ELECTROWEAK BARYOGENESIS WITHOUT THE PHASE TRANSITION

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Radiation domination at the electroweak epoch is a simplifying assumption, but one for which there is no observational basis. Treating the expansion rate as a variable, I re-examine electroweak baryogenesis in various scenarios. At a first order phase transition the main effect is on the sphaleron bound, which becomes a lower bound on the expansion rate in any given theory. At a second-order or cross-over phase transition, the created baryon asymmetry is directly proportional to the expansion rate. I sketch an alternative post-inflationary cosmology, in which the kinetic energy of a scalar field dominates the Universe until shortly before nucleosynthesis, and argue that the observed baryon asymmetry could be produced in this case even at an analytic cross-over.

1 Introduction

A loose analogy between nucleosynthesis and electroweak baryogenesis is often drawn in arguing the particular merit of the latter - that, in the not too distant future, our experimental knowledge of the relevant physics may be on the same firm basis as our knowledge of the nuclear and particle physics on which nucleosynthesis rests. It has some hope of being a truly testable theory which may provide us with some solid probe of the pre-nucleosynthesis universe, potentially as compelling as that provided by nucleosynthesis of its epoch. The analogy is worth pushing a little further. One of the great successes of nucleosynthesis is how it has been able to constrain the number of effective relativistic degrees of freedom. This constraint is actually just one on the expansion rate at freeze-out of the weak interactions, and affects (to a first approximation) only the abundance of Helium. It is a constraint on the cosmology (albeit stated as one on particle physics), given this single observational input. In this contribution I address the analogous question of electroweak baryogenesis: What constraint does the *requirement* that the observed baryon asymmetry be generated at the electroweak scale place on cosmology at that scale? Having answered this question I sketch one simple cosmology in which baryon generation could occur at the electroweak scale in a regime in which it is usually assumed to be impossible. The results sketched here are from recent work by myself [1] and in collaboration with T. Prokopec [2], in which the reader can find greater detail and a fuller set of references.

A trivial but important point to emphasize is that we actually know *nothing* of cosmology at the electroweak scale: We have at present no convincing experimental probe of that epoch. In red-shift the electroweak scale is approximately as far from nucleosynthesis as the latter is from the transition to matter domination, and the simplest backward extrapolation is certainly worth calling into question. Here I assume as usual that the Universe is indeed a hot plasma back to temperatures $T > T_{ew} \sim 100\text{GeV}$, but allow the expansion rate to be a variable H_{ew}. To determine the relics from this epoch we need to know the expansion rate as a function of temperature around T_{ew} and take

$$H = H_{ew}(\frac{T}{T_{ew}})^p \tag{1}$$

where p is some number, of which the results below are essentially independent.

2 EWB at a first order phase transition

The asymmetry created as envisaged in the standard scenario as bubble walls sweep through the unbroken phase is essentially insensitive to a change, even of many orders of magnitude, in the expansion rate away from its radiation dominated value $H_{rad} \sim 10^{-16}T$. The main effect is on the sphaleron bound for the preservation of the created asymmetry which requires that

$$\mathcal{D} \equiv -\ln\frac{B_{\text{final}}}{B(T_b)} = \int_{t_b}^{\infty} dt\Gamma_{sph}(t) = H_b^{-1}\int_0^{T_b} dT\frac{\Gamma_{sph}}{T}(\frac{T_b}{T})^p < 1 \tag{2}$$

where $B(T_b)$ is the baryon asymmetry at the completion of the transition, at temperature T_b when the expansion rate is H_b, and Γ_{sph} is the appropriately normalized rate of sphaleron processes. The latter form of this expression follows from (1) which gives $t \propto T^{-p}$. This integral is dominated by temperatures very close to T_b so that the result is essentially independent of p. The bound can be cast to a good approximation in the form of an expansion rate dependent correction to its familiar form as a lower bound on $\frac{\phi_b}{T_b}$ (the ratio of the vev to the temperature at T_b):

$$\frac{\phi_b}{T_b} > \left(\frac{\phi_b}{T_b}\right)_{rad} - \frac{0.06}{\mathcal{B}}\ln\frac{H_b}{H_{rad}} \tag{3}$$

where $(\frac{\phi_b}{T_b})_{rad}$ is the appropriate critical value (typically $1-1.2$) in the radiation dominated universe, and $\mathcal{B} \in [1.5, 2.7]$ is the usual monotonic function of m_H^2/m_W^2 which appears in relating the sphaleron energy to the vev. For $m_H \sim 80\text{GeV}$, for example, this means a change of 0.08 in the bound per order of magnitude in the expansion rate. In any particular model (3) can alternatively be used to state the sphaleron bound for preservation of the baryon asymmetry as a lower bound on the expansion rate of the Universe.

3 EWB in a homogeneous Universe

Under this rubric I include any case where the electroweak sphaleron processes freeze out at a time when the evolution of the plasma is well approximated as homogeneous. This includes the case of a very weak first order phase transition (too weak, that is, to satisfy the sphaleron bound), a second-order phase transition and an analytic cross-over (where there is, strictly speaking, no phase transition at all). I consider a two doublet Higgs model (and the special case which is the minimal supersymmmetric standard model), and calculate the baryon production due to a CP odd term [3] in the one loop effective action $\mathcal{L}_{eff} = (g^2/16\pi^2)\chi F\tilde{F} = \dot{\chi}B$, where F and $\tilde{F}$ are the $SU(2)$ field strength tensor and its dual, and the latter form follows from the anomaly equation and the assumption of homogeneity. The time dependent parameter is given by [3]

$$\dot{\chi} = 7\zeta_3 \left(\frac{m_t}{\pi T}\right)^2 \frac{v_2^2}{v_1^2 + v_2^2}\dot{\theta} \tag{4}$$

where v_1 and v_2 are the ratios of the two vevs (the first of which is assumed to couple to the top quarks), θ is the CP odd relative angle between them, and $\zeta_3 \approx 1.2$. Performing a simple local thermal equilibrium calculation in the presence of this term (which is simply a potential for baryon number) one obtains the baryon to entropy ratio

$$\frac{B}{s} = -\frac{45 c_n}{2\pi^2 g_*}\left(\frac{H}{T}\right)_{freeze}\left(T\frac{d\chi}{dT}\right)_{freeze} \tag{5}$$

where $c_n \approx 0.44$, g_* is the number of relativistic degrees of freedom and the subscript means that the quantities are evaluated at freeze-out of the sphaleron processes.

In order that (5) be in the range compatible with nucleosynthesis we therefore require the expansion rate at freeze-out to satisfy

$$\left(\frac{H}{T}\right)_{\rm freeze} \simeq (2-12)\times 10^{-11}\, g_* \,\frac{1}{|(T d\chi/dT)_{\rm freeze}|}. \tag{6}$$

4 An alternative cosmology

The previous section clearly motivates consideration of the possibility that the expansion rate at the electroweak scale is considerably greater than usually assumed. One simple way in which this can come about is if there was some component of the energy density *scaling faster than radiation* which dominated prior to nucleosynthesis (but red-shifts away sufficiently by that time). One does not need to look far beyond the usual framework of early universe cosmology to find such a component, for consider a real scalar field ϕ with potential $V(\phi)$ for which the equation of motion (of the zero mode) is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{7}$$

Defining $\zeta(t) = \frac{V(\phi)}{\rho(\phi)}$ where $\rho(t) = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ is the total energy density of the scalar field, one gets on integration that

$$\rho(t) = \rho(t_o)e^{-\int_{t_o}^{t} 6(1-\zeta(t))H(t)dt} = \rho(t_o)e^{-\int_{a_o}^{a} 6(1-\zeta(a))\frac{da}{a}} \tag{8}$$

Attention is usually focussed on the case $\zeta \to 1$ since it gives inflation with $\rho \approx const$. It is the other limit, of kinetic energy domination, when $\zeta \to 0$ and $\rho \propto 1/a^6$ which is of interest here. A phase of the universe dominated by such a mode I term *kination*.

A very simple way in which such an epoch could follow inflation is if the inflaton potential becomes asymtyotically exponential with $V(\phi) = V_o e^{-\lambda\phi/M_P}$ where $M_P = 2.4 \times 10^{18}$GeV is the reduced Planck mass. Such potentials can in fact arise quite generically in theories involving compactified dimensions, such as superstring and supergravity theory. For sufficiently large λ the field will run into a mode which scales as $1/a^6$. How then is the Universe reheated? It was pointed in [4] that in a model of this sort the energy density in particles created by the inflationary expansion would come to dominate at some later stage. Typically at the end of inflation, when the expansion rate is H_i and the energy density in the inflaton ρ_i, the energy density created in radiation is $\rho_{radn} \sim H_i^4$, and $\frac{\rho_{radn}}{\rho_i} \sim \frac{\rho_i}{M_P^4} << 1$. The Universe after inflation is filled with radiation (which rapidly thermalizes), but dominated until a lower temperature $\sim H_i^2/M_P$ by the kinetic energy scaling as $1/a^6$. During such a phase we have

$$H^2 = \frac{1}{3M_P^2}\frac{\rho_e}{2}\left[\left(\frac{a_e}{a}\right)^6 + f(a)\left(\frac{a_e}{a}\right)^4\right], \tag{9}$$

where a_e is the scale factor when the density in the mode becomes equal to that in radiation and ρ_e is the total energy density at that time. The factor $f(a) = [g_*(a_e)/g_*(a)]^{1/3}$ describes the effect of decouplings. Nucleosynthesis provides a lower bound on a_e, which can easily be converted to an upper bound on the expansion rate at freeze-out:

$$\left(\frac{H}{T}\right)_{freeze} \leq 2 \times 10^{-11}\left(\frac{T_{\text{freeze}}}{100\text{GeV}}\right)^2 \tag{10}$$

Allowing this bound to be saturated (i.e. taking the mode to dominate as long as is consistent with nucleosynthesis) then gives the requirement

$$\left| T \frac{d\chi}{dT} \right|_{\text{freeze}} \geq g_* \left(\frac{100GeV}{T_{\text{freeze}}} \right)^2 \tag{11}$$

A simple examination of the two Higgs doublet model shows that in a typical part of the parameter space, one has $T \frac{d\theta}{dT} \approx \frac{1}{\tan\theta} \frac{T}{v} \frac{dv}{dT}$, where v is one of the vevs. Using the perturbative effective potential for the minimal standard model(MSM), one finds that $\frac{d\phi}{dT} \sim 50$ in a narrow temperature range below T_c where the vev changes by about 50%. This will typically be where the sphaleron processes freeze out. Examining the data from lattice studies of the MSM[5] this sort of behaviour appears to survive in the regime of analytic cross-over - physical quantities change continuously, but 'sharply' over a range of a few GeV (with T_f which can be as large as $\sim$ 250GeV). Thus the constraint (11) may indeed be satisfied in parts of the parameter space of models such as the two Higgs model with a CP odd source term like (4).

I conclude by returning to the analogy with nucleosynthesis. I began by comparing the calculation here to fitting the Helium abundance predicted by nucleosynthesis by varying the number of relativistic degrees of freedom. To go further and emulate nucleosynthesis, we need other probes of the expansion rate. What might be our deuterium? An interesting possibility is the relic density of any dark matter particle which freezes out prior to nucleosynthesis, the sensitive dependence of which on cosmology has been previously discussed in the literature[6]. A more indirect root is through observables consequences at other epochs of alternative cosmologies prior to nucleosynthesis e.g. the existence of an exponential field of the type required in the model outlined has effects on structure formation in the Universe, which should be discernible in future observations of the cosmic microwave background[7].

References

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