{eff}} \sim H_{(p)} \wedge F + \mathcal{L}_{\text{YM}} + DT \cdot \overline{DT} , \quad (7)$$

as well as the terms we are looking for. Here \mathcal{L}_{YM} is the Yang-Mills Lagrangian and in keeping with the spirit of the rest of this note all constants have been omitted. At low energies ($-t = p_a^2 \sim 0$) the prefactor of Eq. (6) may be expanded

$$\pi \frac{\Gamma[-2t]}{\Gamma[\frac{1}{2}-t]^2} = \frac{-1}{2t} + 2 \log 2 + O(t) .$$

The first term corresponds to the RR particle decaying into a massless open string (via the first term in Eq. (7)) which propagates (resulting in the pole) and decays into two tachyons (via the third term in Eq. (7)). Because the second term is non-zero, the effective action contains a coupling between the RR field, the tachyon and the antitachyon.

In summary, in this subsection we have shown that the effective action contains the term

$$S_{\text{eff}} = \int_{\Sigma_{(p+1)}} H_{(p)} \wedge \text{Tr} \bar{T} dT .$$

which is Eq. (2) after integrating by parts and covariantising. As an aside, if there is no antitachyon (in the case of the non-supersymmetric brane) the integration by parts gives zero.

3 Summary and Discussion

In coincident brane-antibrane systems we have shown, by calculating tree-level string amplitudes, that in addition to the usual Wess-Zumino terms the world-volume effective action contains

$$\int_{\Sigma_{(p+1)}} C_{(p-1)} \wedge d \text{Tr} (T \bar{D} T) ,$$

to $O(T\bar{T})$. After tachyon condensation the correct charges for decay products are obtained. We propose that the full result (to all orders in the tachyon) can be written in terms of the curvature of the superconnection:

$$\int_{\Sigma_{(p+1)}} C \wedge \text{STr} \exp \mathcal{F} .$$

It is amusing to note the connection of our proposal with noncommutative geometry. It would be very interesting to show that a similar sort of substitution $F \rightarrow \mathcal{F}$ could be made in the Dirac-Born-Infeld part of the effective action.

Acknowledgments

We thank Siddhartha Sen and Jim McCarthy for commenting on various drafts. This work was supported financially by the Enterprise Ireland grant F01121.

References

- [1] A. Sen *Tachyon condensation on the brane antibrane system* JHEP **9808** 012 (1998) hep-th/9805170
- [2] P. Horava *Type IIA D-branes, K-theory and Matrix theory* CALT-68-2205 hep-th/9812135
- [3] O. Bergman and M. R. Gaberdiel *Stable non-BPS D-particles* hep-th/9806155
- [4] A. Sen *Type I D-particle and its interactions* hep-th/9809111

- [5] A. Sen *BPS D-branes on non-supersymmetric cycles* JHEP **9812** 021 (1998) hep-th/9812031
- [6] A. Sen *Descent relations among bosonic D-branes* hep-th/9902105
- [7] M. Frau, L. Gallot, A. Lerda and P. Strigazzi *Stable non-BPS D-branes in type I string theory* DFTT-14-99 hep-th/9903123
- [8] A. Sen *SO(32) spinors of type I and other solitons on brane-antibrane pair* JHEP **9809** 023 (1998) hep-th/9808141
- [9] E. Witten *D-branes and K-theory* hep-th/9810188
- [10] O. Bergman, E. G. Gimon and P. Horava *Brane transfer operations and T-duality of non-BPS states* hep-th/9902160
- [11] A. Sen *Stable non-BPS states in string theory* JHEP **9806** 007 (1998) hep-th/9803194
- [12] A. Sen *Stable non-BPS bound states of BPS D-branes* JHEP **9808** 010 (1998) hep-th/9805019
- [13] J. Polchinski *Dirichlet-branes and Ramond-Ramond charges* Phys. Rev. Lett. **75** 4724–4727 (1995) hep-th/9510017
- [14] M. Li *Boundary states of D-branes and dy-strings* Nucl. Phys. **B460** 351–361 (1996) hep-th/9510161
- [15] M. Bershadsky, C. Vafa and V. Sadov *D-branes and topological field theories* Nucl. Phys. **B463** 420–434, (1996) hep-th/9511222
- [16] M. Green and J. Harvey and G. Moore *I-brane inflow and anomalous couplings on D-branes* Class. Quant. Grav. **14** 47–52, (1997) hep-th/9605033
- [17] YK. Cheung and Z. Yin *Anomalies, branes, and currents* Nucl. Phys. **B517** 68–91, (1998) hep-th/9710206
- [18] I. Pesando *On the effective potential of the $D_p\overline{D}_p$ system in type II theories* DFTT-7/99 hep-th/9902181
- [19] D. Quillen *Superconnections and the Chern character* Topology **24** 89–95, (1995)
- [20] N. Berline, E. Getzler and M. Vergne *Heat Kernels and Dirac operators*, Springer-Verlag (1991)
- [21] G. Roepstorff *Superconnections and the Higgs field* hep-th/9801040
- [22] S. S. Gubser, A. Hashimoto, I. R. Klebanov and J. M. Maldacena *Gravitational lensing by p-branes* Nucl. Phys. **B472** 231–248 (1996) hep-th/9601057
- [23] M. R. Garousi and R.C Myers *Superstring scattering from D-branes* Nucl. Phys. **B475** 193–224, (1996) hep-th/9603194

- [24] A. Hashimoto and I. R. Klebanov *Decay of excited D-branes* Phys. Lett. **B381** 437–445 (1996) hep-th/9604065
- [25] A. Hashimoto and I. R. Klebanov *Scattering of strings from D-branes* Nucl. Phys. Proc. Suppl. **B55** 118–133 (1997) hep-th/9611214
- [26] M. R. Garousi *Superstring scattering from D-branes bound states* JHEP **9812** 008 (1998) hep-th/9805078
- [27] M. R. Garousi and R. C. Myers *World-volume interactions on D-branes* Nucl. Phys. **B542** 73–88 (1999) hep-th/9809100
- [28] V. A. Kostelecký, O. Lechtenfeld and S. Samuel *Covariant string amplitudes on exotic topologies to one loop* Nucl. Phys. **B298** 133–177 (1988)
- [29] M. Billó, B. Craps and F. Roose *Ramond-Ramond couplings of non-BPS D-branes* KUL-TF-99/17 hep-th/9905157