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Turning Around the Sphaleron Bound: Electroweak Baryogenesis in an Alternative Post-inflationary Cosmology

Michael Joyce* and Tomislav Prokopec‡

* School of Mathematics, Trinity College, Dublin 2, Ireland
‡ Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen, Denmark

Abstract

The usual sphaleron bound and the statement of the impossibility of baryon production at a second order phase transition or analytic cross-over are reformulated in the first part of the paper as requirements of the expansion rate of the Universe at the electroweak scale. With an (exact or effective) additional contribution to the energy density scaling as $1/a^6$, which dominates until just before nucleosynthesis, the observed baryon asymmetry may be produced at the electroweak scale in simple extensions of the Minimal Standard Model, even in the case that the phase transition is not first order. We focus our attention on one such cosmology, in which the Universe goes through a period termed *kination* in which its energy is dominated by the kinetic energy of a scalar field. The required kinetic energy dominated modes can occur either as a field rolls down an exponential (or steeper) potential, or in the oscillation of a field about the minimum of a steep power-law potential. We implement in detail the former case with a single exponential field first driving inflation, and then rolling into a kinetic energy dominated mode. Reheating is achieved using an alternative to the usual mechanism due to Spokoiny [1], in which the Universe is 'reheated' by particle creation in the expanding background. Density perturbations of the magnitude required for structure formation may also be generated. We show that the analogous model for the power-law potential cannot be consistently implemented. In models with inflation driven by a second field and the usual mechanism of reheating (by decay of the inflaton) the required kinetic energy dominated cosmology is viable in both types of potential.

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*email: joyce@maths.tcd.ie
‡email: prokopec@nbi.dk

1 Introduction

Nucleosynthesis provides a role model for electroweak baryogenesis to whose impressive heights it can still only aspire. The great attraction of the idea that the baryon asymmetry of the Universe (BAU) may have been created at the electroweak epoch lies in the possibility that one day an *ab initio* calculation to rival that of nucleosynthesis may be possible, and that it will give a definitively positive or negative answer. Rather than simply providing an alternative to scenarios for baryon creation at the GUT scale, it has the fundamental interest of relying on physics at a scale directly accessible to experiments. We can realistically hope to know the correct theory of physics at the electroweak scale, in particular the structure of the CP violating and symmetry breaking sectors. Just as in nucleosynthesis it is then a question of putting this theory in an expanding universe and finding the output. Electroweak baryogenesis however faces more substantial obstacles on the road to a reliable calculation than did nucleosynthesis, *e.g.* the determination of the baryon asymmetry involves all the details of departure from equilibrium at the phase transition (if there is one), the crucial baryon number violating processes arising from the chiral anomaly at finite temperature involve many difficult and still unresolved questions, *etc.* Much progress has however been made, recently in particular using lattice methods to study the phase transition [2], and the problems seem not to be insurmountable.

The approach of this paper is somewhat orthogonal to the direction of investigation of most work on electroweak baryogenesis. Rather than investigating some aspect of the particle physics, we consider the cosmological side of the problem. The standard and indeed most natural assumption about cosmology at the electroweak epoch is that it is what one gets by the simplest backward extrapolation from nucleosynthesis: a homogeneous and isotropic radiation dominated universe. In nucleosynthesis the assumption of such a universe is relaxed to place limits on, for example, the contribution of a magnetic field or of a cosmological "constant" to the energy density. In this paper we ask the analogous question of electroweak baryogenesis: how is the standard scenario for production of the observed baryon asymmetry at the electroweak epoch affected if we consider cosmologies other than the standard one? And are there simple alternative cosmologies which lead to significantly different results for electroweak baryogenesis?

The same sort of question has been previously addressed in the context of calculations of the relic densities of weakly interacting particles in work of Barrow [3] and Kamionkowski and Turner (KT) [4]. These relic densities depend on the temperature at freeze-out which occurs (approximately) when the annihilation cross-section of the particular species drops below the expansion rate of the Universe. Barrow discussed the particular case of a non-anisotropic universe, in which the average (volume) expansion rate which determines the freeze-out has an extra component driving it which scales as $1/a^6$ (a is the average scale factor). Consistent with the requirement of radiation domination at nucleosynthesis, the expansion rate can thus be very much greater in the anisotropic universe prior to nucleosynthesis when dark matter relics typically freeze-out ($T \sim 100MeV$), and the requirement that such a particle be the cosmological dark matter may in principle place a bound on the anisotropy. The important idea - that relic densities can provide a probe of the Universe prior to nucleosynthesis, which might be other than the standard radiation dominated one - was considered in a more general way by KT, who discussed the case of an anisotropic universe, as well as various others, including a Brans-Dicke-Jordan theory of gravity. In this latter case the effect can also be modelled as an extra contribution to the energy density scaling as $1/a^6$, producing in the same way a speeded up expansion rate before nucleosynthesis without violating the nucleosynthesis constraints. KT also mention an example (which they describe as "exotic") of a scalar field ϕ which oscillates in the minimum of a potential ϕ^n, for which the energy density scales as $a^{-6n/(n+2)}$, *i.e.* faster than radiation for $n > 4$. Again the energy in such a mode can contribute significantly before nucleosynthesis without disrupting the latter. As discussed in [5] the relevant feature of this model is that it is the kinetic energy of the scalar field which gives the dominant contribution to the energy density of the Universe. As well as the oscillating mode of the power-law potential, the scaling applies to a scalar field rolling down a simple exponential potential. Rather than being exotic (compared to the models which [3] and [4] focus on), such models are minimal in the sense that they leave Einstein gravity intact and are consistent with the inflationary explanation of the homogeneity and isotropy of the Universe. In this paper we construct and study in detail a model for each of the two cases in which the single field (exponential or power-law) both inflates the Universe and then rolls into the kinetic energy dominated mode. Reheating is achieved using a simple alternative model of reheating proposed by Spokoiny several years ago [1]. In the power-law potential density perturbations are produced which are too large and the model is not viable. Furthermore the coherent oscillating mode is unstable to decay due to parametric resonance. We also discuss less constrained viable models in which the inflaton is a different field and reheating proceeds in the usual way (by decay of the condensate). The required potentials do in fact arise in many particle physics models: Power-law potentials have been discussed, for example, in the context of supersymmetry motivated inflationary models in [6, 7]. (The lower order terms can be excluded by imposing a discrete $Z_{n/2+1}$ symmetry on the superpotential). Exponential potentials arise quite generically in theories involving compactified dimensions, such as supergravity and higher dimensional theories of gravity (for specific examples, see [8, 9, 10]). The latter are also interesting in that they can play an important (potentially observable) role in the late-time cosmology of structure formation [11, 12].

In the first part of the paper we address the question of how the expansion rate of the Universe affects the baryon asymmetry produced at the electroweak scale, without reference to any particular cosmological model. The two distinct cases - a first order phase transition, and a second-order phase transition or a cross-over - are treated separately. In the first case the expansion rate enters (i) in determining when the transition occurs, since this depends on the cooling rate of the Universe below the critical temperature, and (ii) in determining the depletion of the baryon asymmetry produced by sphaleron processes (and hence in determining the sphaleron bound). In this case the baryon asymmetry is actually (at least in certain extensions of the standard model) produced on or near the bubble walls as they propagate through the plasma, and does not depend directly on the expansion rate. In the second case the expansion rate is the sole parameter which controls the departure from equilibrium and the baryon asymmetry which can be generated depends directly on it.

In the second part of the paper we turn to the discussion of alternative cosmologies, first reviewing those considered in previous work and then turning to the detailed consideration of cosmologies dominated by the kinetic energy of a scalar field for a period between inflation and nucleosynthesis, concentrating in particular on the case when this phase (which, following [5] we term *kination*) persists until after the electroweak scale. In this case the expansion rate at the electroweak scale is increased, producing the modifications to calculations for electroweak baryogenesis discussed in the first part. As outlined above, the two types of models we consider are exponential potentials and power-law ϕ^n $(n > 4)$ potentials. For both cases we first discuss a one field model, in which the field both inflates the Universe and causes kination. Instead of decaying as in the standard explanation of reheating, the inflaton rolls into a kinetic mode and simply red-shifts away. The radiation created by the superluminal expansion of the Universe at the transition between the two phases thermalizes and comes to dominate the energy density of the Universe at a later time determined by the expansion rate at the end of inflation [1]. We show that in the exponential potential one can have (i) a transition to radiation domination as late as nucleosynthesis and (ii) thermalization of the radiation well prior to the electroweak scale, and further, (iii) density perturbations of the right amplitude for structure formation. In the power-law potential, however, we find that the requirement that the Universe become radiation dominated before nucleosynthesis leads to the production of density perturbations which are much larger than is consistent with observations. In any case the oscillating mode in this model typically decays non-perturbatively (through parametric resonance) unless the self-couplings of the field are extremely small, and the energy does not stay in the kinetic mode for long enough. We conclude section 5 with a brief discussion of two fields models in which one field is the inflaton and reheats the Universe in the standard way, and the second field is the 'kinaton' which comes to dominate for a phase subsequent to inflation. In this case the second field can be either exponential or power-law (provided the couplings are such that decay by parametric resonance does not occur until after nucleosynthesis). In the last section we discuss the implications of our results for the testability of theories of electroweak baryogenesis, and consider briefly other ways in which pre-nucleosynthesis cosmology might be probed.

2 Dependence on the Expansion Rate

In electroweak cosmology the assumption is generally made that the Universe is flat, homogeneous, isotropic, and radiation dominated. Hence all cosmological information is encoded in the expansion rate $H_{\rm rad}$, which is given as a function of the plasma temperature T:

$$H_{\rm rad} = h\frac{T^2}{M_{\rm P}}\,, \qquad h = \left(\frac{\pi^2 g_*}{90}\right)^{\frac{1}{2}} \tag{1}$$

where $g_* \sim 10^2$ is the number of relativistic degrees of freedom in the plasma and $M_{\rm P} = (8\pi G)^{-1/2} \simeq 2.4 \times 10^{18}$GeV is the reduced Planck mass. The clean separation between the purely particle physics and cosmological calculations occurs because of the adiabaticity of the expansion. It is adiabatic because the timescale characterizing the expansion – $\tau_{\rm expansion} = M_P/(hT^2) \simeq 10^{16}(100{\rm GeV}/T)T^{-1}$, taking $g_* \simeq 10^2$ – is much greater than the timescales associated with the thermalization processes which have typical rates $\sim \alpha^2 T$ (where $\alpha > 1/50$ for all the interactions well above 100GeV). The phase transition can thus be studied using equilibrium methods - the expansion of the Universe enters only in determining the cooling rate and, hence, when the transition occurs (if it does). In general we could of course consider any cosmology at the electroweak scale, with the sole requirement that it be consistent with nucleosynthesis. We have no probe of the electroweak scale except that provided by electroweak physics, and methodologically it makes sense to ask how changing the standard assumption about the Universe at this phase affects the predictions of the remnants which result. Here we limit ourselves to relaxing only the assumption that the expansion rate is related to the temperature by (1). Instead we take

$$H = H_{\rm ew}\left(\frac{T}{T_{\rm ew}}\right)^p \tag{2}$$

where p is a number and the subscript 'ew' means the quantities are evaluated at some temperature characteristic of the electroweak phase transition. Using $H = \dot{a}/a$ this corresponds to the time dependence $a \propto t^{1/p}$ for the scale factor a. All our results concerning baryogenesis are, we will see, essentially independent of p because they depend only on temperatures very close to $T_{\rm ew}$. We will treat $H_{\rm ew}$ as a free parameter, only taking it to be such that the assumption of adiabaticity is valid, which allows it to be different form the standard value by orders of magnitude. We will review in section 5 some of the non-standard cosmologies which can be described by these assumptions. The model which we will discuss in detail is a homogeneous and isotropic universe dominated by a kinetic mode of a scalar field rather than by radiation.

Baryogenesis, the creation of baryons from an initial zero baryon state, requires a departure from thermal equilibrium. In the Big Bang Universe this is provided by the expansion which causes the Universe to cool. At the electroweak scale this cooling can lead to two very different effects, depending on whether the electroweak phase transition is of first order or not. Recall that at a first order phase transition as the Universe cools it becomes thermodynamically favourable for the system to be in the "broken" state. The "broken" and "unbroken" phases are separated by a potential barrier which decreases as the Universe cools. Once the barrier is low enough, the transition proceeds by the nucleation and propagation of true vacuum bubbles. This departure from equilibrium is characterized by timescales which are much shorter than that associated with the expansion. Almost all proposed mechanisms for baryogenesis at the electroweak scale make use of this dramatic departure from equilibrium, using the interaction between the plasma and propagating walls to generate the baryon asymmetry. In the case that the transition is second order or cross-over there is no such effect. Everything evolves continuously and the departure from equilibrium is controlled directly by the expansion rate. This usually leads one to conclude that anything but a first order phase transition is inimical to baryogenesis at the electroweak scale. Once one relaxes the assumption that the expansion rate is its standard radiation dominated value, this conclusion does not follow and needs to be examined more carefully. We will thus treat these two cases separately in some detail.

3 First Order Phase Transition

In this case the baryons are created as the bubbles of the true vacuum propagate through the false vacuum. The net effect of the propagation of the bubble through the medium in all proposed mechanisms is the creation of a flux of baryons into the broken phase. The expansion rate enters only indirectly through other parameters involved in this calculation - through the temperature at which the transition occurs which it determines, and, in certain regimes, through the bubble wall velocity. On the other hand, it enters directly in the determination of the amount of the created asymmetry which survives once it is in the broken phase. We consider these two dependences separately.

3.1 Bubble Nucleation

In this section we investigate how the bubble nucleation temperature depends on the expansion rate of the Universe. We also briefly discuss how the bubble wall velocity may depend on this parameter. As the Universe supercools below the critical temperature $T_{\rm c}$, the fraction f filled by nucleated bubbles at a time t is given by (see [2] and references therein):

$$f(t) = 1 - e^{-\Delta(t)}\,, \qquad \Delta(t) = \int_{t_{\rm c}}^{t} dt' \frac{4\pi}{3} v^3 (t-t')^3 \mathcal{R}(t')\,, \quad \mathcal{R} = I_0 T^4 \mathrm{e}^{-S_{\rm b}/T}\,, \tag{3}$$

where $S_{\rm b}$ is the bounce action, $\mathcal{R}$ the nucleation rate per unit volume, I_0 is a prefactor which is a slowly varying function of temperature of order one, given in more detail below, and v is the bubble wall velocity. Changing the integration variable to $x = (T_{\rm c} - T)/T_{\rm c}$ and using the time-temperature relation $t \propto T^{-p}$ which follows from (2), one finds

$$\Delta(x) = \frac{4\pi v^3}{3} I_0 \left(\frac{T_{\rm c}}{H_{\rm c}}\right)^4 \int_0^x dx' (1-x')^{3-p} \left(\frac{1}{p(1-x)^p} - \frac{1}{p(1-x')^p}\right)^3 \exp\left(-\frac{S_{\rm b}(x')}{T_{\rm c}(1-x')}\right) \tag{4}$$

where $H_{\rm c}$ is the expansion rate at $T_{\rm c}$. We will see below that the nucleation temperature $T_{\rm nucl}$ defined by $\Delta(x_{\rm nucl}) = 1$ is always very close to the critical temperature so that we can take $0 < x' \leq x \ll 1$ and expand to linear order in (4) to get

$$\Delta(x) = \frac{4\pi v^3}{3} I_0 \left(\frac{T_{\rm c}}{H_{\rm c}}\right)^4 \int_0^x dx'\, (x-x')^3 \exp\left(-\frac{S_{\rm b}(x')}{T_{\rm c}}\right)\,. \tag{5}$$

Keeping the first term in a derivative expansion of the bounce action about x, *i.e.* taking $S_{\rm b}(x') = S_{\rm b}(x) + (dS_{\rm b}(x)/dx)(x'-x) + \mathcal{O}(x'-x)^2$, the integral can be performed with the assumption that $xd(S_{\rm b}/T)/dx(x \approx x_{\rm c}) \gg 1$, and gives the nucleation temperature implicitly as

$$\frac{S_{\rm b}(T_{\rm nucl})}{T_{\rm c}} = \ln\left[8\pi v^3 I_0 \frac{(T_{\rm c}/H_{\rm rad})^4}{(dS_{\rm b}/dT_{\rm nucl})^4}\right] - 4\ln\frac{H_{\rm c}}{H_{\rm rad}}\,, \tag{6}$$

where $H_{\rm rad} \simeq 1.2\times 10^{-16}(T_{\rm c}/100{\rm GeV})T_{\rm c}$ is the expansion rate at $T_{\rm c}$ in the standard radiation dominated cosmology.

To check the consistency of our assumptions and evaluate this expression to give the nucleation temperature, one must calculate the bounce action near the critical temperature in the particular model of interest. We consider the Minimal Standard Model (MSM) in the regime where it is described by the effective potential

$$V(\phi,T) = \frac{\gamma}{2}(T^2 - T_0^2)\phi^2 - \frac{\alpha}{3}T\phi^3 + \frac{\lambda_T}{4}\phi^4 \tag{7}$$

with the one-loop ring improved values [13, 14]

$$\begin{aligned}
\alpha &= \frac{1}{2\pi}\frac{2m_W^3 + m_Z^3}{v_0^3} + \frac{1}{4\pi}\left(3 + 3^{\frac{3}{2}}\right)\lambda_T^{\frac{3}{2}}\,,\\
\gamma &= \frac{2m_W^2 + m_Z^2 + 2m_t^2}{4v_0^2} + \frac{1}{2}\lambda_T\,,\\
\lambda_T &= \frac{m_H^2}{2v_0^2} - \frac{3}{16\pi^2 v_0^4}\left(2m_W^4\ln\frac{m_W^2}{a_B T^2} + m_Z^4\ln\frac{m_Z^2}{a_B T^2} - 4m_t^4\ln\frac{m_t^2}{a_F T^2}\right)\,,\\
T_0^2 &= \frac{m_H^2 + 8\beta v_0^2}{2\gamma}\,, \qquad \beta = \frac{3}{64\pi^2 v_0^4}\left(4m_t^4 - 2m_W^4 - m_Z^4\right)\,,
\end{aligned} \tag{8}$$

where $v_0 = 246{\rm GeV}$, $a_B = (4\pi)^2 {\rm e}^{-2\gamma_E} \simeq 50$, $a_F = (\pi)^2{\rm e}^{-2\gamma_E} \simeq 3.1$, and γ_E is Euler's constant. This treatment of the MSM is reasonably accurate up to $m_H \sim 60{\rm GeV}$, when nonperturbative effects become important. With this effective potential the critical temperature $T_{\rm c}$ is given by

$$T_{\rm c} = \frac{T_0}{\left(1 - \frac{2}{9}\frac{\alpha^2}{\lambda_T\gamma}\right)^{1/2}}\,, \tag{9}$$

and the latent heat L and surface tension σ by

$$L = V(\phi_*,T) + T\frac{dV(\phi_*,T)}{dT}\,, \qquad \sigma = \int_0^{\phi_*} d\phi\sqrt{2V}\,, \tag{10}$$

where ϕ_*, defined by degeneracy of the minima ($V(\phi_*,T) = V(0,T)$), is

$$\frac{\phi_*}{T} = \frac{2\alpha - \left[4\alpha^2 - 18\lambda_T\gamma(1-(T_0/T)^2)\right]^{1/2}}{3\lambda_T}\,. \tag{11}$$

The (spherical) bounce action is given by

$$S_{\rm b} = 4\pi\int r^2 dr\left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi,T)\right] \tag{12}$$

with the boundary conditions $\phi(r=0) = \phi_*$, $d\phi/dr(r=0) = 0$, and $\phi(r=\infty) = 0$ ($r = |\vec{x}|$ is the radial coordinate). Rather than solving this exactly (which is numerically expensive), or in the thin wall approximation (which is inaccurate for strong phase transitions when $\phi_* \sim T$), we will use an approximation for $S_{\rm b}$ developed in [13]:

$$
\begin{aligned}
\frac{S_{\rm b}(T)}{T} &= 9\pi \frac{\gamma^{3/2}}{\alpha^2}\left[1-\left(\frac{T_0}{T}\right)^2\right]^{\frac{3}{2}} f(\mathcal{A})\,, \\
f(\mathcal{A}) &= 1+\frac{\mathcal{A}}{4}\left[1+\frac{2.4}{1-\mathcal{A}}+\frac{0.26}{(1-\mathcal{A})^2}\right]\,, \quad \mathcal{A}=\frac{1-(T_0/T)^2}{1-(T_0/T_{\rm c})^2}\,,
\end{aligned} \tag{13}
$$

which is valid for $0 \le \mathcal{A} \le 0.95$.

The prefactor I_0 in (3) can be written as

$$
I_0 = \frac{\kappa_{\rm dyn}}{2\pi}\left(\frac{S_{\rm b}}{2\pi}\right)^{\frac{3}{2}} \lambda_-^{-1/2}\mathcal{K}_{\rm bubble}^{-1/2}\,, \qquad \kappa_{\rm dyn} = \left[\frac{2\sigma\rho}{R^3L^2}\right]^{\frac{1}{2}} \tag{14}
$$

where $\kappa_{\rm dyn}$ is the dynamical prefactor as given in [15]. R is the radius of the nucleating bubble, which can be estimated in the thin wall approximation to be $R \simeq R_{\rm nucl} \sim 2\sigma/Lx_{\rm nucl}$, $x_{\rm nucl} = 1 - T_{\rm nucl}/T_{\rm c}$, $\rho = \pi^2 T^4 g_*/30$ is the energy density of the plasma with g_* relativistic degrees of freedom. The one loop fluctuation determinant consists of the 'negative' mode $\lambda_- \simeq 0.05 g^{1/2}\phi(T)$, and we take $\mathcal{K}_{\rm bubble} = 1$ (for a more accurate value, see [16, 17]).

We have solved (6) numerically to find the nucleation temperature $T_{\rm nucl}$, using these values and approximations. We also used $m_t = 175$GeV, $m_W = 81$GeV, $m_Z = 91$GeV, and took the bubble wall velocity $v = 0.4$ [14] [1]. In Figure 1(a) we show a plot of $\zeta_{\rm nucl} = (T_{\rm c} - T_{\rm nucl})/(T_{\rm c} - T_0)$ against the logarithm of the expansion rate $H_{\rm c}$ at the critical temperature, for a range of Higgs masses m_H. We see clearly that the usual result in a radiation dominated universe is qualitatively unchanged by varying the expansion rate over orders of magnitude: $\zeta_{\rm c}$ is small, so the nucleation temperature is very close to the critical temperature, and much closer to $T_{\rm c}$ than T_0. There is a small quantitative change, $\zeta_{\rm c}$ varying by about 40% as the expansion rate varies over five orders of magnitude, but the change in absolute terms is tiny since $\Delta T_{\rm nucl} = -(T_{\rm c} - T_0)\Delta\zeta_{\rm c} \le 5 \times 10^{-3} T_{\rm nucl}$. Typically there is about the same change to $\zeta_{\rm c}$ over this range at a fixed Higgs mass as is brought about by decreasing the Higgs mass by about 25GeV in a radiation dominated universe[2].

The results are easy to understand both quantitatively and qualitatively. Varying the bounce action (13) and assuming $1-\mathcal{A} \le 0.2$ (which is satisfied for most of the parameter space in figure 1(a)) so that the last term in the expression dominates, one obtains $\delta(S_{\rm b}/T) \sim -2(\delta\zeta_{\rm nucl}/\zeta_{\rm nucl})(S_{\rm b}/T)^3$, and hence (using (6))

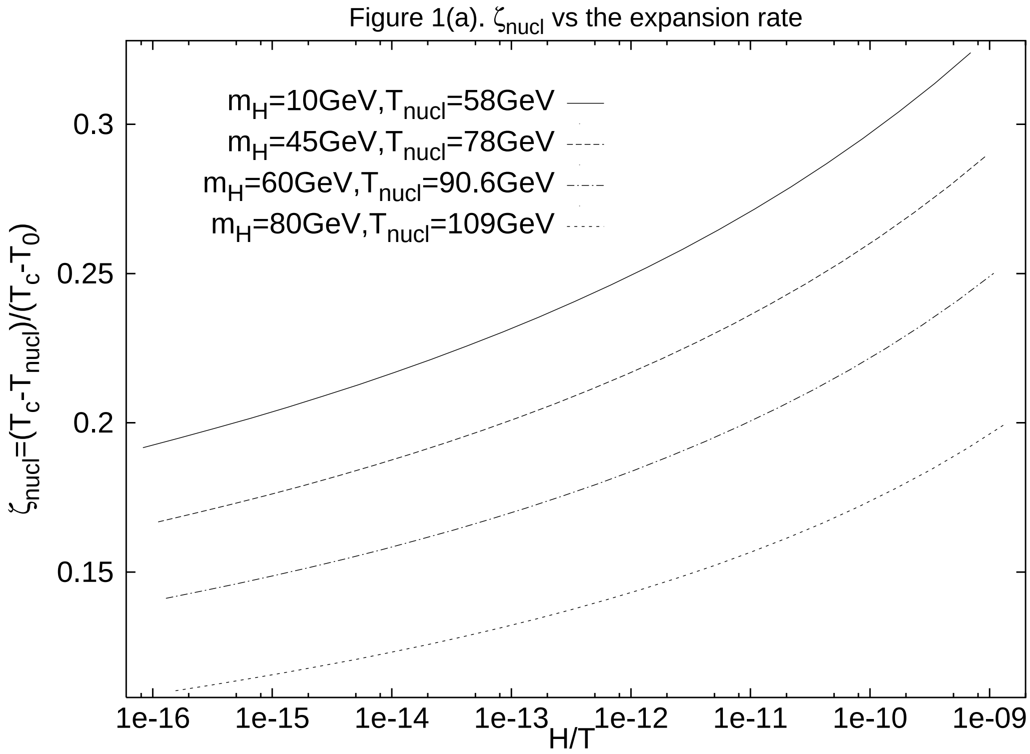
$$
\frac{\Delta\zeta_{\rm nucl}}{\zeta_{\rm nucl}} \approx \frac{2\ln(H_{\rm c}/H_{\rm rad})}{S_{\rm b}/T}\,. \tag{15}
$$

The bounce action $S_{\rm b}$ approximately halves, going from $100T$ to $50T$ as the expansion rate changes from $H_{\rm rad} \to 10^5 H_{\rm rad}$. Taking an average value for it in (15) gives good agreement with the estimates we made above from the figure. Qualitatively the reason the expansion rate changes the nucleation temperature so little is that, as long as the Universe expands on a time-scale much longer than $T_{\rm nucl}^{-1}$, the transition always

[1] This is a friction limited upper bound. The results here are of course not very sensitive to the details of the prefactor in the nucleation rate. As we will discuss below, this assumes that that the Universe is not reheated significantly by latent heat release, which is a reasonable approximation in the MSM. In the case that significant reheating occurs, so that bubble nucleation stops, the correct value of v in the early stages of nucleation would be the speed of sound $v_s \approx 1/\sqrt{3}$ at which the shock fronts propagate.

[2] $\zeta_{\rm nucl}$ increases as the Higgs mass m_H decreases because the phase transition gets stronger, and therefore more supercooling occurs.

[3] This estimate is just that obtained in the thin wall approximation in which $S_{\rm b,thin} \propto \zeta_{\rm nucl}^{-2}$.



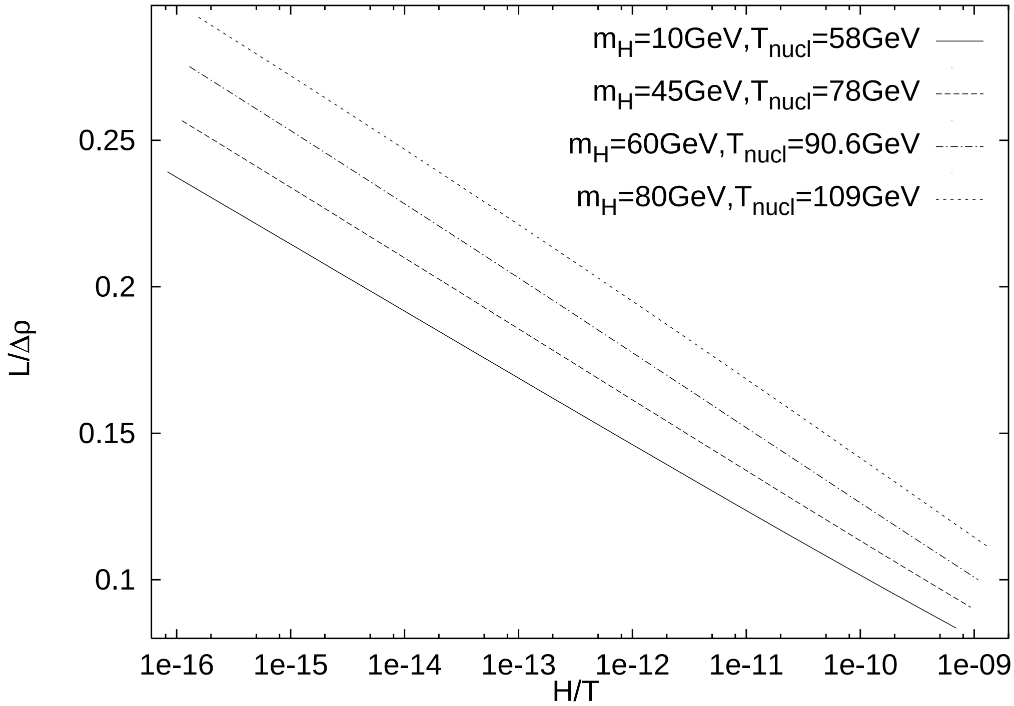
proceeds when the nucleation rate is very suppressed, where the bounce action is an extremely sensitive function of temperature. The nucleation temperature decreases as the expansion rate increases because the Universe must supercool more to attain a less suppressed nucleation rate.

Such a small change to the nucleation temperature leads to minor changes to the quantities which determine the baryon asymmetry generated. We will see in section 3.2 that between T_c and T_0 the VEV of the Higgs field changes by 50%, and its derivative with respect to T by about a factor of three. From figure 1(a) this would mean an increase in the VEV at nucleation of 1% or a little more per order of magnitude increase in the expansion rate. We would expect that this result will hold true in any typical electroweak model and not just the MSM in the regime we have studied it here. These minor changes to the VEV(s) and the other macroscopic parameters which determine the baryon asymmetry (bubble wall thickness, profile *etc.*) are essentially negligible in their effect on the baryon asymmetry generated.

One condition must be attached to this conclusion, however: Other macroscopic effects can come into play as the bubbles propagate. When the propagating bubbles begin "bathing" in the hydrodynamic shock waves of the neighboring bubbles, *i.e.* when $\Delta(t) \sim 1$ in (3), the plasma can heat up and slow down the propagation of the bubbles [18]. To determine how big this effect can be one compares the latent heat release L with the difference in the thermal energy density $\Delta\rho = 4\rho(T_{\rm c} - T_{\rm nucl})/T_{\rm c}$ between the nucleated phase and the unbroken phase. If $L/\Delta\rho \geq 1$ the system can reheat all the way back up to $T_{\rm c}$. If such reheating occurs the main effect on baryon generation at the bubble walls is through the slowing down of the bubble walls.

In figure 1(b) we show a plot of this ratio $L/\Delta\rho$ as a function of the expansion rate in the MSM with the same values and range of Higgs mass as above. As the expansion rate increases the amount of re-heating decreases - simply a result of the increased supercooling. In all the parameter space the ratio is less than one, but of order one, so the effect of reheating may be significant, leading potentially to a slow-down of the bubble walls relative to the friction dominated regime. To determine the effect precisely is a very involved problem, and we will not attempt to tackle here the even more involved one of looking at the effect on

Figure 1(b). L/Δρ vs the expansion rate



such slow-down of changing the expansion rate. We limit ourselves to the qualitative observation that the amount of reheating decreases slightly as the expansion rate increases, and that modifying the expansion rate by orders of magnitude is likely to lead to only small corrections.

The one-loop effective action which we use here becomes an increasingly poor approximation for $m_H >$ 60GeV, and the latent heat is one of the quantities it estimates very inaccurately. Lattice studies [2] have shown that there is a regime in the MSM where $L/\Delta\rho > 1$. In other models also this is certainly a condition we can envisage being satisfied. As mentioned above, in this case the bubble walls slow down to a final velocity determined by the expansion rate of the Universe, which has been estimated to be $v \sim 10^{-2} - 10^{-3}$ [18, 2] in a radiation dominated universe. These estimates show that this velocity depends on the expansion rate only through the combination HR where R is the average bubble radius, which is the radius at which $\delta f/\delta R$ of (3) peaks, *i.e.* $d^2\Delta/dR^2 \approx 0$. A little algebra gives

$$HR \simeq \frac{3v_s}{|dS/dT|}, \tag{16}$$

where $v_s = 1/\sqrt{3}$ is the speed of sound. Now using the expression (6) above we see that $|dS_{\rm b}/dT| \propto S_{\rm b}/T$ decreases by about 10% – and hence the final wall velocity increases by the same amount – per order of magnitude change in the expansion rate. If $L/\Delta\rho > 1$ for different expansion rates, this will be the only change to the final bubble velocity. Given the behaviour we have observed of $L/\Delta\rho$ it is clear that in certain models increasing the expansion rate considerably relative to its radiation dominated value could make the difference between this final wall velocity and a (typically much larger) friction limited one corresponding to $L/\Delta\rho < 1$. In this particular case the change to the bubble wall velocity may not be so small.

The determination of how the baryon asymmetry would be affected by such changes in the velocity of the bubble wall is a model dependent problem. The velocity dependence of the baryon asymmetry depends on what precise mechanism is operative, which depends on both the microscopic and macroscopic physics. At low velocities ($v < 10^{-3}$) the result always goes to zero at least linearly, and at larger velocities the most sensitive dependence is $\sim 1/v^2$. Using this dependence and assuming the greatest possible effect due to a change in the expansion rate (from an upper bound in the friction dominated regime $v \sim 0.4$ to the lower bound of the adiabatic 'complete reheat' regime with $v \sim 10^{-3}$) would give a change in the calculated BAU by (at most) 10^5. As we have discussed however, in most models the change will be much smaller and probably very small. A more detailed investigation of this question would however be required to draw stronger quantitative conclusions.

3.2 Washout and the Sphaleron Bound

The baryons created at the bubble walls are subject to decay after they enter the broken phase if the baryon number violating processes are not sufficiently suppressed. The requirement that this attenuation not reduce the created asymmetry to less than that required for nucleosynthesis leads to the sphaleron bound [19] in a radiation dominated universe. Here this bound will be restated as a requirement of the expansion rate of the Universe in a given theory. In the course of this discussion we will also draw attention to imprecisions in commonplace statements of the sphaleron bound (with the usual assumption of radiation domination) which can be of considerable importance.

Since the time-scale associated with the baryon number violating sphaleron processes is much longer than the time-scale for thermalization processes, the baryon number after the completion of the electroweak phase transition is given by its local equilibrium value

$$\langle B \rangle = \frac{1}{Z} Tr \left[B \mathrm{e}^{-\beta(H - \mu_B B - \sum_A \mu_A Q_A)} \right] \tag{17}$$

where μ_B and μ_A are chemical potentials for baryon number and the other charges Q_A conserved on the relevant time-scale, *i.e.* exactly conserved, or violated at a rate slower than the baryon number violating processes. On the time scale over which the violation of baryon number is relevant, the system relaxes to equilibrium at a rate $\dot{B} = -\bar{\Gamma}_{\mathrm{sph}}(\Delta F/T)\Delta B$, where $\bar{\Gamma}_{\mathrm{sph}}$ is the rate per unit volume of sphaleron processes in which the Chern-Simons number N_{cs} changes by *one* unit, and ΔF is the free energy change per process. Since $\Delta B = N_F \Delta N_{\mathrm{cs}}$ per process (N_F is the number of fermion families), we get

$$\dot{B} = -N_F^2 \frac{\bar{\Gamma}_{\mathrm{sph}}}{T} \mu_B \tag{18}$$

where we assume that other charges are defined so that they are conserved in these processes ($\Delta Q_A = 0$). The sphaleron rate is given by [20]

$$\bar{\Gamma}_{\mathrm{sph}} = \mathcal{C}\, g \frac{\phi^7}{T^3} \mathrm{e}^{-\frac{E_{\mathrm{sph}}}{T}}\,, \quad E_{\mathrm{sph}} = \mathcal{B} \left(\frac{4\pi}{\alpha_w} \right)^{\frac{1}{2}} \phi(T)\,, \quad \mathcal{C} = \frac{\omega_-}{2\pi\, g\phi(T)} \mathcal{N}_{\mathrm{tr}} \mathcal{N}_{\mathrm{rot}} \mathcal{V}_{\mathrm{rot}} \mathcal{K}_{\mathrm{sph}} \tag{19}$$

where $\alpha_w = g^2/4\pi \approx 1/29$, $\mathcal{B}$ is a monotonically increasing function of $\lambda/g^2 = m_H^2/8m_W^2$ ranging between 1.5 and 2.7 as λ/g^2 varies from 0 to ∞ [21] and $\mathcal{C}$ is a temperature independent 'constant', given fully below (with $\omega_- \sim g\phi(T)$ the frequency of the negative mode of the sphaleron, $\mathcal{K}_{\mathrm{sph}}$ the one loop fluctuation determinant, $\mathcal{V}_{\mathrm{rot}} = 8\pi^2$ a group volume factor, and $\mathcal{N}_{\mathrm{tr}}$ and $\mathcal{N}_{\mathrm{rot}}$ the number of translational and rotational degrees of freedom).

The conserved charges Q_A are just the primordial values of the exactly (or, in some cases, approximately) conserved charges in the electroweak model with which we are calculating. In scenarios for electroweak baryogenesis these are always taken to be zero. From (17) it then follows that μ_B can be expressed in terms of B, so that (18) becomes simply

$$\dot{B} = -\alpha_n \Gamma_{\mathrm{sph}} B\,, \qquad \Gamma_{\mathrm{sph}} = 6N_F^2 \frac{\bar{\Gamma}_{\mathrm{sph}}}{T^3} \tag{20}$$

where α_n is a number of order *one* whose precise value depends on the model and its corresponding set of charges Q_A. In section 4 below we carry out the constraint calculation explicitly and find $\alpha_n \approx 0.4$ for typical electroweak models. Integrating (20) gives the baryon asymmetry $B_{\rm freeze}$ which survives to partake of nucleosynthesis:

$$B_{\rm freeze} = B(T_{\rm b}) \exp\left[-\int_{t_{\rm b}}^{\infty} dt \alpha_n \Gamma_{\rm sph}(t)\right] = B(T_{\rm b}) \exp\left[-H_{\rm b}^{-1} \int_0^{T_{\rm b}} dT \frac{\alpha_n \Gamma_{\rm sph}}{T} (\frac{T_{\rm b}}{T})^p\right], \tag{21}$$

where $B(T_{\rm b})$ is the baryon asymmetry at the completion of the transition, at temperature $T_{\rm b}$ and $H_{\rm b}$ is the expansion rate at that time. As discussed in the previous section the appropriate value of $T_{\rm b}$ depends on the details of the of the phase transition and lies in the range $T_{\rm b} \in [T_{\rm c}, T_{\rm nucl}]$. To obtain the latter form of (21) we have used the time-temperature relation $t \propto T^{-p}$ which follows from (2). Changing variables to $y = T_{\rm b}/T$ we can write the *depletion factor* $\mathcal{D}$ as

$$\mathcal{D} \equiv -\ln \frac{B_{\rm freeze}}{B_{\rm b}} = \frac{T_{\rm b}}{H_{\rm b}} \times 6\alpha_n \mathcal{C} N_F^2 \, g \int_1^{\infty} \left(\frac{\phi(T)}{T_{\rm b}}\right)^7 y^{5+p} \mathrm{e}^{-\frac{E_{\rm sph}(T)}{T_{\rm b}} y} dy \,. \tag{22}$$

Over the range of integration the factor $E_{\rm sph}/T_{\rm b}$ in the exponential increases from its minimum value at $y = 1$, which is quite a large number ~ 30. This means that the dominant contribution to this integral comes from temperatures very close to $T_{\rm b}$ with $y \sim 1+1/30$. In fact we will see below that the rate of change of the VEV is typically large enough to narrow the range of temperatures which dominate the integral even more than this. The p dependence in the integral is therefore very weak and the only significant effect of the change in the expansion rate from its radiation dominated value $H_{\rm rad}$ is to change the depletion factor $\mathcal{D}$ by the factor $H_{\rm rad}/H_{\rm b}$. Increasing the expansion rate decreases the depletion because the sphaleron rate decouples at a higher temperature.

Is this change significant? For a given theory (with all parameters determined) the depletion factor is (in principle) calculable. There is essentially no depletion for any expansion rate greater than the expansion rate $H_{\rm sph}$ given by setting $\mathcal{D} = 1$ in (22). For $H < H_{\rm sph}$, however, a baryon asymmetry produced at the first order phase transition is attenuated by a factor $e^{-H_{\rm sph}/H}$. Whether a change in the expansion rate from that in a radiation dominated universe to a different value is important therefore depends on what the critical expansion rate $H_{\rm sph}$ is in the particular model. If a model has $H_{\rm sph} = 10^n H_{\rm rad}$, the baryon asymmetry left behind in the universe with $H \sim H_{\rm sph}$ may be compatible with observation, and that in a radiation dominated universe too small by a factor e^{-10^n}. If, on the other hand, $H_{\rm sph} < H_{\rm rad}$ the asymmetry will survive unattenuated in either universe.

We now turn to determining the effect of treating the expansion rate as a variable on the sphaleron bound in its more familiar forms, in which the requirement $\mathcal{D} \leq 1$ is converted to a bound on parameters in a particular model. The bound is usually stated as a lower bound on the sphaleron energy, or as a lower bound on the ratio of a VEV to the temperature at the nucleation or critical temperature, and then converted into a bound on parameters in the specific model concerned. We will follow through the derivation of such bounds in detail, particularly because we wish to note certain points which are often overlooked in this context. We then analyse the case of the MSM in detail using the same effective potential (7) and (8) used in the previous section.

Using the sphaleron energy $x = \mathcal{B}(4\pi/g)(\phi(T)/T)$ as the variable in (22) we obtain the sphaleron bound in its new form as

$$\begin{aligned} H_{\rm b} \geq H_{\rm sph} &= 6\alpha_n N_F^2 \mathcal{C} \left(\frac{\alpha_w}{4\pi}\right)^4 g \mathcal{B}^{-8} \int_{x_{\rm b}}^{\infty} \left(\frac{T_{\rm b}}{T(x)}\right)^p \left|\frac{d(\phi/T)}{dT}(x)\right|^{-1} dx x^7 \mathrm{e}^{-x} \\ &\approx \left(\frac{\alpha_w}{4\pi}\right)^{\frac{1}{2}} \mathcal{B}^{-1} \left|T \frac{d(\phi/T)}{dT}\right|_{\rm b}^{-1} \alpha_n \Gamma_{\rm sph}(T_{\rm b}) \end{aligned} \tag{23}$$

where, to derive the latter expression, we assumed that over the range of temperatures which contribute to the integral the derivative term is approximately constant, and $(T_b/T(x))^p \approx 1$. Let us assess the validity of this approximation in more detail in the case of the MSM. At any temperature at which the two phases coexist, i.e. between T_c and T_0,

$$T\frac{d}{dT}\left(\frac{\phi}{T}\right) \approx -\frac{2\gamma(T_0/T)^2}{2\lambda_T(\phi/T)-\alpha} - \frac{T}{\lambda_T}\frac{d\lambda_T}{dT}\left[\frac{\phi}{T} + \frac{\gamma\left(1-(T_0/T)^2\right)}{2\lambda_T\phi/T-\alpha}\right], \tag{24}$$

where $Td\lambda_T/dT = -8\beta$, and

$$\frac{\phi}{T} = \frac{\alpha + \left[\alpha^2 - 4\lambda_T\gamma(1-(T_0/T)^2)\right]^{1/2}}{2\lambda_T}. \tag{25}$$

For simplicity, in (24) we neglected the temperature dependence of α and γ, which would result in numerically irrelevant corrections. Even though both ϕ/T and its derivative in (24) and (25) are very sensitive functions of T, in the temperature interval $T_c \geq T \geq T_0$ both are monotonically decreasing, and we can write their lower and upper bounds as follows:

$$\frac{\phi_c}{T_c} = \frac{2}{3}\frac{\alpha}{\lambda_T}, \qquad \frac{\phi_0}{T_0} = \frac{\alpha}{\lambda_T} \tag{26}$$

and

$$\left[T\frac{d}{dT}\frac{\phi}{T}\right]_c = -\frac{6\gamma}{\alpha}\left(\frac{T_0}{T_c}\right)^2 + \frac{4\alpha}{3\lambda_T}\left(1+\frac{8\beta}{\lambda_T}\right) \approx -\frac{6\gamma}{\alpha}, \quad \left[T\frac{d}{dT}\frac{\phi}{T}\right]_0 = -\frac{2\gamma}{\alpha} + \frac{8\alpha\beta}{\lambda_T^2} \approx -\frac{2\gamma}{\alpha}, \tag{27}$$

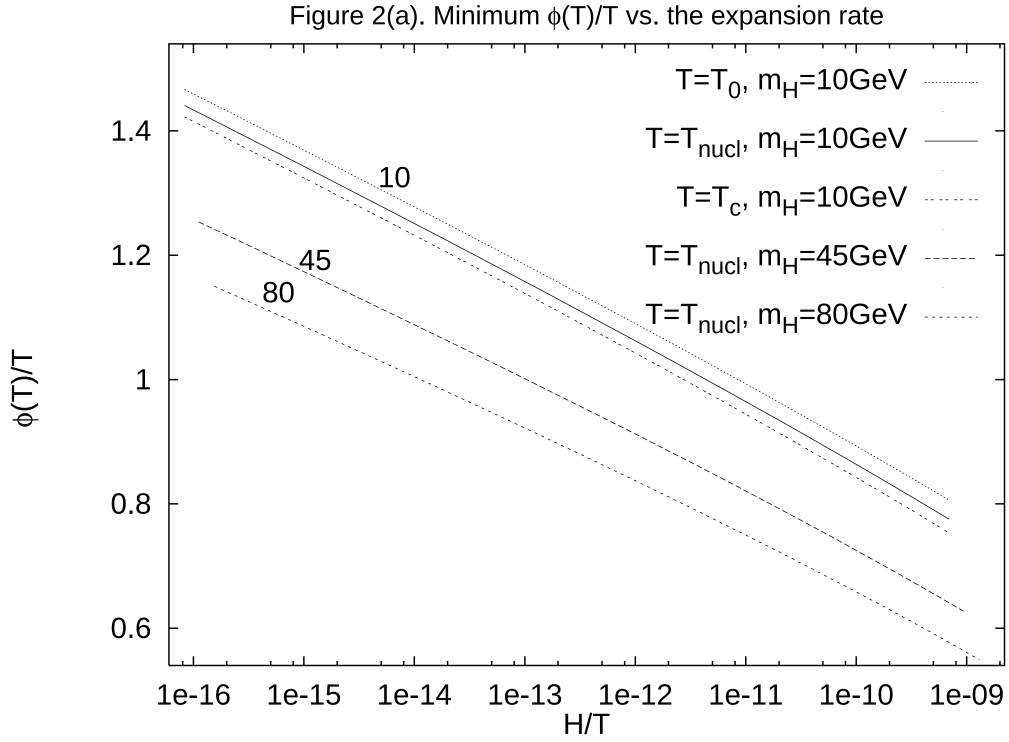
where $\frac{\gamma}{\alpha} \approx 18$. The large value of the derivative means that the pre-factor in front of the sphaleron rate in (23) is $\geq 10^3$. This is essentially just the (inverse) fraction of an expansion time in which the sphaleron freezes out (leading to the difference from the naive freeze-out estimate $H_{\rm sph} \sim \Gamma_{\rm sph}(T_b)$). The range of temperatures which contributes in the integral is therefore much less than between T_c and $T_0 \simeq 0.99T_c$, and the constant derivative approximation used in evaluating it is indeed very accurate. Further, as T_b varies in this range the change in the result associated with the derivative is at most this factor of three. In what follows we will keep track of this dependence of the sphaleron bound on T_b, and quantify it in comparison to the other effects on the bound in which we are interested here.

The sphaleron bound as given in (23) can be converted, for a given expansion rate H_b, into a lower bound on the ratio ϕ_b/T_b (where $\phi_b \equiv \phi(T_b)$). A numerically convenient and instructive way to write the lower bound on this quantity is in the implicit form

$$\frac{\phi_b}{T_b} = \frac{1}{\mathcal{B}}\left(\frac{\alpha_w}{4\pi}\right)^{\frac{1}{2}}\left[\ln\frac{6N_F^2\alpha_n\left(\frac{\omega_-}{2\pi g\phi(T)}\mathcal{N}_{\rm tr}\mathcal{N}_{\rm rot}\mathcal{V}_{\rm rot}\mathcal{K}_{\rm sph}\right)\alpha_w}{\left|T\frac{d}{dT}\frac{\phi}{T}\right|_b\mathcal{B}\frac{H_{\rm rad}}{T_b}} - \ln\frac{H_b}{H_{\rm rad}} + 7\ln\frac{\phi_b}{T_b}\right] \tag{28}$$

In figure 2(a) we show the solutions to this equation obtained from an iterative evaluation of (28), for the MSM. We have taken $\mathcal{V}_{\rm rot} = 8\pi^2$, and fit $\mathcal{N}_{\rm tr}\mathcal{N}_{\rm rot} \simeq 86 - 5\ln(m_H^2/8m_W^2)$ [22]. The one loop result for $\mathcal{K}_{\rm nucl}$ we took from [23]: $\mathcal{K}_{\rm sph} = \{7.54, 5.64, 4.57, 3.89, 3.74\}$ for $m_H = \{0.4, 0.5, 0.6, 0.8, 1\}m_W$, and extrapolated or interpolated for other values $m_H \in [10, 80]$GeV. ω_- we took from [22], where it was found that $\omega_-/g\phi(T) \in [0.4, 0.55]$ for $m_H \in [10, 80]$GeV. We neglected the plasma effects on ω_-. Finally, we took $\alpha_n = 0.4$, and $\mathcal{B} = \{1.52, 1.61, 1.83, 2.10\}$ for $m_H^2/m_w^2 \in \{0, 8\times10^{-3}, 8\times10^{-2}, 0.8, 8\}$ [21], and quadratically interpolated for the intermediate values.

Besides varying the expansion rate over the range shown in the figure, we have taken a wide range of Higgs masses and different values for T_b. It is instructive to do this because the sphaleron bound is



often stated as a bound on this ratio of VEV to temperature as if this were a model-independent and temperature independent statement of it. We see from figure 2(a) that this is very far from being true. For a range of Higgs masses from 10GeV to 80GeV the bound on ϕ/T decreases by about 20%. That most of the dependence comes from the factor $\mathcal{B}$, which varies non-negligibly with the Higgs mass, can be verified easily[4]. This is also clear from figure 2(b), which shows the sphaleron bound as a lower bound on the sphaleron energy $E^{\rm b}_{\rm sph}/T_{\rm b}$ (where $E^{\rm b}_{\rm sph} \equiv E_{\rm sph}(T_{\rm b})$) as a function of the expansion rate. There is however still a significant mass dependence (approximately 8% over the mass range considered) in the bound stated this way. The temperature dependence of the bounds is comparatively smaller - as T increases from T_0 to $T_{\rm c}$, the bound on ϕ/T decreases by $3-4\%$, and for $T_{\rm b} \in [T_{\rm nucl}, T_{\rm c}]$ by less than 1%. This dependence comes from the derivative of the VEV inside the logarithm, which as we saw above can vary by a factor of three over the range from $T_{\rm c}$ to T_0. In analyzing any particular model in detail the parameter dependence of the sphaleron bound stated this way in terms of these quantities should clearly be borne in mind and carefully examined.

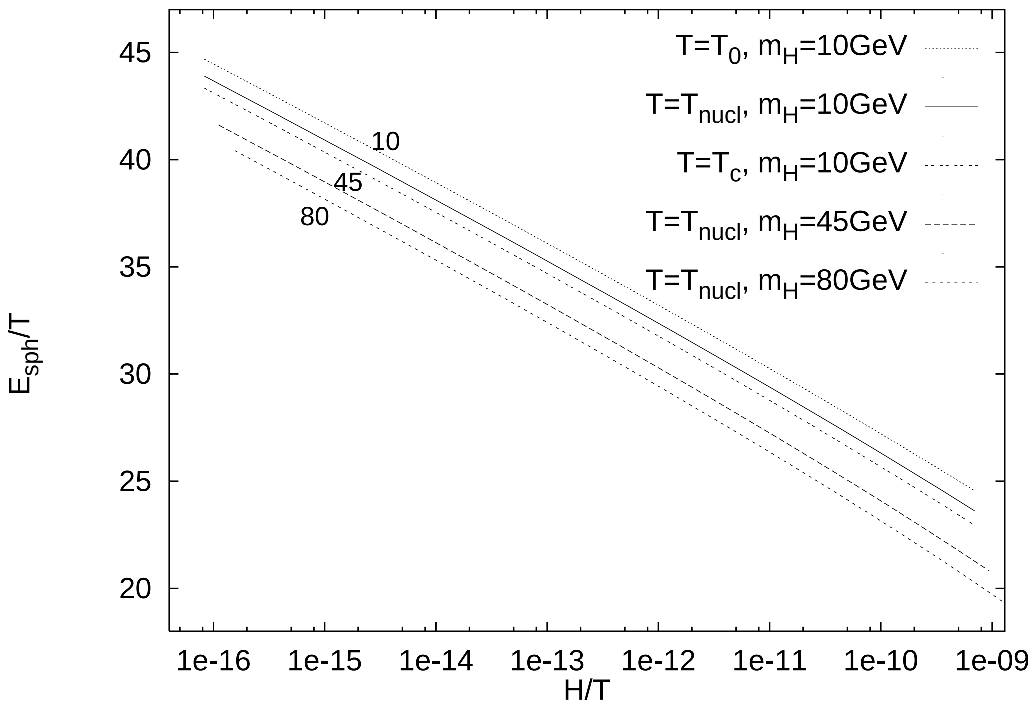
The dependence we are primarily interested in here is that seen in both figures 2(a) and 2(b) on the expansion rate of the Universe. Both $\phi_{\rm b}/T_{\rm b}$ and $E^{\rm b}_{\rm sph}/T_{\rm b}$ show an almost exact linear dependence on the logarithm of H which is evident from (28). For a small fractional change in the bound on $\phi_{\rm b}/T_{\rm b}$ or $E^{\rm b}_{\rm sph}/T_{\rm b}$ due to a change in the expansion rate from $H_{\rm rad}$ we have the approximate formula

$$\mathcal{B}\left(\frac{4\pi}{\alpha_w}\right)^{1/2}\Delta\left(\frac{\phi_{\rm b}}{T_{\rm b}}\right) \equiv \Delta\left(\frac{E^{\rm b}_{\rm sph}}{T_{\rm b}}\right) \approx -\frac{\ln\frac{H}{H_{\rm rad}}}{1-7\left(\frac{E^{\rm b}_{\rm sph}}{T_{\rm b}}\right)^{-1}} \tag{29}$$

which, given $(4\pi/\alpha_w)^{1/2} \approx 20$, agrees well with the numbers read off from the figures. Over five orders of

[4] $\delta(\phi/T)/(\phi/T) \simeq -(\delta\mathcal{B}/\mathcal{B})\left[1-7\mathcal{B}^{-1}(\alpha_w/4\pi)^{1/2}/(\phi/T)\right]^{-1}$. For $m_H = 10$GeV to 80GeV, $\delta\mathcal{B}/\mathcal{B} \simeq 1/8$, and hence $\delta(\phi/T)/(\phi/T) \simeq 1/6$, accounting for most of the dependence on m_H on figure 2(a).

Figure 2(b). Minimum sphaleron energy vs the expansion rate

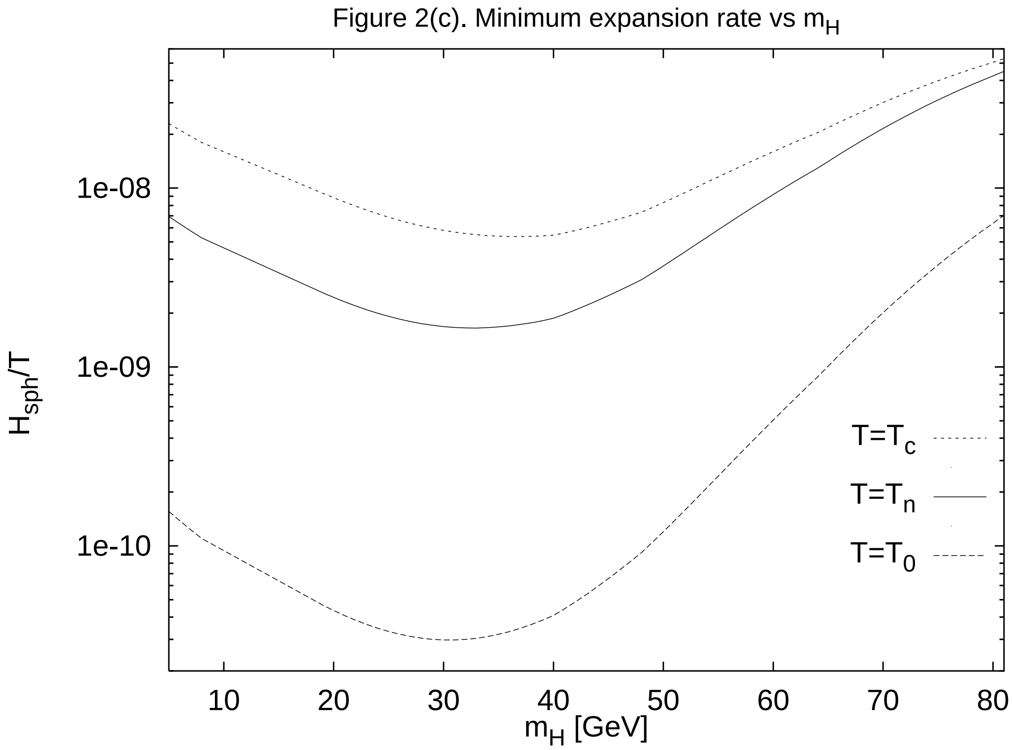


magnitude in the expansion rate we see a decrease in the bound on $\phi_{\rm b}/T_{\rm b}$ by about 0.4, or approximately 0.08 per order of magnitude increase in the expansion rate.

The usual starting point for analysis of most extensions of the standard model departs from the sphaleron bound given as a lower bound on the ratio of the sphaleron energy or the appropriate VEV to the temperature, and then converts this to a bound on the parameters of the model. We have noted that such a procedure should be considered more carefully as there can in fact be significant model dependence in the bounds on these quantities. We have derived nevertheless how such bounds are changed as a function of the expansion rate of the Universe, and the approximate form (29) is essentially model independent. Using this formula one can therefore turn the usual sphaleron bound for any given model into a lower bound on the expansion rate as a function of model parameters, provided one has the correct form of the bounds on $\phi_{\rm b}/T_{\rm b}$ (or $E^{\rm b}_{\rm sph}/T_{\rm b}$) in the radiation dominated case: For each set of parameter values one calculates the value of the given ratio, and then solves using (29) for the expansion rate which reduces (or increases) the radiation dominated value to the calculated critical value.

However, the most direct way to calculate the sphaleron bound as a lower bound on the expansion rate is simply to evaluate the integral (23) directly to find $H_{\rm sph}$ for each value of the parameters in the theory. We have done this for the MSM using the same parameter values and effective potential as above, and for the temperatures $T_{\rm b} = T_{\rm c}, T_{\rm nucl}, T_0$. The result is shown in figure 2(c), where the sphaleron bound is given as a plot of the minimum expansion rate required as a function of the Higgs mass m_H. The dependence on the temperature seen in the figure is greater than in the bound on ϕ/T, since it also enters in relating m_H to ϕ/T, as shown in figure 2(d). Figure 2(c) shows dramatically how badly the usual sphaleron bound is violated in the MSM [5]. For *no* physical Higgs mass is the minimum required expansion rate within orders of magnitude of that in a radiation dominated universe. The discrepancy of this result with the early sphaleron bounds calculated for the MSM [19] is explained by the much larger (now physical) top quark

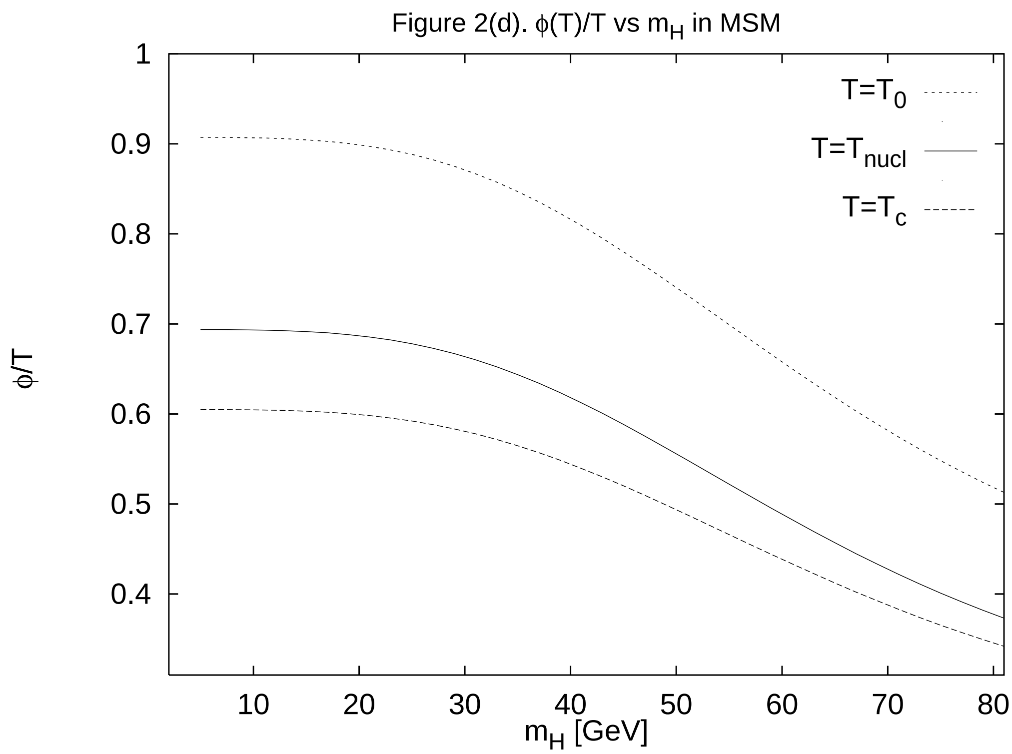
[5]Studies of the two loop effective potential and lattice studies show that the one-loop ring improved effective potential we are using underestimate the strength of the phase transition, but not enough to significantly alter the conclusions drawn here.



mass $m_t = 175$GeV used here. For small m_H one can see from (8) that the one-loop thermal contribution from the top quark dominates λ_T, and therefore, from (25), that ϕ/T stops increasing and levels off as seen in figure 2(d). The increase in the minimum expansion rate seen in figure 2(c) as m_H decreases below this value comes simply from the dependence on m_H of the sphaleron energy through $\mathcal{B}$ (which decreases, increasing the sphaleron rate).

In many extensions of the standard model it has been shown that, in contrast to the MSM, there are physically allowed regions of the parameter space where the usual sphaleron bound is satisfied. The way of stating the sphaleron bound we have illustrated for the MSM can be easily generalized to any such model. Besides being, as we argue in this paper, a more correct way to state the sphaleron bound (given that the expansion rate really is an unconstrained parameter), our discussion also shows that it is an instructive way to state it, because it quantifies how well or badly the bound is satisfied or violated. If we state the sphaleron bound in this way, it is easy to determine the effect on the calculated bounds in any change to input parameters (e.g. to any of the pre-factors in the sphaleron rate).

Having discussed how the sphaleron bound should be restated as a lower bound on the expansion rate, let us ask finally what electroweak baryogenesis at a first order phase transition can potentially tell us about the expansion rate at that epoch. A priori we do not know what it is and can use baryogenesis as a probe. If the correct electroweak theory turns out to be one in which there is a first order phase transition which successfully produces exactly the right amount of baryons during the phase transition, we would have compelling evidence that the expansion rate is greater than the corresponding critical value $H_{\rm sph}$. But it can tell us no more. If the model satisfies the 'old' sphaleron bound with the assumption of radiation domination, but has $H_{\rm sph} < H_{\rm rad}$ (as it typically will), the success of the model provides no evidence that the Universe expands at $H_{\rm rad}$. It could even potentially expand orders of magnitude slower than $H_{\rm rad}$. We will now see that in contrast electroweak baryogenesis in a homogeneous universe provides a much more sensitive probe of the expansion rate at that scale.



4 Baryogenesis in a Homogeneous Universe

Analysis using the effective potential constructed in perturbation theory indicates a first order phase transition but is only of validity for Higgs masses up to about 60GeV. Recent non-perturbative results [24] indicate that for heavier Higgs masses the line of first order phase transitions ends in a second order phase transition at about 80GeV in the Minimal Standard Model. For larger masses the transition is an analytic cross-over, *i.e.* there is actually no phase transition since all physical quantities vary continuously (and differentiably) as a function of temperature. This sort of behavior is typical of a system in which there is no order parameter which can define the symmetry state of the system - the gauge symmetry is never strictly speaking broken or unbroken.

The only departure from equilibrium in this case is that caused directly by the expansion of the Universe. All physical quantities vary on a timescale characterized by the cooling rate $\sim H$. Unlike the case of bubble nucleation there is no separation between the mechanism by which the baryons are created and the part of the calculation involving the expansion rate directly, a separation which allowed us to take the created asymmetry simply as an input without specifying how it was created. Here we must make use of a particular model in order to answer the question of how the baryon asymmetry depends on the expansion rate.

Most work on mechanisms for electroweak baryogenesis has considered extensions of the Standard Model with an additional source of CP violation beyond the KM matrix. On bubble walls formed at a first order phase transition the CP violation produces in various ways a term biasing the anomalous sphaleron processes, causing the creation of baryons on or around the wall. These source terms, which are present when there is space or time dependence of the condensate fields, can equally be used to bias the anomalous processes and produce baryons when the phase transition is not first order. In the case of a second-order or cross-over transition we expect the evolution to be homogeneous with time dependence only of the condensate fields, and we will model the problem this way. In fact the validity of the analysis is broader than just the regime where the phase transition is not first order. It also describes well the period after the completion of a first order phase transition. In particular, as we will discuss below, it describes the case where the phase transition is too weakly first order to satisfy the sphaleron bound.

We will now consider separately two types of source terms for baryogenesis discussed in the literature.

4.1 Potentials for Baryon Number

The first apparently viable mechanisms for electroweak baryogenesis, discussed in [25] and [26], considered potentials for baryon number. The models differ in their particular realisations of this potential. In various theories – two doublet extensions of the minimal standard model [25] and supersymmetric theories with or without an additional singlet [26] – there are CP odd terms in the effective action for the gauge-Higgs sector, of the form $(g^2/16\pi^2)\chi F\tilde{F}$, where F and $\tilde{F}$ are the $SU(2)$ field strength tensor and its dual, χ is some field or combination of fields which acquire VEVs at the phase transition, times a numerical factor (typically a suppression). When these terms are integrated by parts and the anomaly equation $(g^2/16\pi^2)F\tilde{F} = \partial_\mu j_B^\mu$ is used, in the homogeneous case (with time dependence only) they produce terms calculationally equivalent to a chemical potential for baryon number $\dot{\chi}B$. Specifically in two doublet models there are terms with [25]

$$\begin{aligned} \dot{\chi}_B &= -\frac{7\zeta_3}{4}\left(\frac{m_t}{\pi T}\right)^2 \frac{2}{v_1^2} i\left(\Phi_1^\dagger \mathcal{D}_0 \Phi_1 - (\mathcal{D}_0\Phi_1)^\dagger \Phi_1\right) \\ &\approx 7\zeta_3 \left(\frac{m_t}{\pi T}\right)^2 \frac{v_2^2}{v_1^2+v_1^2}\dot{\theta}\,, \qquad \zeta_3 \simeq 1.202\,, \end{aligned} \tag{30}$$

where θ is the relative phase between the two doublets, with VEVs of magnitude v_1 and v_2 (where the former couples to the top quarks). In theories with CP violation characterized by some scale M [26] the equivalent quantity $\dot{\chi}_B$ is

$$\frac{1}{3M^2}\partial_0|\phi|^2\,, \qquad \frac{1}{3M}\partial_0 s\,, \tag{31}$$

where the first case is a theory with doublets only, the second one with a singlet s.

Up to higher derivative corrections to the VEVs the system in this background tries to thermalize to the equilibrium in the presence of this extra term, in which the baryon number is given by the expression in (17), with $\mu_B = 0$ and H including the additional term due to the background.

It is perhaps instructive to note exactly why an equilibrium calculation of this sort can give a baryon asymmetry. The Sakharov requirement of a departure from equilibrium is sometimes shown using this type of expression by acting with Θ =CPT on baryon number as follows

$$\langle B\rangle = Tr\left[\Theta B \Theta^\dagger \Theta \mathrm{e}^{-\beta H}\Theta^\dagger\right] = -\langle B\rangle \tag{32}$$

and therefore $\langle B\rangle = 0$. The same will hold true if we allow non-zero chemical potentials for charges which are CP even and, of course, it will not hold if we impose a chemical potential for CP odd charges like B or $B-L$. In the case we are considering the reason it does not hold is that the additional effective term in H is not CPT invariant, as the time varying condensate field violates CPT spontaneously. The underlying Hamiltonian is of course CPT invariant, but in the expanding Universe this symmetry is spontaneously violated.

The constraints which are to be imposed are those setting all the charges which are conserved over the relevant timescale to zero. Compared to the case of the same source term used to generate a biasing of baryon number on the bubble walls during a first order phase transition, there are thus two important differences:

(i) The 'relevant timescale' on the bubble wall (of thickness L moving with velocity v_w) is the wall passage time L/v_w, typically $\sim 10^2/T$. Here it is that characterizing the time rate of change of the field; in the homogeneous case this will be $\sim H^{-1}$. Thus the numerous processes (*e.g.* chirality flipping processes of the lighter quarks and leptons), which are effectively inoperative on the bubble wall, are equilibrated in the present case. The set of relevant conserved charges is therefore much smaller (and the calculation therefore simpler). For a radiation dominated universe the only conserved charges at the electroweak scale are the exactly conserved charges - hypercharge Y, electric charge Q, and $\frac{1}{3}B-L_i$ (L_i the lepton number in generation i). The charge which is violated slowest is right-handed electron number e_R, since it is coupled to other species only through its small Yukawa coupling y_e, by processes with rate $\sim y_e^2 T \sim 10^{-12}T$ (times a number of order $0.1 - 0.01$ - see [27], [28]). Thus for $H > 10^{-12}T$ we will also need to add e_R as a conserved charge, for $H > 10^{-8}T$ both e_R and μ_R, *etc.*

(ii) On a bubble wall the constraints forcing the conserved charges to zero are appropriate only when negligible charge can be transported onto the wall over the relevant time scale. (The charges are conserved only globally, not locally on the bubble wall unless this is true [29].) This places a condition on the thickness of the wall $L > D/v_w$ for the applicability of this simple form of the calculation. This condition follows from the requirement that the wall passage time be greater that the diffusion time ($L/v_w > D/v_w^2$), in order for transport to be inefficient. In the present case the Universe is (assumed) homogeneous and the global constraints forcing the charges equal to zero are always appropriate.

Because the electroweak phase transition is not a symmetry breaking phase transition, we cannot define an exact criterion for whether the 'broken' or 'unbroken' basis of particle states presents the correct description in the equilibrium calculation. A correct calculation would assume neither basis. A simple example of such a calculation has been discussed recently in [30] and a cross-over from one limit to the other explicitly shown to occur at $m_W \sim m_D$ (the vacuum mass and Debye mass of the gauge bosons respectively). Converting this to a constraint on the ratio of the VEV to temperature, it turns out that the symmetric phase calculation is a better approximation when the sphaleron freezes out. Thus we will calculate here in this approximation, using the 'unbroken' phase classification of the states. In section 4.2 we will also see that either basis of states gives almost identical results.

We take the case when $e_R = 0$ as this will turn out to be appropriate in the regime of expansion rates of interest for the generation of the observed BAU[6]. However, the numerical difference induced by this additional constraint will turn out to be insignificant. Expressing the charges in terms of particles densities n_α, and using the linear approximation $n_\alpha = (T^2/12)k_\alpha\mu_\alpha$, where μ_α is given in terms of the chemical potentials μ_A for the charges Q_A by $\mu_\alpha = q_\alpha^A\mu_A$ (where q_α^A is the Q_A charge of the species α) and k_α is a statistical factor which is equal to 1(2) for fermions (bosons) in the massless approximation[7], we find

$$\begin{aligned}
Y &= \frac{T^2}{6}\left[(10+n)\mu_Y + 2\mu_B + \frac{8}{3}\Sigma_j\mu_j - \mu_{e_R}\right] \\
\frac{1}{3}B - L_i &= \frac{T^2}{6}\left[\frac{8}{3}\mu_Y + \frac{4}{3}\mu_B + \Sigma_j(\frac{4}{9} + 3\delta_{ij})\mu_j - \delta_{1i}\mu_{e_R}\right] \\
e_R &= \frac{T^2}{6}\left[-\mu_Y - \mu_1 + \mu_{e_R}\right]
\end{aligned} \tag{33}$$

[6] We will neglect the potentially interesting effect discussed in [31]. Incorporating it could lead in certain cases to minor changes to the final baryon asymmetry.

[7] Including the lowest order mass correction to this simple formula results in: $n_\alpha = (T^2/6)\mu_\alpha(1-(3/2)(m_\alpha/\pi T)^2)$ for fermions, and $n_\alpha = (T^2/3)\mu_\alpha(1-(3/2)(m_\alpha/\pi T))$ for bosons. In this paper we ignore these mass corrections since they appear as a subleading correction to the result presented in the text.

$$B = \frac{T^2}{6}\left[2\mu_Y + 4\mu_B + \frac{4}{3}\Sigma_j \mu_j\right] \tag{34}$$

where μ_i is the chemical potential for $B - L_i$, and n is the number of Higgs doublets.

Setting the first three charges equal to zero[8] we find

$$B_0 = c_n T^2 \mu_B\,, \qquad c_n = \frac{1}{6}\,\frac{36(29+6n)}{399+82n} \tag{35}$$

Note that $c_n = B_0/(T^2\mu_B)$ is almost insensitive to n, the number of Higgs doublets, varying only between 0.436 and 0.439, as n changes from 0 to ∞ [9]. Taking the source term to be as given in (30) or (31), we set $\mu_B = \dot{\chi}_B$ in (35) to obtain the baryon asymmetry in the "equilibrium" to which the baryon number violating processes will try to bring the plasma in the slowly varying background. To calculate the rate at which these slow processes bring the system to this state, we impose a chemical potential μ_B on baryon number and include the source term. The baryon number B in this state is then given by (35) with the replacement $\mu_B \to \mu_B + \dot{\chi}_B$, and therefore

$$\mu_B = c_n^{-1}\left(\frac{B}{T^2} - c_n \dot{\chi}_B\right)\,. \tag{36}$$

Using (17) this gives the rate at which B approaches its "equilibrium" as

$$\dot{B} = -\alpha_n \Gamma_{\rm sph}(B - c_n\dot{\chi}_B T^2)\,, \qquad \Gamma_{\rm sph} = 6N_F^2\frac{\bar{\Gamma}_{\rm sph}}{T^3}\,, \qquad \alpha_n = \frac{1}{6c_n}\,, \tag{37}$$

where $\alpha_n = 0.382 \to 0.380$ as $n = 0 \to \infty$ and $\bar{\Gamma}_{\rm sph}$ is given in (19).

Before calculating the final baryon asymmetry we consider another treatment of induced source terms for baryogenesis in a time dependent background.

4.2 Potential for Hypercharge

A different treatment of the biasing of baryon number was given in [32][10]. In the broken phase of a two Higgs doublet model the relative phase θ of the neutral components of the Higgs fields enters in the fermionic mass terms. A hypercharge rotation of the fields to remove this phase from the mass term produces *at tree level* a real mass term and an additional term in the Lagrangian which, in the homogeneous case can be written simply as $\dot{\chi}_Y Y$, where Y is the hypercharge operator [11], and

$$\dot{\chi}_Y = -\frac{2v_2^2}{v_1^2+v_2^2}\dot{\theta}. \tag{38}$$

[8] In the 'unbroken' phase one can choose to constrain any two linear combinations of hypercharge Y and isospin T_3. The choice of Y and T_3 is simple, because T_3 is then proportional to its own chemical potential and $T_3 = 0$ is trivial.

[9] If we had not assumed the right-handed electron to be in equilibrium, the change would be small. In this case $c_n = 0.455 \to 0.462$, as $n = 0 \to \infty$.

[10] The account given here is not precisely that of the original version of the idea given in [32], which treated a potential for *fermionic* hypercharge. It was pointed out in [33] that the rotation should also be performed on the Higgs fields, giving a potential for total hypercharge which in the unbroken limit (VEVs $\to 0$) is pure gauge, and therefore can have no physical effect. The leading baryon production is in this case mass-squared suppressed. For a discussion of this point, see also section 3 of [34].

[11] We follow the convention used in [32]. There is nothing special about the choice of hypercharge. The essential element is that it is an anomaly free charge which is spontaneously broken by the mass term. A rotation proportional to isospin, for example, or any charge which is a linear combination of hypercharge and a charge exactly conserved in the broken phase is equally good. It is not difficult to check that the extra induced 'source' term always drops out in the calculations given below.

In the unbroken phase this is just a gauge term, but in the broken phase it can have physical significance because hypercharge is not conserved, being violated by VEV suppressed terms.

Again, as discussed in section 4.1, we can calculate either in the broken phase or unbroken phase, but the latter is probably more appropriate for the temperature range of relevance. In this case of course we must include the information about hypercharge violating processes to get a non-zero answer, so calculating in the unbroken phase means taking the basis of chiral states of the unbroken phase and treating the mass terms as interaction vertices which can violate hypercharge by flipping chirality. The correct constraint calculation is therefore one in which we take the same global conserved charges as in the previous calculation, but instead of the constraint on hypercharge, we must impose the constraint on the (conserved) electric charge Q, and we get

$$
\begin{aligned}
Q &= \frac{T^2}{6}\left[(10+n)\mu_Y + 2(10+n)\mu_Q + 2\mu_B + \frac{8}{3}\Sigma_j\mu_j - \mu_{e_R}\right], \\
\frac{1}{3}B - L_i &= \frac{T^2}{6}\left[\frac{8}{3}\mu_Y + \frac{8}{3}\mu_Q + \frac{4}{3}\mu_B + \Sigma_j(\frac{4}{9} + 3\delta_{ij})\mu_j - \delta_{1i}\mu_{e_R}\right], \\
e_R &= \frac{T^2}{6}\left[-\mu_Y - \mu_Q - \mu_1 + \mu_{e_R}\right], \\
B &= \frac{T^2}{6}\left[2\mu_Y + 2\mu_Q + 4\mu_B + \frac{4}{3}\Sigma_j\mu_j\right].
\end{aligned} \tag{39}
$$

The "chemical potential" μ_Y for hypercharge here is the effective one which arises from the source term for hypercharge, *i.e.* $\mu_Y = -\dot{\chi}_Y$. Setting the conserved charges to zero in (39) we can solve for B to find

$$
B = \frac{T^2}{6}\left(\frac{24(10+n)}{1219+164n}\dot{\chi}_Y + \frac{36(89+12n)}{1219+164n}\mu_B\right). \tag{40}
$$

Using (18) we obtain the equation describing the relaxation of baryon number to the "equilibrium" in presence of the hypercharge source term:

$$
\begin{aligned}
\dot{B} &= -\alpha_n'\Gamma_{\rm sph}(B - c_n'\dot{\chi}_Y T^2), \\
\Gamma_{\rm sph} &= 6N_F^2\frac{\bar{\Gamma}_{\rm sph}}{T^3}, \qquad \alpha_n' = \frac{1219+164n}{36(89+12n)}, \qquad c_n' = \frac{24(10+n)}{6(1219+164n)}.
\end{aligned} \tag{41}
$$

Note that $c_n' = 0.033 \to 0.024$, and $\alpha_n' = 0.3805 \to 0.3796$ for $n = 0 \to \infty$. The rate of relaxation is essentially independent of the number of doublets, and it is almost identical to the rate in the presence of a potential for baryon number in section 4.1.

It is noteworthy that the coefficient c_n' is significantly smaller than the corresponding coefficient in the case of a potential for baryon number ($c_n/c_n' \simeq 13-18$ as $n = 0 \to \infty$). It follows therefore that, even though the baryon number source (30) is suppressed by a factor of mass over temperature squared relative to that in (38), the former may give the dominant source term for baryogenesis. The reason for this is a suppression due to strong sphaleron processes in the case of a hypercharge source term [35]. In the massless quark approximation these force the densities of right and left-handed baryons equal, *i.e.* $B_L = B_R$. On the other hand, it is easy to show that, with source terms for a charge such as hypercharge which is conserved in baryon number violating processes, $\dot{B} \propto (3B_L + L_L)$. With $B - L = 0$ this implies $\dot{B} \propto (\frac{5}{2}B - L_R)$ (where L_R is the density of all right-handed leptons). Therefore, setting $L_R = 0$ would lead to an equilibrium with $B = 0$, *i.e.* a vanishing source term for baryon number. The non-zero result we have obtained is therefore proportional to the charge in the right-handed leptons, which gives a small statistical factor related to the fraction of the total number of degrees of freedom they represent.

When the VEVs approach zero, the result in (41) does indeed vanish, but not explicitly. In this limit the rate of the 'hypercharge violating' processes goes to zero, so the "equilibrium" calculation is no longer appropriate as it is only valid on a time scale longer than one which diverges as one sends the VEVs to zero. It is not difficult to check that one does indeed get zero for the baryon number in the presence of this source term when we impose the extra constraint appropriate in this limit [12].

4.3 The Baryon Asymmetry

The equations (37) and (41) have the same form for both source terms, and so we can analyse them together. Integrating (37) gives the baryon asymmetry as a function of time as

$$B(t) = -\int_{t'}^{t}\int_{t_i}^{t} dt' c_n \dot{\chi}(t') T^2(t') \frac{d}{dt'} \exp\left[-\int_{t'}^{t} dt'' \alpha_n \Gamma_{\rm sph}(t'')\right] \simeq [c_n \dot{\chi} T^2]_{\rm freeze}\,, \tag{42}$$

where t_i is an initial time chosen before the phase transition or cross-over takes place such that the source term may be taken to be zero. This expression is simply the source term integrated against the appropriate Green's function. The freeze-out time (temperature) $t_f(T_f)$ is that at which the integral in the exponent is equal to one, i.e.

$$\int_{t_f}^{t} dt'' \alpha_n \Gamma_{\rm sph}(t'') \equiv 1\,. \tag{43}$$

The approximation in (42) follows since we would expect that the time-scale characterizing the variation in $\dot{\chi}$ should be of the same order as that characterizing the change in ϕ/T. However, as discussed in section 3.2, there is an exponential dependence in the sphaleron rate on ϕ/T with a large pre-factor ($\sim 1/\alpha_w$). This means that the derivative inside the integral in (42) can be approximated by a delta-function at t_f down to a time scale much shorter than that over which $\dot{\chi}$ varies, and the result follows. The sphaleron rate $\Gamma_{\rm sph}$ enters only in determining the freeze-out value for the source $[c_n \dot{\chi} T^2]_{\rm freeze}$. Optimally, the sphaleron processes switch off when the source is at its maximum, leading to an estimate of the maximum production of baryons at a second order phase transition or cross-over: $B_{\rm max} \simeq [c_n \dot{\chi} T^2]_{\rm max}$. To a very good approximation the final baryon to entropy ratio is

$$\frac{B}{s} = -\frac{45 c_n}{2\pi^2 g_*}\left(\frac{H}{T}\right)_{\rm freeze}\left(T\frac{d\chi}{dT}\right)_{\rm freeze}, \tag{44}$$

using the fact that $d\chi/dt = -HT d\chi/dT$ and that the entropy density $s = (2\pi^2/45) g_* T^3$ (g_* the number of relativistic degrees of freedom). The subscript denotes that all these quantities are to be evaluated at the sphaleron 'freeze-out'.

In the case of baryon production in a homogeneous Universe with source terms of this type the final baryon asymmetry is therefore proportional to the expansion rate at freeze-out. This contrasts completely with the case of baryogenesis at a first order transition, for which the baryon asymmetry can be effectively the same for an expansion rate differing by many orders of magnitude.

We can invert (44) to get the range of expansion rates consistent with the baryon to entropy required by nucleosynthesis[13]

$$\left(\frac{H}{T}\right)_{\rm freeze} \simeq (2-12)\times 10^{-11}\, g_* \frac{0.44}{c_n}\frac{1}{|(T d\chi/dT)_{\rm freeze}|} \tag{45}$$

[12] Choosing to impose the conservation on T_3 with chemical potential μ_{T_3}, we get $T_3 = \frac{T^2}{6}(10+n)(\mu_{T_3}+\mu_Q)$ and Q also picks up the extra term $(10+n)\mu_{T_3}$. Imposing $T_3 = 0$ leaves only the linear combination $\dot{\theta} + \mu_Q$ in the other equations. The solution is the trivial unperturbed equilibrium.

[13] This range corresponds to the conservative bounds from direct observations of element abundances given in [36]. Tighter bounds, corresponding roughly to the range $(3-9)\times 10^{-11}$ in (45), are given in [37] and [38]..

How big is $(Td\chi/dT)_{\rm freeze}$ in any given theory? A full treatment of the phase transition in any of the models mentioned would be required to actually calculate this, a task however considerably beyond the methods used to date in the study of the phase transition[14]. A *naive* guess would be

$$T\frac{d\chi}{dT} \sim \frac{d\phi}{dT}\epsilon \sim \epsilon \tag{46}$$

taking the field χ to trace the VEV (or combination of VEVs), which is itself then assumed to evolve roughly in proportion to the temperature (*i.e.* on a timescale given by the expansion rate). The parameter ϵ is one characterizing CP violation, which we might expect to be constrained by CP violation phenomenology of the relevant model.

A full calculation of any given model at finite temperature would be required to turn the bound (45) into a precise one on the expansion rate alone. However, short of such a calculation, we can do better than the very naive estimate given by (46).

(i) In section 3.2 we examined the Minimal Standard Model and saw that, near the critical temperature, the VEV ϕ is a very sensitive function of the temperature, with $Td(\phi/T)/dT \approx d\phi/dT \sim (100-30)$ in the range of temperatures T_c to T_0, and about 60 at the nucleation temperature. Typically the sphaleron will freeze-out in this range of temperatures, as the sphaleron rate changes by many orders of magnitude. The same sort of behavior can be seen to continue at larger Higgs masses in the non-perturbative treatment of the phase transition, in the case when the phase transition is a "sharp - but regular - cross-over" [24]. This means that the range of temperatures over which physical measurables like the susceptibility vary is a small fraction of the temperature at which the change occurs. (It is, of course, this "sharpness" which allows one still to talk about a phase transition when, strictly speaking, there is none.) From the data in [24] we see that there is a range of temperature of a few GeV which compares with a "transition temperature" anywhere between 60 and 200GeV. Thus the standard model estimate of $d\phi/dT \sim 60$ seems reasonable, much larger than our naive estimate in (46).

(ii) We can also learn something about the constraints on ϵ by looking at the effective potential for a particular case. Consider a two Higgs doublet model. One interesting regime is that in which the evolution of the CP violating angle is determined dominantly by terms which break CP *spontaneously.* In a CP invariant Higgs potential [39] only the terms

$$\lambda_5((\phi_1^\dagger\phi_2)^2 + h.c.) \qquad (\lambda_6\phi_1^\dagger\phi_1 + \lambda_7\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2 + h.c.) \tag{47}$$

are functions of the relative angle of the two VEVs. Taking the real parts of the VEVs to be determined by the rest of the potential (*i.e.* working in the approximation that the terms (47) are small) this gives a quadratic potential for the cosine of the relative phase, which (taking $\lambda_5 - \lambda_7$ positive), is minimized at

$$\cos\theta = -\min\left(1, \frac{\lambda_6}{4\lambda_5}\frac{v_1}{v_2} + \frac{\lambda_7}{4\lambda_5}\frac{v_2}{v_1}\right) \tag{48}$$

There are two (CP conjugate) solutions which will be split by additional explicit CP violation. How the angle changes as the VEVs do depends on the values of the ratios of the couplings λ_6/λ_5 and λ_7/λ_5. A necessary condition for $d\theta/dT \neq 0$ at the phase transition is $\cos\theta \neq -1$, which is the case when $\lambda_7/2\lambda_5 < v_1/v_2 < 2\lambda_5/\lambda_6$. For couplings such that the first term in (48) dominates, and v_1 changing faster than v_2 as a function of temperature, we have

$$T\frac{d\theta}{dT} \approx -\frac{1}{\tan\theta}\frac{T}{v_1}\frac{dv_1}{dT}, \quad \text{for} \quad \frac{v_1}{v_2} > \left(\frac{\lambda_7}{\lambda_6}\right)^{\frac{1}{2}}, \quad \frac{d\ln v_1}{d\ln T} > \frac{d\ln v_2}{d\ln T} \tag{49}$$

[14] Perturbative methods apply when the phase transition is fairly strongly first order. The methods which have been employed to describe the opposite regime do not include the evolution of the CP odd fields relevant here.

Typically we have $Td\theta/dT \approx dv_1/dT$, but there are also parts of parameter space (near $\cos\theta = -1$) where the phase can change much faster than this. The only role of the explicit CP violation here is to split the two degenerate minima so that the same sign is chosen everywhere. This illustrates that the constraint on the parameter which we called ϵ from CP violating phenomena at zero temperature may be extremely weak. With a moderate fine tuning it can be considerably larger than one, and not related directly to any small parameter associated with the smallness of CP violation. In fact in theories such as the Minimal Supersymmetric Standard Model (MSSM) it is naturally the case that the terms which break CP spontaneously (which are induced in the plasma through thermal corrections) are dominant over the terms which break CP explicitly (which are suppressed by a loop factor) [40]. We conclude, on the basis of a simple analysis of the two Higgs doublet model, that the naive estimate (46) for $T\frac{d\chi}{dT}$ with $\epsilon \sim 1$ is too small by about two orders of magnitude. A result of this magnitude is obtained for a large portion of the parameter space, without any tuning. With a moderate fine tuning, the effective CP violation can be further enhanced. To make a more precise statement would require a detailed analysis of the Higgs sector of the particular model.

5 Non-Standard Cosmologies

Having established quantitively the dependence of the baryon asymmetry on the expansion rate in two possible scenarios for baryon production at the electroweak scale, we now turn to the discussion of physical mechanisms which could lead to such a different expansion rate at the electroweak scale.

As mentioned in the introduction this kind of question has previously been treated in the context of calculations of relic densities of weakly interacting particles in [3] and [4]. The relic density of a weakly interacting species is determined by the temperature at which the species decouples from the ordinary (visible) matter, which depends, just as in the case of the sphaleron decoupling discussed above, on a comparison between the appropriate interaction rate and the expansion rate of the Universe. In typical models this decoupling occurs before nucleosynthesis, and therefore one is led to consider, just as we are doing here, possible alternatives to radiation domination at that epoch. The alternative which is considered exclusively in [3] and in most detail in [4] is:

(i) An Anisotropic Universe: A universe which is homogeneous but not isotropic is described by a metric with three scale factors, one for each spatial dimension. With an adiabatic approximation (*i.e.* expansion slow enough to allow thermalization) it is the effective volume expansion rate $\overline{H}$ associated with an average scale factor $\overline{a}$ which determines how the temperature changes in the same way as in the isotropic FRW spacetime. The equation of motion for $\overline{H}$ is just that of the FRW space, but with an additional term which is equivalent to a component of the energy density scaling as $1/\overline{a}^6$.

A further possibility considered in [4] is

(ii) Non-standard theories of gravity. The case studied in [4] is a Brans-Dicke theory, which again turns out to effectively produce an extra component in the energy density scaling as $1/a^6$. There are also of course many other variants on standard Einstein gravity which can be considered.

The simple possibility we will concentrate on is:

(iii) Einstein gravity with isotropy and homogeneity, but with an extra contribution to the energy density which is important prior to nucleosynthesis. As noted by one of us (MJ) in [5] any mode of a scalar field dominated by its kinetic energy has the required property, as its energy density can scale as fast as $1/a^6$. The electroweak phase transition could potentially occur during a phase of the Universe dominated by the kinetic energy of a scalar field, termed *kination* in [5], which can end before nucleosynthesis as the kinetic energy density red-shifts away relative to the radiation. Below we will discuss several ways in which such a phase can come about within the context of inflationary cosmology, which explains the assumed isotropy and homogeneity. In particular we will discuss models which come naturally out of an alternative to the usual theory of reheating discussed by Spokoiny in [1].

A clear motivation for considering such models follows from the calculations in the previous section. If we have such a component scaling as $1/a^6$, the expansion rate is given by

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\frac{\rho_e}{2}\left[\left(\frac{a_e}{a}\right)^6 + f(a)\left(\frac{a_e}{a}\right)^4\right], \tag{50}$$

where a_e is the scale factor when the density in the mode becomes equal to that in radiation and ρ_e is the total energy density at that time. The factor $f(a)$ accounts for the effect of decouplings, and, assuming adiabatic decouplings, $f(a) = [g(a_e)/g(a)]^{1/3}$, where $g(a)$ is the number of relativistic degrees of freedom as a function of the scale factor a. Nucleosynthesis constraints place a lower bound on T_e, the temperature at the time of equality of radiation-kinetic energy density, which can be inferred from the corresponding bounds on additional relativistic particle degrees of freedom. This is the case since the predominant effect of such extra degrees of freedom is also in the change they cause to the expansion rate at the beginning of nucleosynthesis, which determines the crucial ratio of neutrons to protons when the weak interactions drop out of equilibrium at $\sim$ 1MeV. We take here the conservative bounds of [36], which allow the equivalent of 1.5 extra Dirac neutrino degrees of freedom over the three degrees of freedom of the Standard Model, *i.e.* we allow an additional energy density at 1MeV which is $3(7/8)/10.75 \approx 0.25$ of the standard model one (with 10.75 degrees of freedom). From (50) this means

$$\frac{1}{\sqrt{f(a_{\rm ns})}}\frac{a_e}{a_{\rm ns}} \leq 0.5 \tag{51}$$

Using $Ta = f(a)T_e a_e$ this gives the upper bound on the expansion rate at the electroweak scale

$$\frac{H}{T} \leq 1.8 \times 10^{-11}\left(\frac{T_{\rm freeze}}{100{\rm GeV}}\right)^2 \tag{52}$$

The result differs only by $\sqrt{3/2}$ if we take the less conservative nucleosynthesis bound of [37] and [38].

Taking $H_{\rm max}$ to be the expansion rate corresponding to the upper bound (52), the requirement (45) for generation of the observed BAU at the electroweak scale in a homogeneous Universe can be expressed as a requirement of the relevant CP violating parameter

$$\left|T\frac{d\chi}{dT}\right|_{\rm freeze} \approx (1-6)g_*\left(\frac{100{\rm GeV}}{T_{\rm freeze}}\right)^2\left(\frac{H_{\rm max}(100{\rm GeV})}{H(100{\rm GeV})}\right) \tag{53}$$

Absorbing the nucleosynthesis limit, *i.e.* taking $H = H_{\rm max}$ we have the strict lower bound [15]

$$\left|T\frac{d\chi}{dT}\right|_{\rm freeze} \geq g_*\left(\frac{100{\rm GeV}}{T_{\rm freeze}}\right)^2 \tag{54}$$

From the analysis in section 4.3 it follows that this bound may indeed be satisfied in extensions of the MSM such as those we have discussed, without any fine tuning. If the upper bound (52) on the expansion rate is not saturated, the CP violation parameter is required to be larger as given by (53). As discussed in section 4.3, with some fine-tuning of parameters in the potential, this parameter can indeed be enhanced to

[15] Note that taking the upper bound on the expansion rate in (52) corresponds to absorbing the upper bound on effective number of degrees of freedom at nucleosynthesis, which is only consistent with the lowest baryon to entropy ratio (increasing the expansion rate increases the fraction of Helium).

considerably greater than the typical value which just satisfies the lower bound (54). An exact statement of how large it can be would require a detailed examination of the model in question.

The important result is that in a cosmology with an additional component scaling as $1/a^6$ which dominates prior to nucleosynthesis, the creation of the baryon asymmetry is possible (*i.e.* consistent with all observations) at the electroweak scale in a homogeneous expanding Universe. The fact that generation of the BAU in this case has generally been dismissed as impossible provides clear motivation for the consideration of such cosmologies in greater detail. Certainly also as experiment pushes the bounds on scalar particles upwards, the usual sphaleron bound for generation of the BAU is becoming increasingly severe and alternative mechanisms for the production of the BAU within the context of electroweak cosmology become more relevant.

5.1 Kination

Consider the dynamics of a real scalar field ϕ with potential $V(\phi)$. Varying the action

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)^\dagger(\partial_\nu\phi) - V(\phi)\right] \tag{55}$$

and taking the FRW metric $ds^2 = dt^2 - a(t)^2 d\vec{x}^{\,2}$ with scale factor $a(t)$, gives the equation of motion for the homogeneous modes which, after multiplication by $\dot{\phi}$, can be written as follows

$$\frac{d}{dt}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] + 3H\dot{\phi}^2 = 0\,. \tag{56}$$

Defining $\zeta(t) = V(\phi)/\rho(\phi)$, where $\rho(t) = \dot{\phi}^2/2 + V(\phi)$, we find

$$\rho(t) = \rho(t_o)\exp - \int_{t_o}^{t} 6[1-\zeta(t)]H(t)dt = \rho(t_o)\exp - \int_{a_o}^{a} 6[1-\zeta(a)]\frac{da}{a}\,. \tag{57}$$

When the kinetic energy dominates, $\zeta \to 0$ and

$$\rho \propto \frac{1}{a^6}\,. \tag{58}$$

If a potential possesses a flat direction, for example, the energy in the associated coherent goldstone mode scales in this way. In this case the scaling can be seen to follow directly from the conservation of the Noether current associated with the symmetry. Consider for example a complex scalar Φ with a potential invariant under the global symmetry $\Phi \to e^{i\theta}\Phi$

$$\lambda(\Phi^\dagger\Phi - v^2)^2\,. \tag{59}$$

The mode $\Phi = ve^{-i\theta}$ with $\dot{\theta} = \mathit{const}$ is a solution of the equations of motion for which the conserved Noether charge is

$$j^o \equiv \rho_\theta = a^3 i\Phi^\dagger \overset{\leftrightarrow}{\partial^o} \Phi = 2a^3v^2\dot{\theta}\,. \tag{60}$$

Thus $\dot{\theta} \propto 1/a^3$ and $\rho = v^2\dot{\theta}^2/2 \propto 1/a^6$.

Such kinetic energy dominated modes represent the opposite limit to inflation [41] which is driven by potential energy so that $\zeta \to 1$ and $\rho(t) \approx \rho(t_o)$. Indeed for any homogeneous mode (assuming only that $V(\phi)$ is positive) we have that

$$\rho(t_o)\left(\frac{a_o}{a}\right)^6 \leq \rho(t) \leq \rho(t_o)\,, \qquad t \geq t_o\,. \tag{61}$$

Instead of superluminal expansion in inflation, a kinetic energy dominated mode of a scalar potential drives a subluminal expansion very similar to that of radiation ($a \propto t^{1/2}$) or matter ($a \propto t^{2/3}$). Writing the stress energy tensor in terms of the pressure p and the energy density ρ in the standard way, the equation of state is $p = \rho$ for the kinetic mode, in contrast to $p = (1/3)\rho$ (radiation), $p = 0$ (matter), and $p = -\rho$ (inflation).

We now consider various ways in which a phase of kination could come about. Inflation is the standard paradigm which explains isotropy and homogeneity of the Universe as it appears today. A scalar field drives a period of inflation and subsequently decays, filling the Universe with radiation and matter. We will assume that a period of inflation produces the isotropic and homogeneous Universe, but ask how it might come about that after inflation a reheated universe would be dominated by a kinetic scalar mode.

Two questions can be separated:

- How can a scalar field potential support a mode that is kinetic energy dominated? This question is twofold. Firstly, what shape must the potential have to keep the kinetic energy dominant? Secondly, what is required of the field in order that energy does not leak out of the coherent mode?
- How can kinetic modes come to dominate the energy density, *i.e.* how can they be excited?

Let us start with the *first* question. The most trivial case of potential energy domination is the example used above of an exactly flat potential. This case is not of interest here, since the energy in such a mode is negligible at the end of inflation, as it also red-shifts away as $1/a^6$ during inflation. Any kinetic energy domination must therefore occur through the roll of a field in a potential after inflation. The dynamics of a homogeneous real scalar field ϕ with potential $V(\phi)$ in an expanding FRW universe are described by the equations

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} + V'(\phi) &= \frac{1}{a^3}\frac{d}{dt}\left(a^3\dot{\phi}\right) + V'(\phi) = 0 && (62)\\ H^2 &= \frac{1}{3M_{\mathrm{P}}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_r\right] && (63)\\ \dot{\rho}_r + 4H\rho_r &= 0 && (64)\end{aligned}$$

where ρ_r is the energy density in radiation, to which we assume the scalar field to be coupled only through gravity. This is a roll damped by the expansion of the Universe and the first question is therefore: how steep must a potential be in order that the roll be more effective in creating kinetic energy than the damping is in attenuating it? A hint of the solution is immediately given by considering again the trivial case $V(\phi) = 0$, which gives the solution

$$\dot{\phi}(t) = \dot{\phi}_o\left(\frac{a_o}{a}\right)^3 = \dot{\phi}_o\left(\frac{t_o}{t}\right), \qquad \phi(t) = \phi_o + \dot{\phi}_o t_o \ln\frac{t}{t_o} \tag{65}$$

when $\rho_r = 0$. If, with this solution for ϕ, the potential is such that the terms which depend on it decrease faster than the other terms in the equations of motion, the kinetic energy domination will continue once established. Given that the time dependence is logarithmic, it is clear that an exponential potential is what is required. That exponential potentials define what is steep enough for kination can be seen by taking the explicit attractor solutions given in [42] for the potential $V(\phi) = V_o \mathrm{e}^{-\lambda\phi}$

$$\phi(t) = \frac{2}{\lambda}\ln t\,, \qquad a \propto t^{\frac{2}{\lambda^2}}\,, \qquad \zeta = 1 - \frac{\lambda^2}{6}\,, \quad \lambda^2 < 6\,, \tag{66}$$

and the origin of ϕ is redefined so that $V_o = 2\lambda^{-2}\left(6\lambda^{-2} - 1\right)$. (For simplicity we took $M_{\mathrm{P}} = 1$.) The context within which [42] discussed this potential was "power-law inflation", for which the superluminal expansion occurs when $\lambda < \sqrt{2}$. From (57) it follows that $\rho \propto a^{-\lambda^2}$ so in the limit $\lambda \to \sqrt{6}$ we recover the scaling of an exactly flat potential. When $\lambda > \sqrt{6}$ the pre-factor cannot be written in this way. There is no single attractor solution, and the ratio ζ rather than being fixed approaches zero asymptotically.

In this analysis we have assumed a simple roll down a potential. Another possibility is that a field oscillates about a minimum. It is easy to see from (57) the well-known result that an oscillation in a quadratic potential gives an energy scaling like matter $\propto 1/a^3$ since ζ can be replaced by its average $\langle\zeta\rangle = 1/2$ over a time-scale of the expansion time. The analysis for the ϕ^n potential is given in [43]. The result is that an oscillating mode scales as $a^{-6n/(n+2)}$ and, correspondingly, $\langle\zeta\rangle = 2/(n+2)$, so that the kinetic energy becomes more dominant as n increases.

We also require that, if such a mode is excited, the energy remains in it, *i.e.* that it does not leak out by decay of the coherent mode into particle excitations of itself or other fields to which it is coupled. In the present context we want to maximize the effect of the mode and therefore need the energy to stay in the kinetic mode from before the electroweak scale until shortly before nucleosynthesis. Potentials which support coherent modes which are so weakly coupled to other fields (or self-coupled in the case of massless fields) that they do not decay before nucleosynthesis are in fact commonplace in particle physics - they are the source of problems like the Polonyi problem. In particular, exponential potentials which arise in theories involving compactifications are typically extremely weakly coupled to other sectors. Accordingly, these sectors are termed "hidden," as they interact with the "visible" matter only (or predominantly) gravitationally. We will thus assume that perturbative decay is negligible. On the other hand non-perturbative decay mechanisms like parametric resonance which have been much discussed recently in the context of the problem of reheating after inflation [47, 55, 54, 53, 52, 48], must be considered in the case of oscillating modes of non-linear potentials. This is a possibility we will consider explicitly below, and the requirement that such a mode survive until nucleosynthesis will place constraints on the potentials we consider.

The *second* question above concerned how such a mode would come to dominate over radiation. In the analysis just given of a field rolling down a potential we set $\rho_r = 0$, and the solutions are therefore valid only if $\rho_\phi \gg \rho_r$ (where ρ_ϕ is the total energy in the scalar field). What happens if $\rho_r \gg \rho_\phi$? Can we end up rolling into the kinetic energy dominated mode with $\rho_\phi \gg \rho_r$ if we are in an exponential with such a mode? In a radiation dominated universe $a \propto t^{1/2}$ ($H = 1/2t$) so that the damping is stronger than in the kinetic energy dominated universe where $a \propto t^{1/3}$ ($H = 1/3t$). If the scalar field is in a kinetic energy dominated mode, we find, assuming radiation domination, that

$$\dot{\phi}(t) = \dot{\phi}_o(\frac{a_o}{a})^3 = \dot{\phi}_o\left(\frac{t_o}{t}\right)^{\frac{3}{2}}, \qquad \phi(t) = \phi_o + 2\dot{\phi}_o t_o\left(1 - (\frac{t_o}{t})^{\frac{1}{2}}\right). \tag{67}$$

The result is that the exponential potential energy will always "catch up" with the kinetic energy and the field will be driven into a mode which scales much slower than radiation, until $\rho_\phi \sim \rho_r$. These quite different behaviours in the two limits $\rho_\phi \gg \rho_r$ ($\rho_\phi \propto 1/a^6$) and $\rho_\phi \ll \rho_r$ ($\rho \sim const$) which tend to drive the system from one regime to the other suggests that there may be an attractor solution with $\rho_\phi \sim 1/a^4$. That such a solution exists and is an attractor has been noted in [44] and [45]. It can in fact be generalized to the case that the non-scalar component scales as $\rho_m \propto 1/a^m$ (*e.g.* non-relativistic matter with $m = 3$), and all components scale as it does with the ratios of their contributions given by

$$\zeta = 1 - \frac{m}{6}, \qquad \frac{\rho_\phi}{\rho_\phi + \rho_m} = \frac{m}{\lambda^2}. \tag{68}$$

In this case one need not assume that the pre-factor in the exponential can be written in the special form required for the solution (66) and λ can take on any value $\lambda > \sqrt{m}$, which is just the requirement that the attractor mode in (66) with $\rho_m = 0$ scale faster than $1/a^m$.

The existence of this attractor means that if we start in a radiation dominated universe (or, more generally, in a universe dominated by energy scaling as $1/a^m$) we will always end up in this solution (68) rather than in the kinetic energy dominated mode of the exponential. In order to realize kination in this potential we must therefore satisfy the condition at the end of inflation, or some time after it, that the kinetic energy dominate over the radiation. The dynamics of the simple exponential alone will not produce kination if we have a radiation dominated universe after inflation. We will examine two possibilities: (i) A non-standard theory of re-heating in which the radiation in the Universe is that created by the expansion of the background during inflation, and radiation is naturally subdominant after inflation, and (ii) standard reheating with a slightly different exponential potential which can first cool the radiation with a short inflationary phase and then roll into a kinetic mode.

We will concentrate on this first kind of model, because in it there must by construction be a phase of kination, and in our view it offers a very attractive (and unjustifiably neglected) alternative to the standard re-heating scenario. What sort of model would lead to this phase ending as late as nucleosynthesis will be the question which interests us in the specific context of electroweak cosmology.

In the oscillatory potential things are slightly different. The scaling was predicated on the assumption that the field oscillated on a time-scale short compared to the expansion time, but not on any assumption about the time dependence of the expansion rate (*i.e.* about which component dominates the energy density). Thus if the Universe is radiation dominated when we enter the oscillatory mode of a potential with $n > 4$, it will always be radiation dominated since the energy in the scalar field red-shifts away faster. To realize kination in this potential we therefore require the radiation to be sub-dominant when the oscillatory phase begins. Just as for the exponential potential we will discuss in this case how this condition can be realized in two ways: (i) in the same alternative standard theory of reheating after a period of inflation driven by the power-law potential itself, and (ii) with ordinary reheating by another field followed by a brief subsequent period of inflation as the field with the power-law potential rolls before it begins oscillating. Again most of our attention will focus on the first case, in which a single field is both inflaton and 'kinaton'.

5.2 Reheating by Kination

Reheating after inflation is required in order to match the 'cold empty' Universe left behind by inflation onto the radiation dominated one which must in the standard cosmology be established by nucleosynthesis at the latest, (and usually, it is assumed, at some temperature high enough to support some theory of baryogenesis). In the standard theory this is achieved by the decay of the inflaton into particles in an oscillatory phase after inflation, the zero entropy coherent state producing the enormous entropy of the radiation dominated universe. That there exists a simple alternative to this scenario has been pointed out by Spokoiny in [1]. The Universe is *not* in fact in an exactly cold zero entropy state after inflation – besides the energy in the inflaton, there is some energy in the particles created by the accelerated expansion. The process which gives rise to the perturbations from homogeneity required for structure formation on large scales creates an energy density which is peaked at the scale H, where H is the expansion rate during inflation, with energy density $\delta\rho^H = \epsilon_\rho H^4$, where the superscript H denotes that this energy density is dominated by the scale $k \sim H$, and $\epsilon_\rho \simeq (\pi^2 g_*^{eff}/30)/(2\pi)^4$ [56], where $g_*^{\rm eff} \sim 10^2$ is the effective number of light (mass $m < H$) degrees of freedom[16]. In a typical inflationary model with energy density $\rho_{\rm i\,,\,end}$ at the end of inflation

$$\frac{\delta\rho^H_{\rm i\,,\,end}}{\rho_{\rm i\,,\,end}} \simeq \frac{\epsilon_\rho}{3}\frac{H^2_{\rm i\,,\,end}}{M_{\rm P}^2} \simeq \frac{\epsilon_\rho}{9}\frac{\rho_{\rm i\,,\,end}}{M_{\rm P}^4}\,, \tag{69}$$

[16] This estimate assumes the same contribution from all particles as from the scalar particles analysed in [56].

which is very small since the energy scale associated with inflation is typically required to be well below $M_{\rm P}$, *e.g.* for chaotic inflationary model in a potential $\lambda\phi^4$, $\delta\rho^H_{\rm i\,,\,end}/\rho_{\rm i\,,\,end} \sim \epsilon_\rho\lambda$, while the requirement that one gets density perturbations of the correct magnitude (on COBE scales $\delta\rho/\rho \sim 5\times 10^{-5}$) gives $\lambda \sim 10^{-13}$. In the context of ordinary re-heating this small fraction is irrelevant as it is swamped by the radiation created by inflaton decay. The possibility envisaged in [1] is one which is easy to see given the observations of the preceeding section on kination: If, instead of decaying, the inflaton rolls into a potential in which its energy density scales as $1/a^s$ with $s>4$, the energy density $\delta\rho^H$ will come to dominate at some time $t_{\rm k\,,\,end}$ after inflation when the scale factor has evolved to $a_{\rm k\,,\,end}$ from $a_{\rm i\,,\,end}$ at the end of inflation with

$$\frac{a_{\rm k\,,\,end}}{a_{\rm i\,,\,end}} \approx \left(\frac{9}{\epsilon_\rho}\frac{M_P^4}{\rho_{\rm i\,,\,end}}\right)^{\frac{1}{s-4}} = \left(\frac{3}{\epsilon_\rho}\right)^{\frac{1}{s-4}} \left(\frac{M_P}{H_{\rm i\,,\,end}}\right)^{\frac{2}{s-4}}, \tag{70}$$

where $a_{\rm k\,,\,end}$ is the scale factor at the end of kination (during which $\rho \propto 1/a^s$), the phase which interpolates between inflation and radiation domination. The energy in the inflaton simply red-shifts away instead of decaying. As discussed in [1], in order to accommodate nucleosynthesis there are two requirements which must be fulfilled: (i) the radiation must thermalize at a temperature above 1MeV and, (ii) the transition to radiation dominance must occur sufficiently long before nucleosynthesis to satisfy the appropriate constraints at that time on the expansion rate. Taking $k_{\rm eff}(a)$ to be the typical energy of the created radiation as a function of scale factor, we have $k_{\rm eff} = H_{\rm i\,,\,end}a_{\rm i\,,\,end}/a$. Assuming that the dominant form of this radiation is in standard model degrees of freedom, the interaction rate for processes coupling them is $\sim \alpha^2 k_{\rm eff}$ (for $k_{\rm eff} \gg M_W$ and $\alpha \sim 1/30 - 1/50$). Comparing this to the expansion rate [17] $H \approx 2/st \approx H_{\rm i\,,\,end}(a_{\rm i\,,\,end}/a)^{s/2}$ (in kination), we get an estimate for the thermalization temperature $T_{\rm reheat}$

$$T_{\rm reheat} \sim H_{\rm i\,,\,end}\left(\frac{30\epsilon_\rho}{\pi^2 g_{*\,,\,\rm i\,,\,end}}\right)^{1/4} \alpha^{\frac{4}{s-2}} \tag{71}$$

where $g_{*\,,\,\rm i\,,\,end}$ is the number degrees of freedom which are relativistic at $k \sim H_{i\,,\,\rm end}$, and we have defined $\delta\rho^H_{\rm i\,,\,end} = g_{*\,,\,\rm i\,,\,end}\pi^2 T^4_{\rm i\,,\,end}/30$, and taken $T \propto 1/a$ [18]. Assuming this temperature $T_{\rm reheat}$ to be attained before the transition to radiation dominance, it follows from (70) that $T_{\rm k\,,\,end}$, the temperature at the beginning of radiation domination, is given approximately by

$$\frac{T_{\rm k\,,\,end}}{M_P} \simeq \left(\frac{\epsilon_\rho}{3}\right)^{\frac{1}{s-4}} \left(\frac{30\epsilon_\rho}{\pi^2 g_{*\,,\,\rm i\,,\,end}}\right)^{\frac{1}{4}} \left(\frac{H_{\rm i\,,\,end}}{M_P}\right)^{\frac{s-2}{s-4}} \tag{72}$$

Requiring this to be above the nucleosynthesis temperature 1MeV places a lower bound on $H_{\rm i\,,\,end}$. For $s=6$ we find that $H_{\rm i\,,\,end} > 10^7$ GeV, which corresponds to $T_{\rm reheat} > 10^6$GeV, consistent with the assumption that $T_{\rm reheat} > T_{\rm k\,,\,end}$. For $s=5$ both $H_{\rm i\,,\,end}$ and $T_{\rm reheat}$ are greater by a factor of $\sim 10^4$. In both cases a late transition to radiation dominance implies that the energy scale at the end of inflation and thermalization scale are well below the GUT scale.

In standard inflationary models the usual constraint on $H_{\rm i\,,\,end}$ or the energy density at the end of inflation comes from the requirement that the amplitude of perturbations be that required for structure formation. In the models which we discuss below we will consider how this non-trivial constraint is satisfied in this model of reheating (a question not considered in [1]), and in particular how it fits with the particular type of realization of this model we are interested in, where the transition to radiation

[17] A far from equilibrium system may in fact need many rescatterings (*i.e.* $N_{\rm scatt} \gg 1$) to fully thermalize. Modifying the estimate in (71) to incorporate this gives $T_{\rm reheat}$ smaller by a factor $N_{\rm scatt}^{-2/(s-2)}$.

[18] Here and below we neglect the effect of possible decouplings between $T_{\rm reheat}$ and $T_{\rm k\,,\,end}$, *i.e.* we assume the number of relativistic degrees of freedom to be fixed.

domination does actually occur close to nucleosynthesis with the potentially important consequences for electroweak baryogenesis discussed in the first part of this paper.

5.3 Inflation-Kination in an Exponential Potential

As discussed in section 5.1, a simple exponential which gives rise to the kinetic energy dominated mode required for kination does not itself accommodate an inflationary solution. We need to have a potential which is flatter in some region (for inflation) and sufficiently steep (for kination) in the part of the potential the field rolls into after inflation. An example is an exponential $\sim e^{-\lambda\phi}$ where λ varies as a function of ϕ. As a simple case of this, which we can treat analytically, we consider[19]

$$V(\phi) = V_o \mathrm{e}^{-\lambda\phi}\,, \quad \text{where} \qquad \begin{array}{ll} \lambda < \sqrt{2} & \text{for} \quad \phi < \phi_{\mathrm{i\,,\,end}} \\ \lambda \equiv \lambda' > 2 & \text{for} \quad \phi > \phi_{\mathrm{i\,,\,end}} \end{array} \tag{73}$$

where we set $M_{\mathrm{P}} = 1$. As discussed above, one solution to the equations of motion for this potential is a power-law inflationary attractor (66) with $\phi = \frac{1}{\lambda}\ln\left(\frac{V_o\lambda^4 t^2}{2(6-\lambda^2)}\right)$, and $a \propto t^{2/\lambda^2}$. We assume the field to evolve in this attractor in inflation from $\phi \ll \phi_{\mathrm{i\,,\,end}}$. When the field reaches $\phi_{\mathrm{i\,,\,end}}$ inflation ends and after a transient period it will roll, for $\lambda' < \sqrt{6}$, into the new attractor in the steeper potential. If $\lambda' \geq \sqrt{6}$, there is no single attractor, but the field will run after a few expansion times into a solution in which the kinetic energy is very dominant. In either case we will neglect the details of the few expansion times in which this transition occurs.

We calculate first the cosmological perturbations generated in the usual way by the amplification of quantum fluctuations during inflation. The amplitude of the perturbation in a mode with comoving momentum k when it re-enters the horizon after inflation at time $2X$, is given by the usual formula

$$\frac{\delta\rho}{\rho}(k) \approx \epsilon_\delta \frac{H_{1X}^2}{\dot{\phi}_{1X}}\,, \tag{74}$$

where $1X$ denotes the time when the perturbation k exits the horizon in inflation, and $\epsilon_\delta = 3/5\pi \simeq 0.2$ [41] is a constant. The formula is valid provided the slow-roll approximation holds at this time, which in the case of the exponential potential corresponds to $\lambda \ll \sqrt{2}$. The evolution of scales is illustrated on figure 3, where $k = k_{\mathrm{physical}}a$ is plotted versus Ha (both on the logarithmic scale). Since the comoving scale is fixed, it follows that

$$k \equiv k_{1X} = k_{2X} \quad \Leftrightarrow \quad (Ha)_{1X} = (Ha)_{2X}\,, \tag{75}$$

and therefore

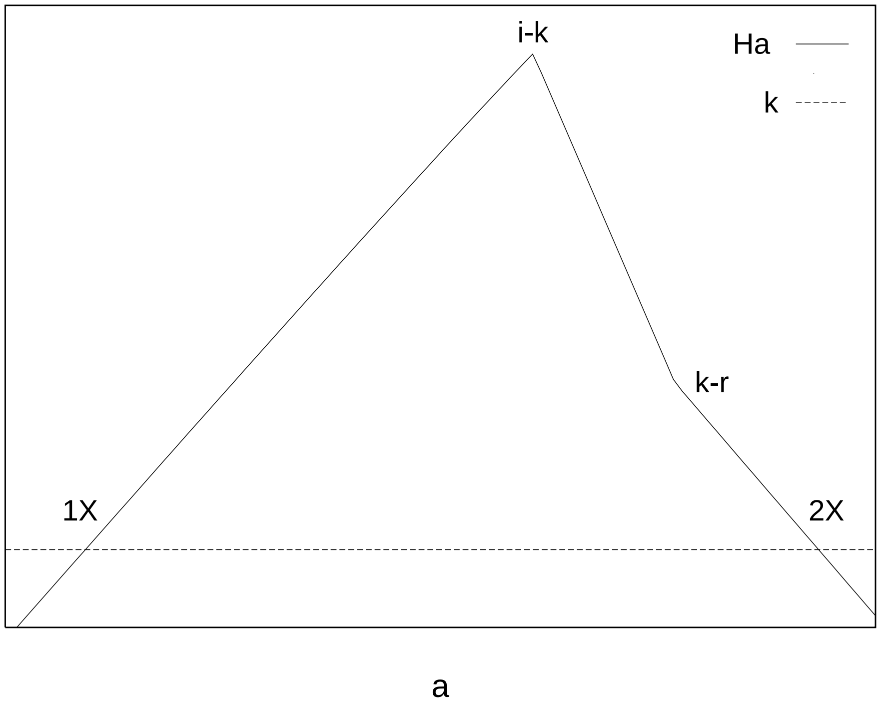
$$\frac{(Ha)_{1X}}{(Ha)_{\mathrm{i\,,\,end}}}\,\frac{(Ha)_{\mathrm{k\,,\,beg}}}{(Ha)_{\mathrm{k\,,\,end}}}\,\frac{(Ha)_{\mathrm{r\,,\,beg}}}{(Ha)_{\mathrm{r\,,\,end}}}\,\frac{(Ha)_{\mathrm{m\,,\,beg}}}{(Ha)_{2X}} = 1\,, \tag{76}$$

where the indices *i, k, r,* and m denote *inflation, kination, radiation* and *matter*, respectively, and we have assumed that $(Ha)_{\mathrm{i\,,\,end}} = (Ha)_{\mathrm{k\,,\,beg}}$, $(Ha)_{\mathrm{k\,,\,end}} = (Ha)_{\mathrm{r\,,\,beg}}$, $(Ha)_{\mathrm{r\,,\,end}} = (Ha)_{\mathrm{m\,,\,beg}}$, and that the relevant perturbation enters the horizon in the matter era. In writing (76) we assumed sharp transitions $i \to k \to r \to m$. Within our approximation we keep a continuous, but its derivative exhibits a jump ($H = 2/\lambda^2 t$ in inflation matches onto $H = 2/\lambda'^{\,2} t$ in kination). With the attractor solutions for (73) in (74) we obtain

$$\frac{\delta\rho}{\rho}(k) = \epsilon_\delta \frac{2}{\lambda^3}\frac{1}{t_{1X}}\,. \tag{77}$$

[19]We could of course consider any potential which accommodates inflation in some region and is asymptotically a sufficiently steep exponential. Motivation for an exponential with varying λ is given in [44].

Figure 3. Evolution of scales in the inflation-kination model



Using (76) and calculating Ha in each of the eras, we find that

$$\frac{t_{1X}}{t_{\mathrm{i\,,\,end}}} = \left[\left(\frac{a_{\mathrm{i\,,\,end}}}{a_{\mathrm{k\,,\,end}}} \right)^{\frac{\Lambda'^{\,2}-2}{2}} \left(\frac{a_{\mathrm{k\,,\,end}}}{a_{2X}} \right) \right]^{\frac{\lambda^2}{2-\lambda^2}} , \qquad \Lambda' = \min\left[\sqrt{6}, \lambda'\right] , \tag{78}$$

where, for simplicity, we take $2X$ to be in the radiation era. The behavior of (77) is the usual one, with an overall amplitude set by the expansion rate at the end of inflation and, for sufficiently small λ (required for consistency of our slow-roll approximation), a fairly flat spectrum of perturbations over the scales relevant to structure formation. Indeed, using the standard expression $\mathtt{n} \approx 1 + \left[-3(V'/V)^2 + 2V''/V\right] M_{\mathrm{P}}^2$ [46], we obtain $\mathtt{n} \approx 1 - \lambda^2$. Assuming the reheating scenario of [1] discussed in the previous section, we can express (77) in terms of the radiation temperature at the end of kination, $T_{\mathrm{k\,,\,end}}$. Taking the radiation energy density at the inflation-kination transition to be $\rho_{\mathrm{i\,,\,end}} = \epsilon_\rho H^4_{\mathrm{i,\,end}}$, and using (70) and (72) allows us to express the temperature and time at the end of inflation as follows

$$T_{\mathrm{i\,,\,end}}^{\Lambda'^{\,2}-2} = \left(\frac{270}{\pi^2 g_{*\,,\,\mathrm{i\,,\,end}}\epsilon_\rho} \right)^{\frac{1}{2}} T_{\mathrm{k\,,\,end}}^{\Lambda'^{\,2}-4} , \qquad H_{\mathrm{i\,,\,end}} = \left(\frac{\pi^2 g_{*\,,\,\mathrm{i\,,\,end}}}{30\epsilon_\rho} \right)^{\frac{1}{2}} T_{\mathrm{i\,,\,end}} = \frac{2}{\lambda^2 t_{\mathrm{i\,,\,end}}} , \tag{79}$$

so that

$$\frac{\delta\rho}{\rho}(k_{2X}) \approx \frac{\epsilon_\delta}{\lambda} \left(\frac{270}{\pi^2 g_{*\,,\,\mathrm{i\,,\,end}}\epsilon_\rho} \right)^{\frac{1}{4}\left[\frac{2}{\Lambda'^{\,2}-2} + \frac{\lambda^2}{2-\lambda^2}\right]} T_{\mathrm{k\,,\,end}}^{\frac{\Lambda'^{\,2}-4}{\Lambda'^{\,2}-2}} \, T_{2X}^{-\frac{\lambda^2}{2-\lambda^2}} . \tag{80}$$

This is the desired expression for the amplitude of fluctuations at the comoving scale k_{2X} which re-enters the horizon when the temperature is T_{2X}.

This result depends on three unknown parameters - the temperature at the end of kination $T_{\mathrm{k\,,\,end}}$, and the parameters λ and λ' in the potential. The COBE experiment provides us with a constraint on the amplitude ($\delta\rho/\rho(k) \approx 5 \times 10^{-5}$) and the spectral index of density perturbations ($0.7 \leq \mathtt{n} \leq 1.3$). The extra constraints we impose are those required by our consideration of electroweak baryogenesis: The phase of kination must persist well past the electroweak phase to have an important effect on the expansion at that scale. For example, for $T_{\mathrm{k\,,\,end}} = T_{\mathrm{ns}}$, and $T_{2X} = 1\mathrm{eV}$, we find $\lambda = \sqrt{0.33} = 0.57$ for $\lambda' \geq \sqrt{6}$, and $\lambda = \sqrt{0.11} = 0.33$ for $\lambda' = \sqrt{5}$. These lead to the tilt in the power spectrum $\mathtt{n} \approx 0.7$ for $\lambda' \geq \sqrt{6}$, and $\mathtt{n} \approx 0.9$ for $\lambda' = \sqrt{5}$, which are consistent with the constraint from COBE. Following the discussion in section 4 we know that in order to create the observed BAU at a second order or cross-over electroweak phase transition we need to have very close to $1/a^6$ scaling in kination, *i.e.* $\lambda' \geq \sqrt{6}$. This requirement therefore leads in this model to a prediction of the spectral index $\mathtt{n} \approx 0.7$. Using (79) we can also compute $T_{\mathrm{i\,,\,end}} \sim H_{\mathrm{i\,,\,end}}$, $t_{\mathrm{i\,,\,end}}$, $\phi_{\mathrm{i\,,\,end}}$, *etc.* In particular, for $\lambda' \geq \sqrt{6}$ we have $T_{\mathrm{i\,,\,end}} = 6 \times 10^7\mathrm{GeV}$, and for $\lambda' = \sqrt{5}$ we have $T_{\mathrm{i\,,\,end}} = 2 \times 10^{11}\mathrm{GeV}$ (independent of λ in inflation).

What we have illustrated with this analysis is the observational adequacy (and even potential predictivity) of a model of this type. The 'prediction' we derived here is of course particular to a model we have invoked in its specific form in an *ad hoc* way. It would be of interest to study models which are derived in detail from a well motivated particle physics model. We will limit ourselves here to one qualitative comment on the sort of model which motivated our choice (see [44]) in which the parameter λ varies slowly (logarithmically) as a function of ϕ. It is not difficult to see that the constraint on the spectral index may be much weaker when λ interpolates between our limiting values: We were constrained to increase λ as Λ' increased (to give $1/a^6$ scaling) to make the amplitude of perturbations sufficiently large. The effect of having an interpolating scaling between that in inflation and $1/a^6$ scaling will be to increase $H_{\mathrm{i\,,\,end}}$ (at fixed $T_{\mathrm{i\,,\,end}}$), which also increases the density perturbations, allowing a spectral index closer to one.

5.4 Inflation-Kination in a Power-Law Potential

In this section we consider another one field model in which an inflation-kination-radiation domination cosmology can be realized. Again we assume the mechanism of reheating through particle creation in inflation discussed in section 5.2. The potential we study is simply the non-renormalizable power-law potential

$$V(\phi) = \frac{\lambda_n}{n} M_P^4 \left(\frac{\phi}{M_P}\right)^n \tag{81}$$

where $n > 4$ is an integer, which is taken to be even for stability reasons. As discussed in section 5.1 this potential has an oscillatory solution in which the energy density in the field scales as $a^{-6n/(n+2)}$, *i.e.* faster than radiation for $n > 4$. It also has, as we will discuss below, "slow-roll" inflationary solutions which will precede such an oscillatory phase for appropriate initial conditions, just as in "chaotic" inflation in a ϕ^4 potential. As mentioned in the introduction potentials of this type have been studied in the context of inflation motivated by F and D flat directions of supergravity theories (see, for example, [6, 46]). Lower order perturbative terms are forbidden by a discrete symmetry imposed on the superpotential.

There are several important differences between the exponential we have considered in the previous section and this potential. There the potential was made up of two pieces, one with an inflationary attractor the other with a 'kinationary' attractor. Here we also have solutions of the two types in different regions of the potential, but the cross-over from one to the other is dynamically determined rather than an independent input (*i.e.* specified by $\phi_{\mathrm{i\,,\,end}}$). Therefore once λ_n and n are specified, the potential and $T_{\mathrm{k\,,\,end}}$ and $\delta\rho/\rho(k)$ are completely determined.

The second difference between the two potentials is that in the power-law potential kination is associated with an oscillatory mode, which can decay non-perturbatively *via* parametric resonance [47]. Only if such decay occurs after the transition to radiation domination, is the scenario we have envisaged possible. If it occurs a little earlier, but still sufficiently close to nucleosynthesis that the reheat temperature resultant from the decay of the field is below the electroweak temperature, there will be some minor effect on the predictions of electroweak cosmology. We will not consider this marginal case and simply require the stability of the oscillatory mode until after the transition to radiation domination, which we require below the electroweak scale in order to have an effect on electroweak cosmology. Later in this section we will investigate in more detail the consequences of the resonant inflaton decay.

In analogy to our treatment of the exponential potential in section 5.3, we now determine how the potential (81) is constrained by the requirement that ϕ generates cosmological perturbations of the required magnitude for structure formation, and that kination – driven by the oscillatory mode – ends in radiation domination (by the mechanism of [1]) before the temperature $T_{\rm ns}$ at which nucleosynthesis occurs. We will see that these two requirements cannot be simultaneously satisfied by a suitable choice of the two variables in the potential λ_n and n.

To determine the amplitude of the cosmological perturbations we follow exactly the analysis of the previous section, taking the perturbations to be given by (74). Furthermore, we will make use of (75) and (76) to determine Ha in inflation and kination.

The equations of motion for the homogeneous mode can be written as

$$\ddot{\phi} + 3H\dot{\phi} + \lambda_n \phi^{n-1} = 0 \tag{82}$$

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_\phi}{3}, \qquad \rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{\lambda_n}{n}\phi^n \tag{83}$$

setting $M_P = 1$, *i.e.* with the rescaling

$$\phi \to \frac{\phi}{M_{\rm P}}, \quad t \to tM_P, \quad H \to \frac{H}{M_{\rm P}}. \tag{84}$$

In the standard "slow roll" approximation we take

$$\ddot{\phi} \ll 3H\dot{\phi}, \lambda_n \phi^{n-1} \qquad \frac{1}{2}\dot{\phi}^2 \ll \frac{\lambda_n}{n}\phi^n, \tag{85}$$

and find from (82) that

$$\dot{\phi} = -\left[\frac{n\lambda_n}{3}\right]^{\frac{1}{2}} \phi^{\frac{n}{2}-1}. \tag{86}$$

Putting this expression back in (85) it is easy to show that the "slow roll" condition is

$$\phi^2 \gg \frac{n^2}{6}. \tag{87}$$

Furthermore during the slow roll (83) gives

$$\frac{H}{\dot{\phi}} \equiv \frac{d\ln a}{d\phi} = -\frac{\phi}{n} \tag{88}$$

and hence

$$a = \exp -\frac{\phi^2}{2n}, \tag{89}$$

where we chose for the integration constant $a_0 = \exp[-\phi_0^2/2n]$. For completeness, we also write the solution to (86)

$$\phi = \left[\frac{12}{(n-4)^2 n\lambda_n}\right]^{\frac{1}{n-4}} t^{-\frac{2}{n-4}}, \tag{90}$$

where we chose $t = 0$ such that $\phi \to \infty$ as $t \to 0$.

We can now write the desired expression in inflation:

$$Ha = \left(\frac{\lambda_n}{3n}\right)^{\frac{1}{2}} \phi^{\frac{n}{2}} \exp -\frac{\phi^2}{2n} \qquad \text{(inflation)}\,, \tag{91}$$

which allows us to compute the first term in (76).

Next we compute Ha during kination. It is convenient for this analysis to change the field and time variables to rescaled variables τ and φ given by

$$dt = \left(\frac{a}{a_0}\right)^{\frac{3(n-2)}{n+2}} d\tau\,, \qquad \phi = \varphi \left(\frac{a_0}{a}\right)^{\frac{6}{n+2}} \tag{92}$$

in terms of which (82) and (83) become

$$\frac{d\varphi}{d\tau^2} + \frac{6}{n+2}\left[-\frac{1}{a}\frac{d^2a}{d\tau^2} + \frac{n-4}{n+2}\left(\frac{1}{a}\frac{da}{d\tau}\right)^2\right] + \lambda_n \varphi^{n-1} = 0 \tag{93}$$

$$\mathcal{H}^2 \equiv \left(\frac{1}{a}\frac{da}{d\tau}\right)^2 = \frac{\rho_0(\tau)}{3}\left(\frac{a_0}{a}\right)^{\frac{12}{n+2}}\,, \tag{94}$$

$$\rho_0(\tau) = \frac{1}{2}\left(\frac{d\varphi}{d\tau} - \frac{6}{n+2}\mathcal{H}\varphi\right)^2 + \frac{\lambda_n}{n}\varphi^n\,.$$

The approximation of a sharp transition from inflation to kination consists in ignoring the explicit time dependence of $\rho_0(\tau)$, which is equivalent to

$$\mathcal{H} \ll \frac{n+2}{6}\frac{1}{\varphi}\frac{d\varphi}{d\tau} \simeq \frac{n+2}{6}\omega_n\,, \tag{95}$$

where $\omega_n^2 \sim \lambda_n \varphi_0^{n-2}$ is the average frequency squared of φ (see (97) below), and $\varphi_0 = \phi_0$ is the inflaton amplitude at the beginning of kination. Within this approximation, $d^2a/ad\tau^2 = (n-4)\mathcal{H}^2/(n+2)$, and hence the term in the square brackets of (93) vanishes. It is this feature of the damping term in these variables which made their choice appropriate. Thus all of the time dependence in (93) and (94) drops out and the equations can be easily integrated. The first integral of (93) leads to

$$\rho_0 \equiv V(\varphi_0) = \frac{1}{2}\left(\frac{d\varphi}{d\tau}\right)^2 + \frac{\lambda_n}{n}\varphi^n\,, \tag{96}$$

which is just the energy conservation law for φ.

The oscillatory solution for φ can be then expressed in terms of an elliptic integral with the frequency

$$\omega_n \equiv \frac{2\pi}{\tau_n} = \frac{\pi}{\sqrt{2n}c_n}\lambda_{\text{eff}}^{\frac{1}{2}}\,\phi_0\,, \tag{97}$$

$$\lambda_{\text{eff}} = \lambda_n \phi_0^{n-4}\,, \qquad c_n = \int_0^1 \frac{dx}{\sqrt{1-x^n}}$$

where τ_n is the oscillation period. Note that ω_n decreases exponentially with n, when the initial amplitude $\phi_0 < 1$. In the limit of a large n, c_n approaches unity. Finally, the solution to (94) is

$$\frac{a}{a_0} = \left[\frac{6}{n+2}H_0\tau\right]^{\frac{n+2}{6}}\,, \tag{98}$$

(where we used $\mathcal{H}_0 = H_0$) and

$$\frac{Ha}{H_0 a_0} = \left(\frac{a_0}{a}\right)^{\frac{2(n-1)}{n+2}} \qquad \text{(kination)}\,. \tag{99}$$

The results for radiation and matter era are given by setting $n = 4$ and $n = 2$ in (99) respectively, so that (76) can be recast as

$$\phi_{1X}^2 - \phi_{\mathrm{i\,,\,end}}^2 = 2n \left\{ \ln \left[\left(\frac{a_{\mathrm{k\,,\,end}}}{a_{\mathrm{k\,,\,beg}}}\right)^{\frac{2(n-1)}{n+2}} \frac{a_{\mathrm{r\,,\,end}}}{a_{\mathrm{r\,,\,beg}}} \left(\frac{a_{2\mathrm{X}}}{a_{\mathrm{m\,,\,beg}}}\right)^{\frac{1}{2}} \right] + \frac{n}{2} \ln \frac{\phi_{1X}}{\phi_{\mathrm{i\,,\,end}}} \right\}. \tag{100}$$

The ratio of the scale factors at the beginning and the end of kination is given by (70) in section 5.2. Using $s = 6n/(n+2)$ we have (with $M_P = 1$)

$$\left(\frac{a_{\mathrm{k\,,\,end}}}{a_{\mathrm{k\,,\,beg}}}\right)^{2\frac{n-4}{n+2}} = \frac{\rho_{\phi\ \mathrm{i\ end}}}{\rho_{\mathrm{i\ end}}} = \left(\frac{270}{\epsilon_\rho g_{*\,,\,\mathrm{i\,,\,end}} \pi^2}\right)^{\frac{1}{2}} \frac{1}{T_{\mathrm{i\ end}}^2}\,. \tag{101}$$

and recalling that $Ta = const$, we have

$$\left(\frac{a_{\mathrm{k\,,\,end}}}{a_{\mathrm{k\,,\,beg}}}\right)^{2\frac{n-1}{n+2}} = \left(\frac{270}{\epsilon_\rho g_{*\,,\,\mathrm{i\,,\,end}} \pi^2}\right)^{\frac{1}{4}} \frac{1}{T_{\mathrm{k\ end}}} \tag{102}$$

so that (100) becomes

$$\phi_{1X}^2 = \phi_{\mathrm{i\,,\,end}}^2 + 2n \left\{ \ln \left[\left(\frac{270}{\epsilon_\rho g_{*\,,\,\mathrm{i\,,\,end}} \pi^2}\right)^{\frac{1}{4}} \frac{1}{T_{\mathrm{r\,,\,end}}} \left(\frac{T_{\mathrm{m\,,\,beg}}}{T_{2\mathrm{X}}}\right)^{\frac{1}{2}} \right] + \frac{n}{2} \ln \frac{\phi_{1X}}{\phi_{\mathrm{i\,,\,end}}} \right\}. \tag{103}$$

Taking this expression with

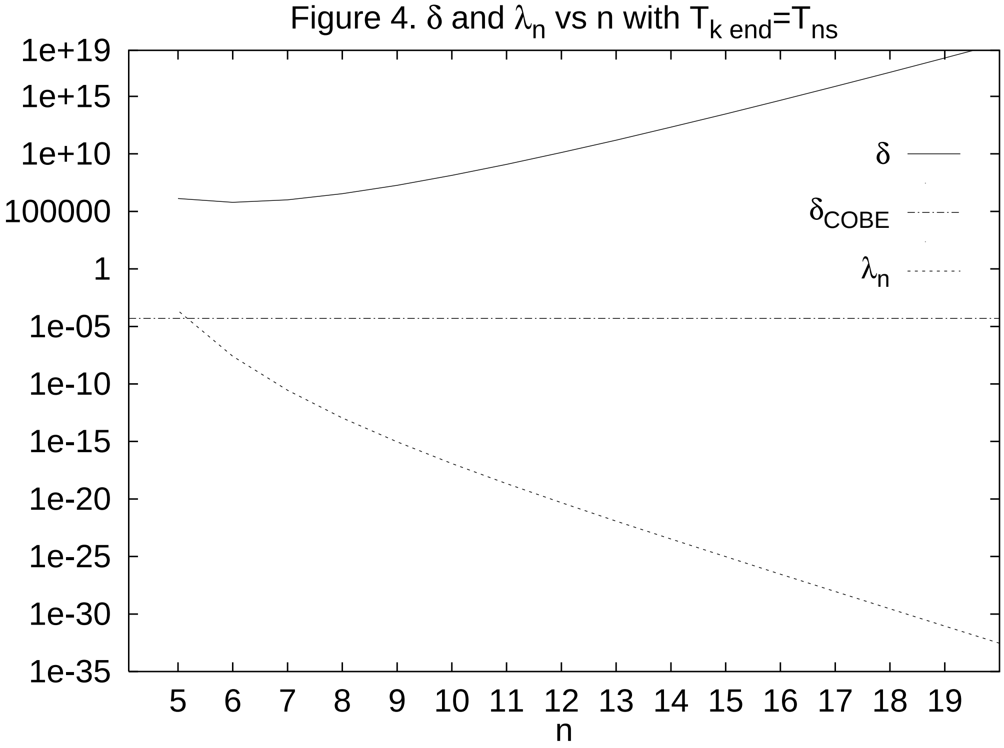
$$\frac{\delta\rho}{\rho}(k_{1X}) = \epsilon_\delta \frac{\lambda_n^{1/2}}{3^{1/2} n^{3/2}} \phi_{1X}^{(n+2)/2}\,, \quad \epsilon_\delta = \frac{3}{5\pi} \tag{104}$$

specifies the amplitude of density perturbations in the model implicitly in terms of the parameters in the potential λ_n and n. Comparing this to the requirement of COBE provides the first constraint on the model. The second constraint is the requirement that kination ends before nucleosynthesis, *i.e.* at a temperature $T_{\mathrm{k\,,\,end}} > T_{\mathrm{ns}}$. After some algebra we obtain the simple relation

$$T_{\mathrm{k\,,\,end}} = \left(\frac{270}{\epsilon_\rho g_{*\,,\,\mathrm{i\,,\,end}} \pi^2}\right)^{\frac{1}{4}} \left(\frac{\epsilon_\rho}{9} \frac{\lambda_n}{n} \phi_{\mathrm{i\,,\,end}}^n\right)^{\frac{n-1}{n-4}}\,. \tag{105}$$

On figure 4 we show a plot of $\delta \equiv \delta\rho/\rho(k_{1X})$ and of λ_n as a function of n with $T_{\mathrm{k\,,\,end}} = T_{\mathrm{ns}}$ and $T_{2X} = 1\mathrm{eV}$. It is clear that everywhere the amplitude is too large (by several orders of magnitude) to satisfy the constraint from COBE. Further, to increase $T_{\mathrm{k\,,\,end}}$ at fixed n we require a larger λ_n, which results in a larger δ. That kination end by nucleosynthesis thus forces the energy density in radiation at the end of inflation to be sufficiently large, which forces λ_n to be so large that the density perturbations produced are too large, for any n. A single field model of this type is therefore ruled out.

This analysis neglects the possible decay of the oscillating mode by parametric resonance into either its own fluctuations, or other fields that it couples to. If such decay occurs when the energy density of the oscillating mode has red-shifted to be sub-dominant relative to the radiation, then such decay is



irrelevant and the model is simply ruled out by nucleosynthesis constraints (and the requirement that density perturbations not be too large). If, on the other hand, it decays when the energy density of the oscillating mode is still dominant over the radiation, the model may be viable if the decay products can thermalize with ordinary matter. In this case, however, there will always be large production of entropy (with corresponding dilution of the baryon to entropy ratio) and hence the model is not of much interest in the context of the question of how an increased expansion rate at the electroweak could lead to production of the BAU at that scale in scenarios when it is usually assumed to be impossible.

It is nevertheless of interest to study the non-perturbative decay of this mode (i) to see whether such models are really ruled out by the observations above, and (ii) because this decay channel is relevant to a scenario where another field plays the role of inflaton, which we will discuss briefly below.

We wish to compare the resonant decay time $\tau_{\rm decay}$ of the inflaton-kinaton to the time at the end of kination $\tau_{\rm i\,,\,end}$. As we show in the appendix, the field can decay either into its own fluctuations, or to other fields that it couples to. In both cases the decay time $\tau_{\rm decay}$ can be approximated by

$$\tau_{\rm decay} \sim \frac{1}{2\omega_n\mu}\ln\frac{n_{\rm scatt}}{n_0}\,, \tag{106}$$

where $n_0 \simeq 1/2$ is the initial occupation number, and $n_{\rm scatt}$ is the 'late time' occupation number. The resonance is usually terminated by the back reaction effects from the created particles [48], and $n_{\rm scatt}$ can be estimated to be $n_{\rm scatt} \sim 1/\lambda_{\rm eff} = 1/(\lambda_n\phi_{\rm i\,,\,end}^{n-4})$ for the inflaton-kinaton decay into its own fluctuations, and $n_{\rm scatt} \sim 1/g$ when ϕ couples to a scalar field ζ, where g is the coupling constant of the interaction term $g\zeta^2\phi^2/2$. In the appendix we also show that $\mu = \mu_n \simeq 0.16/n$ for the decay into its own fluctuations, and for the decay into another field $\mu \simeq 0.1 - 0..2$ when $q \geq 1$, and $\mu \ll 1$ when $q \ll 1$.

In order to get an expression for $\tau_{\rm k\,,\,end}$, we make use of (95), (98), and (101) to get

$$\tau_{\rm k\,,\,end} \simeq \frac{1}{\omega_n}\left(\frac{9}{\epsilon_\rho}\frac{n}{\lambda_n}\frac{1}{\phi_{\rm i\,,\,end}^n}\right)^{\frac{3}{n-4}}\,. \tag{107}$$

Setting $\tau_{\mathrm{k,\,end}} = \tau_{\mathrm{decay}}$ results in the following constraint on the instability coefficient

$$\mu^{\mathrm{constr}} \simeq \frac{1}{2}\left(\frac{\epsilon_\rho}{9}\frac{\lambda_n}{n}\phi^n_{\mathrm{i,\,end}}\right)^{\frac{3}{n-4}} \ln\frac{n_{\mathrm{scatt}}}{n_0}\,, \tag{108}$$

such that when the inflaton decays only into its own fluctuations, $n_{\mathrm{scatt}}/n_0 \sim 1/(\lambda_n \phi^{n-4}_{\mathrm{i,\,end}})$, and $\mu_n^{\mathrm{constr}} \simeq 0.049$, significantly larger than any μ_n for $n > 4$. This implies $\tau_{\mathrm{decay}} \simeq (\mu_n^{\mathrm{constr}}/\mu_n)\tau_{\mathrm{k,\,end}}$, typically greater than τ_{ns}, where we took $T_{\mathrm{k,\,end}} \simeq 2T_{\mathrm{ns}}$. When ϕ couples to another scalar field *via* a quartic term of form $g\phi^2\zeta^2/2$, then $n_{\mathrm{scatt}}/n_0 \sim 1/g \simeq 1/(4q\lambda_{\mathrm{eff}})$, where $q = g\phi^2/4\omega_n^2$ (*cf.* (114) in the appendix), and we see that for $q \geq 1$, $1/g < 1/\lambda_{\mathrm{eff}}$, and hence $\mu^{\mathrm{constr}} < 0.049$. Recall that in this case μ is typically of order 0.1. This means that when $q > 1$, the inflaton decays somewhat before $\tau_{\mathrm{k,\,end}}$ *via* parametric resonance. If, on the other hand, $q \ll 1$, then $\mu \ll 1$, implying a late inflaton decay, $\tau_{\mathrm{decay}} > \tau_{\mathrm{ns}}$.

In summary, the oscillating mode in the power-law potential decay *via* parametric resonance before nucleosynthesis if it couples to another scalar field with $q \geq 0.1$, or equivalently $g \geq 10^{-20}$. When $g \geq 10^{-20}$ it is not immediately obvious whether the resonance shift slows down the decay or not, and, although the discussion in the appendix suggests that it does not, further analysis is required to establish this definitively. In any case we can conclude that this single power-law potential driving inflation with reheating of the type we have discussed (as in [1]), is therefore only ruled out as a viable cosmological model for $g \leq 10^{-20}$.

5.5 Two Field Models

Finally we consider briefly models with ordinary re-heating (through the decay of the inflaton). In this case the field which supports the kinetic energy dominated mode cannot also be the inflaton, but is a second field which comes to be important after inflation and ordinary re-heating. Again we consider the two cases of an exponential with its rolling mode and the power-law potential with its oscillating mode.

Case 1: Inflaton + Exponential Potential

As discussed in section 5.1 a simple exponential $V(\phi) = V_o e^{-\lambda\phi/M_P}$ with λ such that it supports, when dominant over radiation, a mode scaling faster than radiation, will not come to dominate over radiation irrespective of the initial conditions on the field. (If the initial condition gives a scaling slower than radiation, it will bring the system to the attractor (68) in which the scalar field contributes at most an amount comparable to the radiation.) Therefore, just as in the one-field case, a potential is required which is only asymptotically this simple exponential. If the field lies initially in a part of the potential which is flatter - flat enough to support an inflationary type solution - a period of inflation will occur once the radiation cools so that its energy density is comparable to that in the scalar field. The initial conditions and details of the potential will determine what the final ratio of the energy in the scalar field and radiation energy is when the scalar field enters its asymptotic kinetic mode. If this second period of inflation occurs at intermediate energy scales (after 'full' inflation at the GUT scale, say) and is of a small number of e-foldings, the ratio will be such that the kinetic energy domination may end before nucleosynthesis.

Such a short period of inflation at an intermediate scale occurs in so-called 'thermal' inflation [50]. A scalar field is trapped in a false minimum by its coupling to the plasma and comes to dominate for a short period until the inflation it drives cools the plasma and allows it to roll away. In the present context all that is required is that the field, rather than rolling into an oscillating mode and decaying, roll into a potential which is asymptotically exponential.

Another way in which such a transient period of inflation which cools the radiation and leaves a kinetic mode dominant could occur is by special initial conditions in certain potentials, *e.g.* if the field ϕ with potential $V(\phi) = V_o e^{-\lambda\phi^2/M_P^2}$ sits initially close to $\phi = 0$, a period of inflation will occur when ρ_{rad} becomes comparable to V_o, the duration of which will depend on how close to $\phi = 0$ the field is initially. Without significant fine-tuning there will be a few e-foldings of inflation followed by a period of kination.

And lastly, we mention a variant of hybrid inflation. Recall that in hybrid inflation, one field (ϕ) is held at the false vacuum minimum by a large expectation value of a second field (ψ), and hence it drives inflation. When ψ becomes sufficiently small, ϕ rolls down to its true minimum. The roll in a steep potential, *e.g.* $g\phi^2\psi^2/2 + V_o e^{-\lambda\phi}$, leads to kination. Since the shape of the ψ potential determines the amplitude of density perturbations, we have more freedom to tune parameters of the model than in the one field case. In particular there is no need for variation of λ.

Case 2: Inflaton + Power-Law Potential

The various examples just given can be carried over in an obvious way to the case of a power-law potential. The difference is parallel to that in the one-field case: If the field is initially sub-dominant relative to the radiation, the oscillatory mode about the minimum could end up being dominant depending on the initial conditions. If the field lies initially at $\phi > nM_{\mathrm{P}}$ there will be a period of inflation which brings the field to dominance over the radiation. For a small number of e-foldings the radiation produced by ordinary re-heating (by decay of the inflaton) may be dominant over any radiation produced by particle production as in the mechanism we discussed in the one-field models. The constraints which we derived in the one field model in section 5.4, and which we found could not be satisfied, are circumvented simply because the initial radiation density is not specified by the potential, and the relation (105) no longer holds. For the model to work we also require that the field decay *via* parametric resonance occur after the mode has become sub-dominant relative to the radiation, *i.e.* after nucleosynthesis, which will translate into some upper bound on the couplings of the field. The precise bound would have to be derived in analogy to the treatment given above for any particular model (which will specify $\tau_{\mathrm{k\,,\,end}}$).

6 Testability: Connections to Other Observables

We have shown in the preceeding section that there are simple post-inflationary cosmologies quite different from the standard scenario in which radiation domination begins straight after inflation. Instead a phase interpolates between the two in which a kinetic energy dominated mode, red-shifting more rapidly than radiation, drives the expansion of the Universe. As we discussed in the earlier sections of the paper, if such a phase continues until after the electroweak scale, the implications for electroweak cosmology would potentially be extremely important. To conclude we wish to discuss some of the broader implications of the observations we have made. In particular, we began this paper with the usual motivation for the consideration of electroweak baryogenesis: It promises to follow nucleosynthesis in making firm and observable predictions about the cosmological remnants from an epoch at which temperatures are such that we can have experimental knowledge of the relevant physics. It promises to be a testable theory. What is left of this testability now that we have effectively turned one crucial parameter, which is usually assumed to be known, into an unknown?

In contrast to nucleosynthesis there is in this case only one "observable" - the baryon to entropy ratio - produced by a calculation. Our analysis shows that, at least in certain particle physics theories, it will be possible to "fit" the observed asymmetry by an appropriate expansion rate. Does making H a variable make the theory intrinsically untestable? The answer is negative for two reasons. Firstly, it is an extremely non-trivial requirement that one can produce the observed BAU in any given electroweak model, even if the expansion rate is a variable. In a first order phase transition, for example, the requirement of various parameters - most importantly on CP violating parameters - are typically extremely strong, independently of the expansion rate (without the sphaleron bound). As we have seen in this paper, it is conceivable that it could turn out that the scalar sector indicates an analytic cross-over or a weakly first order transition and CP violation sufficiently large that the BAU could be produced if the expansion rate is greater by about five orders of magnitude than usually assumed. Would we then take it this to tell us that this is the case or that we are unlikely to be able to draw a definite conclusion as to whether the BAU was created at this scale? This brings us to the second answer to the question: The theory is truly testable only if we can find other observables which depend on pre-nucleosynthesis cosmology. If we do indeed find that the BAU can be generated with a different cosmology, that would provide a major incentive to pursue this possibility.

One possibility is exactly the relic densities of dark matter particles discussed in [3] and [4]. The discovery of a candidate dark-matter particle would allow one to determine the expansion rate at its time of decoupling from the requirement that it be the cold dark matter in the Universe. For example, from figure 2 in [4] we see that the relic density of a Majorana neutrino changes by several orders of magnitude as the expansion rate at its decoupling does. If this indicated an expansion rate different from the standard value and consistent with that required at the electroweak scale for generation of the observed BAU, one would have compelling evidence that cosmology is indeed different. Another possible way of probing pre-nucleosynthesis cosmology is with magnetic fields, which in certain models are produced at or before the electroweak scale, or at the QCD phase transition. This seems a more remote possibility for a firm constraint in that the connection to observed fields is itself very indirect. However, it is one worth bearing in mind. For example, in the mechanism discussed in [31] in which fields are generated by an instability related to the abelian anomaly, the expansion rate enters in determining when perturbative processes come into equilibrium. This depends on the expansion rate, and for a significantly different expansion rate the results would be different.

Further there is also the possibility of probing cosmology at the electroweak scale indirectly by its connections to other epochs. A good example of this is in fact the scalar field cosmology we have discussed, in particular the exponential potential. In this case the same coherent mode which dominated in kination can in fact play an important role again at later times. We noted the existence of an attractor solution with energy densities given as in (68) for the exponential in the presence of a component of matter or radiation. How soon this will be established after the end of kination depends essentially how much the ratio of kinetic to potential energy at the end of kination differs from its value in the attractor (68), and this will vary depending on the model. In [11] the case is treated in which this transient period between the two attractors is assumed to end well before the beginning of matter domination, and details of the observable consequences on structure formation in a flat CDM dominated universe are studied; in [12] the case of entry into the attractor well into the matter era at a red-shift $z \sim 70$ is treated. With the forthcoming satellite experiments which will measure the properties of the microwave sky, such models will become testable in detail.

7 Appendix: Resonant inflaton decay in the ϕ^n potential

In this appendix we study the decay of the inflaton via parametric resonance. First we address the decay into its own fluctuations, and then we discuss how it decays into other fields. We start with writing the evolution equation for small fluctuations around the inflaton zero momentum mode: $\phi \to \phi + \delta\phi$ in (82). After a Fourier transform and setting $M = M_{\rm P} = 1$ as in (84), we get the following linearized mode equation

$$\frac{d^2\delta\phi_{\vec{k}}}{dt^2} + 3H\frac{d\delta\phi_{\vec{k}}}{dt} + \left[\left(\frac{a_0}{a}\right)^2 \vec{k}^2 + \lambda_n(n-1)\phi^{n-2}\right]\delta\phi_{\vec{k}} = 0\,. \tag{109}$$

Rescaling to the new variables as in (92) and assuming pure kination, *i.e.* that the field amplitude is small (95), we obtain

$$\frac{d^2\delta\varphi_{\vec{k}}}{d\tau^2} + \left[\left(\frac{a}{a_0}\right)^{\frac{4(n-4)}{n+2}} \vec{k}^2 + \lambda_n(n-1)\varphi^{n-2}\right]\delta\varphi_{\vec{k}} = 0\,. \tag{110}$$

Notice the scale dependence next to $\vec{k}^2$ which means that, even though the zero-mode equations are time independent, the mode equations are *not.* Assuming adiabatic variation of a/a_0, Eq. (110) becomes the famous Hill equation

$$\begin{aligned} \frac{d^2\delta\varphi_{\vec{k}}}{d\tau'^{\,2}} + [A + 2qf(\tau')]\,\delta\varphi_{\vec{k}} &= 0\,, \\ A = \left(\frac{a}{a_0}\right)^{\frac{4(n-4)}{n+2}} \frac{k^2}{\omega_n^2} + 2q\,, \quad q &= \frac{(n-1)\lambda_n\varphi_0^{n-2}}{4\omega_n^2}\,, \end{aligned} \tag{111}$$

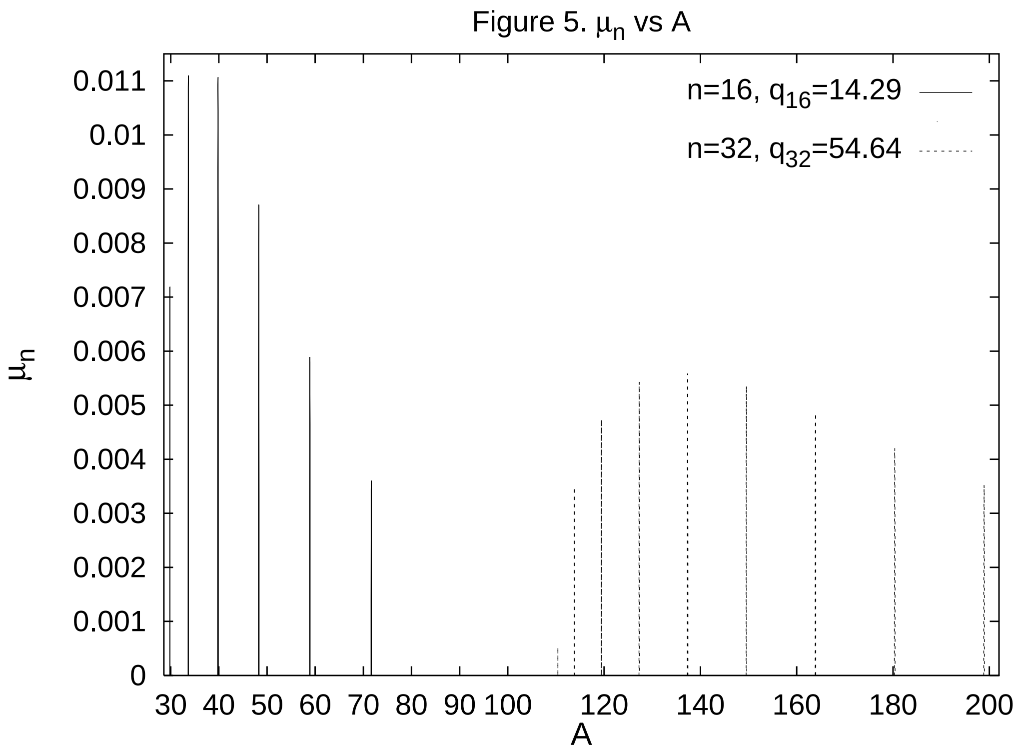
where $\tau' = \omega_n\tau$ and $f = 2(\varphi(\tau')/\varphi_0)^{n-2} - 1$ is defined so that $\max|f| = 1$, $\langle f\rangle = 0$, $f(\tau'+\pi) = f(\tau')$. The general solution of Hill's equation is of the form $e^{\pm\mu\tau'}P(\tau')$, where $P(\tau'+\pi) = P(\tau')$, and it is often given as the stability chart. The unstable regions are specified by the curves of constant μ in the $\{q, A\}$ plane, and the stable regions are bounded by $\mu = 0$. The instability chart is important since the field decays exponentially into the modes with $\mu > 0$, preferably so to the ones with large μ. The special case of the Hill equation when $n = 4$ - the Lamé equation - is extensively studied in the literature on inflaton decay [53, 54], [55]. The instability chart exhibits unstable regions which branch off from $A = n^2$ at $q = 0$. For $q \ll A$ one is in the narrow resonance regime, since the bands are narrow and $\mu \ll 1$. The chart is symmetric under $q \to -q$. On the other hand, for $1 < 2q \le A$ the resonance bands become broad and μ "large." Typically, when $2q \simeq A$, μ peaks at $\sim (2\pi)^{-1}$. In this case the field decays very fast, characteristically in a few dozens oscillations.

Notice that in general for a given n, $q_n = (n-1)\varphi_0^{n-2}/(4\omega_n^2)$ is specified. Consequently, to get a rough estimate of the decay time, it suffices to plot the one dimensional slice $q = q_n$ of the chart. As the field decays, q stays constant, unless the backreaction of the created particles is large enough to change ω_n. Numerical simulations [52, 48] show that for the $\lambda_4\varphi^4$ potential the backreaction from created particles grows to about $\delta m^2 \equiv 3\lambda\langle\delta\varphi^2\rangle \sim \lambda_4\varphi_0^2/4$, changing the effective frequency $\omega_n^2 \to \omega_n^2 + \delta m^2$, and consequently reducing q to about half of its original value and $A \to A + \delta m^2/(\omega_n^2 + \delta m^2)$. The growth of δm^2 is terminated by narrowing the resonance as a consequence of the backreaction on A and q, and intensifying scatterings of the resonant particles off the zero mode, as a consequence of increasing resonant amplitudes. By then a significant portion of the field has decayed. We will assume that a similar scenario holds for a generic ϕ^n case. This is plausible since, as we will see below, the instability charts are quite similar.

We have evaluated numerically [51] the instability charts for some of the models. The results for $n = 4, 8, 16, 32$ are plotted in figures 5. The corresponding initial values for q_n are $1.045, 3.84, 14.29, 54.64$. Note that in all cases to a very good approximation the first instability band terminates at $A = 2q$, so that the field decays into the second instability band. The values for μ are $\mu_4 = 0.0425$ [20], $\mu_8 = 0.023$, $\mu_{16} = 0.011$, $\mu_{32} = 0.0056$, so that $\mu_n \simeq 4\mu_4/n \simeq 0.16/n$. For the following analysis the details of the chart are not that important. It is sufficient to keep in mind that for n larger, μ_n decreases.

As discussed in section 5, an inflaton that decays into its own fluctuations and does not couple to other fields leads to disastrous consequences for nucleosynthesis. Indeed, since the inflaton decay products scale as radiation and eventually dominate the energy density of the Universe, and also decouple from the rest of

[20] Note that this value is a bit higher from $\mu_4 = 0.0359$, the value quoted in [55].



matter, they behave effectively as many additional massless degrees of freedom, leading to a very different expansion rate than predicted by nucleosynthesis. If, on the other hand, the inflaton predominately decays into another scalar field, which consequently thermalizes, producing thus standard radiation and matter particles, nucleosynthesis may be unaffected by late inflaton-kinaton decay, as long as the decay occurs comfortably before nucleosynthesis.

Before we start discussing the inflaton decay time, we outline the physics of the inflaton decay into another scalar field ζ. We assume a standard quartic coupling g to ζ (that itself couples to Standard Model particles) of the form $g\zeta^2\phi^2/2$ such that the linearized mode equations of motion are

$$\frac{d^2\zeta_{\vec{k}}}{dt^2} + 3H\dot{\zeta}_k + \left[\left(\frac{a_0}{a}\right)^2 \vec{k}^2 + g\phi^2\right]\zeta_{\vec{k}} = 0\,. \tag{112}$$

With the rescaling (92)[21] and $\zeta = \tilde{\zeta}(a_0/a)^{6/(n+2)}$, this becomes

$$\frac{d^2\tilde{\zeta}_{\vec{k}}}{d\tau^2} + \left[\left(\frac{a}{a_0}\right)^{\frac{4(n-4)}{n+2}} \vec{k}^2 + \left(\frac{a}{a_0}\right)^{\frac{6(n-4)}{n+2}} g\varphi^2\right]\tilde{\zeta}_{\vec{k}} = 0\,, \tag{113}$$

and can be recast as

$$\begin{aligned} \frac{d^2\tilde{\zeta}_{\vec{k}}}{d\tau'^{\,2}} + \left[A_\zeta + 2q_\zeta f_\zeta\right]\tilde{\zeta}_{\vec{k}} &= 0 \\ A_\zeta = \left(\frac{a}{a_0}\right)^{\frac{4(n-4)}{n+2}} \frac{k^2}{\omega_n^2} + 2q_\zeta\,, \quad q_\zeta &= \left(\frac{a}{a_0}\right)^{\frac{6(n-4)}{n+2}} \frac{g\varphi_0^2}{4\omega_n^2} \end{aligned} \tag{114}$$

[21] In this case conformal rescaling might seem more appropriate since it would get rid of all dependence on a. Nevertheless, we stick to the rescaling in (92), in order to be able to make direct comparison of decay times.

where $\tau' = \omega_n \tau$ and $f_\zeta = 2(\varphi(\tau')/\varphi_0)^2 - 1$ is defined so that $\max|f_\zeta| = 1$, $\langle f_\zeta \rangle = 0$, $f_\zeta(\tau' + \pi) = f_\zeta(\tau')$. As above in (111), in adiabatic limit, this reduces to Hill's equation. There are however two differences: first, q_ζ can assume a wide range of values depending on g, and, second, q_ζ is a (growing) function of a. The corresponding Mathieu equation for $n = 2$ is studied in great detail in the literature, and shows that for $q_\zeta > 1$ the field decays with an average value $\mu \sim 0.1$ [57]. For the conformal case with $n = 4$ a similar value for μ is obtained. Here we will assume that, for any $n > 4$, $\mu \sim 0.1$ as well.

Now we present an estimate of the decay time $\tau_{\rm decay}$. For a moment we assume that the resonance shift does not drastically affect particle production. Later on we comment on the plausibility of this assumption. The typical initial mode amplitudes are such that the corresponding initial "occupation numbers" $n_k \propto \omega_k \varphi_{\vec{k}} \varphi_{-\vec{k}}$ (where ω_k is the energy of the mode k) are of order $n_k^{\rm initial} \simeq n_0 \simeq 1/2$. Since the resonant mode amplitudes grow as $\delta\varphi_k \propto \exp \mu_k \omega_n \tau$, one can estimate the field decay time as follows. The field decays when the energy in fluctuations become comparable to the energy in the zero momentum mode, *i.e.* when the occupation numbers $n_k \simeq n_0 \exp 2\mu_k \omega_n \tau$ become of order $n_k \sim 1/\lambda_{\rm eff}$ ($\lambda_{\rm eff} = \lambda_n \phi_0^{n-4}$), implying that the decay time can be approximated by

$$\tau_{\rm decay} \sim \frac{1}{2\omega_n \mu_k} \ln \frac{n_{\rm scatt}}{n_0} . \tag{115}$$

This same equation applies for the field ϕ decaying into other scalar fields. The only difference is that the maximum occupation number is in this case $n_{\rm scatt} \sim 1/g$. As a caveat to (115), the authors of [48] showed that one should expect longer decay times if the self-coupling of the second field (ζ) is large, *i.e.* $\lambda_\zeta \gg g$. In this paper we do not dwell on these complications, and assume the couplings such that the simple estimate (115) is valid.

Finally we comment on how the time dependence of A in (111) and (114) can affect the decay time (115). We first discuss the decay into a second scalar field. The (comoving) resonant momentum is specified by $\delta A \sim \sqrt{q}$, which in (114) gives

$$k_{\rm res}^2 \simeq \frac{\sqrt{g}\phi_0 \omega_n}{2} \left(\frac{a_0}{a}\right)^{\frac{n-4}{n+2}} . \tag{116}$$

This agrees with the well known result that for $n = 4$ the resonance is static. On the other hand, for $n = 2$ (Mathieu case) the resonant momenta grow rather fast as the Universe expands and, for q large, adiabatic approximation breaks down, leading to "stochastic resonance" [57]. However, it turns out that the instability exponent is rather robust and maintains the value $\mu \sim 0.1$. What happens when $n > 4$? In this case the resonant momentum decreases with time and again for $q > 1$ we expect breakdown of adiabatic approximation. Just like in the $n = 2$ case we expect μ to be robust and be of order $\mu \sim 0.1$. This should not in any case be considered as proof, but conjecture.

In the case when the field decays into its own fluctuations, $\delta A \sim \sqrt{q}$ gives (*cf.* (111))

$$k_{\rm res}^2 \simeq \frac{1}{2}\sqrt{(n-1)\lambda_n}\; \phi_0^{\frac{n-2}{2}}\, \omega_n \left(\frac{a_0}{a}\right)^{\frac{4(n-4)}{n+2}} \tag{117}$$

which again leads to a shift in the resonant momentum. Unfortunately, the conformal case ($n = 4$), in which the resonance is static, is the only case studied in the literature, so we cannot make any analogy as we did in the former case. Since in this case the resonance is rather narrow, the resonant momentum redshift may significantly slow down the decay. An implication would be that the effective μ decreases, leading to somewhat less stringent bounds on λ_n and n than indicated in (108).

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