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Supersymmetric Electroweak Baryogenesis in the WKB Approximation

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Abstract

We calculate the baryon asymmetry generated at the electroweak phase transition in the minimal supersymmetric standard model, treating the particles in a WKB approximation in the bubble wall background. A set of diffusion equations for the particle species relevant to baryon generation, including source terms arising from the CP violation associated with the complex phase δ of the μ parameter, are derived from Boltzmann equations, and solved. The conclusion is that δ must be $\gtrsim 0.1$ to generate a baryon asymmetry consistent with nucleosynthesis. We criticize several recent computations of the effect, arguing that they are overestimates.

Although theories to explain the baryon asymmetry of the Universe (BAU) abound, only electroweak baryogenesis has the hope of short-term testability. To calculate the asymmetry in detail one needs a specific model of electroweak physics beyond the standard model, and supersymmetry is the most popular idea for such new physics. Several groups have recently estimated the BAU coming from the minimal supersymmetric standard model (MSSM) at the electroweak phase transition (EWPT), assuming it is first order and so proceeds by bubble nucleation [1]-[5]. Despite the fact that the physics of the mechanism is essentially agreed upon—particles in the plasma interact with the bubble wall in a CP-violating manner, leading to a chiral asymmetry of quarks in front of the wall, which in turn biases sphalerons to produce the BAU—there is considerable disagreement on both methods and results.

While it is straightforward to formulate diffusion equations for particle transport away from the wall, it is less obvious how to derive the CP-violating source terms (or equivalently the boundary conditions at the wall) for these equations, and it is here that most of the difficulty lies. Several methods have been proposed in which the source terms are first derived and then inserted into a set of diffusion equations, but without any first principles justification for the prescription used. In the present work we generalize a method which was introduced for the treatment of the non-supersymmetric two doublet model in [6], in which the diffusion equations and the source terms are *derived together* starting from a general set of classical Boltzmann equations. This provides a systematic and controlled approximation valid for bubble walls significantly thicker than the inverse temperature, which is the case in the MSSM. Here we will describe the results as concisely as possible; finer details will be presented in a longer publication [7].

The method is based on a WKB quasi-particle approximation to the particle dynamics and interactions in the background of the bubble wall, which is an expansion in derivatives of the Higgs field VEV's that vary continuously in the wall. Such an expansion is appropriate because it well-describes the majority of particles in the plasma, whose wavelength ($\sim 1/T$) is much shorter than the thickness of the wall ($\sim 20/T$). When there is CP violation in the theory, such as is provided by the complex μ and A_t parameters of the MSSM, this approximation shows that particles and anti-particles experience a different force when interacting with the bubble wall, both dynamically as free WKB particles and in their interactions with other particles in the plasma. These two effects—the classical dynamical force on the particles, and the CP biasing of their collisions—appear in the Boltzmann equations for the WKB particles in a quite straightforward way, through the force term and collision term, respectively. These equations determine the distribution of left-handed quark number (q_L) in front of the bubble wall, which is the source term for baryon number generation. An appropriate fluid-type truncation leads to a set of diffusion equations of a familiar form, together with the source terms, which predict the q_L and hence the baryon asymmetry.

In what follows we will concentrate on charginos as the source of the chiral quark asymmetry, because we find they make the dominant contribution. The corresponding analysis for the subdominant squark and quark contributions will be presented in ref. [7]. To find the effect of CP violation on charginos at the bubble wall, we first solve the Dirac equation in the WKB approximation.

WKB approximation for charginos. The mass term for charginos can be written as $\overline{\Psi}_R M_\chi \Psi_L +$ h.c., where in the basis of Winos and Higgsinos, $\Psi_R = (\tilde{W}^+_R, \tilde{h}^+_{1,R})^T$, $\Psi_L = (\tilde{W}^+_L, \tilde{h}^+_{2,L})^T$ the mass matrix is

$$M_\chi = \begin{pmatrix} m_2 & g v_2/\sqrt{2} \\ g v_1/\sqrt{2} & \mu \end{pmatrix}, \tag{1}$$

with the spatially varying Higgs field VEV's v_i. The corresponding Dirac equation is $(i\gamma_3\gamma_0\partial_t + i\partial_z - \gamma_3(M_\chi P_L + M^\dagger_\chi P_R))\Psi = 0$. In the WKB approximation the local eigenstates of mass, energy and helicity are related to the flavor states through a transformation

$$\begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} = \begin{pmatrix} V & 0 \\ 0 & U \end{pmatrix} \begin{pmatrix} \cosh(X) & e^{i\theta}\sinh(X) \\ e^{-i\theta}\sinh(X) & \cosh(X) \end{pmatrix} \begin{pmatrix} \Psi_\downarrow \\ \Psi_\uparrow \end{pmatrix} e^{-iEt + i\int^z p(z)dz}. \tag{2}$$

Here U and V are the unitary transformations that locally diagonalize the mass matrices, $U^\dagger M_\chi V = \mathcal{M}_\chi$ and $V^\dagger M^\dagger_\chi U = \mathcal{M}^*_\chi$, and X is defined by $\tanh(2X) = |\mathcal{M}_\chi|/E$, which determines the mixing between the chirality and the helicity states. θ is a diagonal matrix, consisting of the phases of the eigenvalues $\mathcal{M}_\chi$, whose spatial variation is responsible for all the CP violating effects we will be concerned with. Finally, $p(z)$ is also a diagonal matrix that gives the local momentum of the two mass eigenstates. The explicit expressions for the mass eigenvalues, unitary matrices and CP-violating phases can be given in terms of the quantities

$$\begin{aligned}
\tilde{m}^2 &= (m_2^2+|\mu|^2+u_1^2+u_2^2)/2; \qquad u_i = gv_i/\sqrt{2};\\
\Delta &= (m_2^2-|\mu|^2-u_1^2+u_2^2)/2; \qquad a = m_2u_1+\mu^* u_2\\
\bar{\Delta} &= (m_2^2-|\mu|^2+u_1^2-u_2^2)/2; \qquad \bar{a} = m_2u_2+\mu u_1\\
\Lambda &= (\Delta^2+|a|^2)^{1/2} = (\bar{\Delta}^2+|\bar{a}|^2)^{1/2}; \qquad (3)
\end{aligned}$$

then $|\mathcal{M}_{\chi,\pm}|^2 = \tilde{m}^2 \pm \Lambda$, and

$$U = \frac{1}{\sqrt{2\Lambda(\Delta+\Lambda)}}\begin{pmatrix} \Delta+\Lambda & -a \\ a^* & \Delta+\Lambda \end{pmatrix}, \qquad V = \frac{1}{\sqrt{2\Lambda(\bar{\Delta}+\Lambda)}}\begin{pmatrix} \bar{\Delta}+\Lambda & -\bar{a} \\ \bar{a}^* & \bar{\Delta}+\Lambda \end{pmatrix}, \qquad (4)$$

and

$$|\mathcal{M}_{\chi,\pm}|^2\partial\theta_\pm = \pm g v_c \mathrm{Im}\,(\mu) m_2 \cos\beta \sin\beta u \partial_z u/\Lambda, \qquad (5)$$

where ∂_z denotes the spatial derivative normal to the bubble wall, $v_c = \sqrt{v_1^2+v_2^2}$ is the magnitude of the VEV at the critical temperature, $u = \sqrt{u_1^2+u_2^2}$ and $\tan\beta = v_2/v_1$. In what follows the phase θ will never be needed except in the combination shown above. Finally, to lowest nontrivial order in derivatives and CP-violation, the local momenta are $p(z) = \mathrm{sign}(p_z)\left(\sqrt{E^2-|\mathcal{M}_\chi|^2} \mp \sinh^2(X)\partial_z\theta\right)$, for particles with helicity $\pm$, and in the Lorentz frame where the particle has no momentum parallel to the bubble wall. For the antiparticles, the same equation holds but with an overall change of sign for the CP-violating part.

The important thing to notice in these expressions is that the CP-violating angle θ appears in two places: in the particle momenta (as a derivative) and in the transformation matrices to the WKB basis. The former leads to the dynamical force effect and the latter to source terms of the "spontaneous" type, which have previously been discussed in the literature in various contexts [8]. In what follows it will be convenient to have the dispersion relation for energy in terms of momentum, for the two mass eigenstates i:

$$\begin{aligned}
E_\pm &= \sqrt{\vec{p}^2+|m_i|^2} \pm \mathrm{sign}(p_z)\frac{\partial_z\theta_i}{2\sqrt{\vec{p}^2+|m_i|^2}}\left(\sqrt{p_z^2+|m_i|^2}-|p_z|\right) \equiv E_0 \pm \Delta E;\\
\pm &= +\text{ for } R, \bar{L}, \; -\text{ for } L, \bar{R}. \qquad (6)
\end{aligned}$$

To derive this we used the identity $\sinh^2(X) = \left(\sqrt{p_z^2+|m|^2} - |p_z|\right)/(2|p_z|)$ and transformed to a general Lorentz frame with nonzero momentum parallel to the wall.

Diffusion equations with sources. Our starting point is the Boltzmann equation $(\partial_t + \dot{\vec{x}}\cdot\partial_{\vec{x}} + \dot{\vec{p}}\cdot\partial_{\vec{p}})f_i = C[f_i]$ for the local distributions $f_i(\vec{p},\vec{x})$ of the WKB quasi-particles with the on-shell dispersion relation just derived. The on-shell approximation is valid since the particle widths Γ are small compared to their energies. To truncate these equations we use the ansatz

$$f_i(p,x) = \frac{1}{e^{\beta(E_i - v_i p_z - \mu_i)} \pm 1}, \tag{7}$$

for the distribution functions in the rest frame of the plasma, where E_i contains the perturbation $\Delta E_i \sim \partial\theta$, *i.e.,* we allow a chemical potential and velocity perturbation in each fluid. The validity of the ansatz requires that $v_w < L/3D$ [6], in order that whatever other perturbations are produced are indeed small compared to those of the chemical potential, the latter being damped only by the slower inelastic processes. The velocity perturbation is required to model the anisotropic response to the force term. Expanding to linear order in the perturbation ΔE_i and taking μ_i and v_i to be of this order gives $f'_\pm \partial_z\Big((v_w - p_z/E_0)(p_z v_i + \mu_i) - v_w \Delta E_i\Big) = -C\,[f_i]$, where $f'_\pm = df_\pm/dE$ denotes the derivative of the unperturbed Fermi-Dirac or Bose-Einstein distribution, $f'_\pm = (e^{\beta E_0} \pm 1)^{-1}$. Then averaging over momentum, weighting the equation respectively by 1 and p_z, we obtain the two coupled equations

$$\begin{aligned} -\langle p_z^2/E_0\rangle v_i' + v_w \mu_i' &= \langle C_i\rangle; &(8)\\ v_w\langle p_z^2\rangle v_i' - \langle p_z^2/E_0\rangle \mu_i' - v_w\langle p_z \Delta E_i'\rangle &= \langle C_i p_z\rangle &(9)\end{aligned}$$

where $\langle\cdot\rangle \equiv \int d^3p f'_\pm(\cdot)/\int d^3p f'_+ \equiv \kappa_i \int d^3p f'_\pm(\cdot)/\int d^3p f'_\pm$ and the primes in eqs. (8-9) denote ∂_z. Here we have divided by the distribution function for a massless fermion even if the numerator is for a boson, because this ensures that the collision integral on the r.h.s. of the Boltzmann equation gives the same value for the rate of a given process, regardless of whether one is writing the transport equation for a boson or for a fermion involved in that process. The result of this choice is to introduce the factor of κ_i on the l.h.s. of the Boltzmann equation, which is 1 for fermions and 2 for bosons, in the massless approximation.

To evaluate the collision terms in eq. (8-9) one must substitute the distributions (7) into $\int d\Pi_j \delta^{(4)}(p_i + p_2 \ldots - p_{n-1} - p_n)|\mathcal{M}|^2(f_i f_2 \cdots (1 \pm f_{n-1})(1 \pm f_n) - f_n f_{n-1} \cdots (1 \pm f_2)(1 \pm f_i))$, which vanishes for the equilibrium distibutions. Thus the collision terms are linear in the perturbations: $\langle C_i \rangle = \sum_j \Gamma^d_{i\to j}\mu_j$ and $\langle p_z C_i \rangle = \langle p_z^2 \rangle \sum_j \Gamma^e_{ij\to ij} v_i$, where $\Gamma^{d,e}$ are respectively the inelastic (decays) and elastic interaction rates for processes coupling states i and j. There is further another important contribution to $\langle C_i \rangle$ coming from the fact that the interactions of the flavor eigenstates, when reexpressed in terms of the local mass eigenstates, have factors of $e^{i\theta(z)}$ coming from eq. (2). In fact that equation is derived in the rest frame of the bubble, so that in the wall frame $e^{i\theta(z)} \to e^{i\theta(z-v_w t)}$, which to leading order in derivatives can be written as $e^{i(\theta(z)-v_w\partial\theta)}$. The effect is to spoil energy conservation of the interaction so that $\delta(E_i + E_2 \ldots - E_{n-1} - E_n) \to \delta(E_i - v_w\partial\theta + E_2 \ldots - E_{n-1} - E_n)$. This gives an additional term in $\langle C_i \rangle \sim v_w \partial\theta$, which is of the type referred to as a spontaneous baryogenesis source term. In the case of Higgsinos, the interaction $y\bar{q}_L \tilde{t}_R \tilde{h}_2$ can be rewritten as $y\bar{q}_L \tilde{t}_R (V_{2+}\sinh(X_+)e^{i\theta_+}\Psi_{\uparrow+} + V_{2-}\sinh(X_-)e^{i\theta_-}\Psi_{\uparrow-})$ leading to $\langle C_{\tilde{h}_2} \rangle = v_w \Gamma_y |V_{2\pm}|^2 \langle \sinh^2(X_\pm)\rangle \partial\theta_\pm$, where $\pm$ is whichever sign is needed to make the associated mass eigenstate go smoothly into the pure Higgsino state in front of the wall (see below).

Finally, the two coupled equations can be reduced to a single one by differentiating (9) and eliminating v_i in favor of μ_i. Defining the diffusion constant $D_i = \langle p_z^2/E\rangle^2/(\langle p_z^2\rangle \Gamma^e_i)$ and ignoring the ratio of inelastic to elastic scatterings, we obtain the diffusion equation

$$
\begin{aligned}
&-\kappa_i(D_i\mu_i'' + v_w\mu_i') + \Gamma^d_i \sum_j \mu_j = S_i; \\
&S_i = \frac{D_i v_w}{\langle p_z^2/E\rangle}\langle p_z \Delta E_i'\rangle' - v_w \sum_j \Gamma^d_{j\to i}|V_{ji}|^2 \langle \sinh^2(X_i)\rangle \theta_i', \qquad (10)
\end{aligned}
$$

to leading order in the wall velocity.

Solution of diffusion equations. From this complicated network of equations coupling all species in the plasma we need to determine the distribution of left-handed fermions, which bias the unsuppressed baryon-number-violating processes in front of the wall. It can be simplified by using conservation laws and neglecting processes which are too slow to play any role on the relevant timescales. Parametrically this means comparing the time spent by a particle diffusing in front of the wall $\sim D/v_w^2$ with the decay time Γ^{-1}. The processes we do take account of are then (i) the supergauge interactions, (ii) those described by the following terms in the interaction Lagrangian

$$V_y = y h_2 \bar{u}_R q_L + y \bar{u}_R \tilde{h}_{2L} \tilde{q}_L + y \tilde{u}_R^* \tilde{h}_{2L} q_L - y\mu h_1 \tilde{q}_L^* \tilde{u}_R + y A_t \tilde{q}_L h_2 \tilde{u}_R^* + \text{h.c.} \tag{11}$$

and (iii) strong sphaleron interactions. Taking the supergauge interactions to be in equilibrium, the chemical potentials of all gauginos are zero and the chemical potential of any particle is equal to that of its superpartners. Defining a reduced set of chemical potentials for each generation and chirality of baryons and each Higgs doublet, $H_{1,2}$, $Q_{1,2,3}$, U, D, S, C, B, T, with $H_1 = \frac{1}{4}(\mu_{h_1^0} + \mu_{h_1^-} + \mu_{\tilde{h}_1^0} + \mu_{\tilde{h}_1^-})$, $Q_1 = \frac{1}{4}(\mu_{u_L} + \mu_{d_L} + \mu_{\tilde{u}_L} + \mu_{\tilde{d}_L})$, $U = \frac{1}{2}(\mu_{u_R} + \mu_{\tilde{u}_R})$, etc., the interaction terms are

$$\begin{aligned} &(\Gamma_y + \Gamma_{yA})(-H_2 + Q_3 - T), \quad \Gamma_{y\mu}(H_1 - Q_3 + T), \quad \Gamma_{hf}(H_1 + H_2), \\ &\Gamma_{ss}(2Q_3 + 2Q_2 + 2Q_1 - U - D - C - S - B - T). \end{aligned} \tag{12}$$

Further conservation laws for various unsourced linear combinations give us that $Q_2 = Q_1$ and $U = D = S = C = B$. Now putting all the Yukawa interactions in equilibrium gives the constraints $-H_2 + Q_3 - T = 0$ and $H_1 - Q_3 + T = 0$, and using the conservation of total baryon number (the weak sphalerons are slow and enter only at the end of the calculation) allows the full network to be reduced to just two coupled equations for Q_3 and $H \equiv H_1 + H_2$:

$$\begin{aligned} -3\mathcal{D}_h H - 3\mathcal{D}_q Q_3 + \Gamma_{hf} H + \Gamma_{ss}(28Q_3 - 4H) &= S_H \equiv \frac{1}{2}(S_{H_1} + S_{H_2}) \\ +\frac{3}{2}\mathcal{D}_q H - 9\mathcal{D}_q Q_3 + \Gamma_{ss}(28Q_3 - 4H), &= 0. \end{aligned} \tag{13}$$

where $\mathcal{D}_i \equiv D_i \frac{d^2}{dz^2} + v_w \frac{d}{dz}$. We must keep the strong sphaleron and helicity-flip rates explicitly because in the limit that either becomes infinitely fast, they drive the left-handed quark number, essential for baryogenesis, to zero. The Yukawa interactions, by contrast, simply redistribute the asymmetries between the species, transferring the chemical potential from Higgsinos, where the source is, into the left-handed quark sector. It is well known that the strong sphalerons tend to drive the asymmetry to zero in the massless approximation we are working in [9] and, since $B_L = Q_1 + Q_2 + Q_3 = 14Q_3 - 2H$, this is manifest in eq. (13). For the helicity flips, which are due to the Dirac mass term $\mu\tilde{h}_1\tilde{h}_2$ that connects the two species of Higgsinos, the potential suppression arises from the fact that $S_{H_1} = S_{H_2}$, since $\tilde{h}_1$ and $\tilde{h}_2^c$ are the right- and left-handed components of a Dirac fermion, and thus come with equal and opposite CP phases, as can be seen in eq. (6). If helicity-flipping interactions were in equilibrium this would force $H_1 = -H_2$ and the source for $H_1 - H_2$ would be $S_{H_1} - S_{H_2} = 0$, leading to the trivial solution for all the particle densities.[1]

The equations (13) can be solved by finding the appropriate Green's function, of the form $\mathbf{G}(z-z') = \int \mathbf{G}(p)e^{ip\cdot(z-z')}dp/2\pi$, which is a matrix such that $(H(z), Q_3(z))^T = \int_{-\infty}^{\infty}\mathbf{G}(z-z')(S_H(z'), 0)^T dz'$. However, we do not need the full matrix $\mathbf{G}$, because we are only interested in the linear combination of Q_3 and H that gives the chemical potential for left-handed baryon number, $B_L = 14Q_3 - 2H$. The latter can be determined from the single-component equation

$$B_L(z) = \int_{-\infty}^{\infty} G_B(z-z')S_H(z')dz', \tag{14}$$

where $G_B = -2\mathbf{G}_{11} + 14\mathbf{G}_{21}$. Carrying out the thermal averages in the relativistic limit (for simplicity of presentation–we do not make this simplification to obtain the numerical results), and using the Maxwell-Boltzmann approximation for the distribution functions, the source can be written as

$$S_H = v_w\left(\frac{D_h}{4\langle p_z^2/E\rangle}\Big((m^2\theta')'\langle 1/E\rangle\Big)' - \frac{\Gamma_y}{8}\langle 1/E^2\rangle m^2\theta'\right), \tag{15}$$

where to a good approximation $\langle p_z^2/E\rangle = T$, $\langle 1/E\rangle = 1/(2T+m)$, and $\langle 1/E^2\rangle = (T+m)^{-1}(2T+m)^{-1}$. The appropriate mass m to use here is $|\mathcal{M}_{\chi\pm}|$ (see above eq. (4)), with $\pm$ equal to sign$(\mu - m_2)$: this is the appropriate sign for the mass eigenvalue that goes continuously into that of the chargino ($m = \mu$) in the symmetric phase outside the bubble.

The Green's function is determined by the poles of $G_B(p)$, which are the roots of a quartic polynomial. For $v_w < \sqrt{\Gamma D}$ we can solve perturbatively in v_w, and the result to $O(v_w)$ is

$$G_B(z) = \mathcal{C}\sum_{\pm} \pm k_\pm e^{-k_\pm|z|}\left(1 + v_w(\mathrm{sign}(z)\delta g_\pm - \delta k_\pm z)\right) + O(v_w^2) \tag{16}$$

[1] This statement is true for the force-term contribution to S_H, but the unequal interactions of H_1 and H_2 prevent the spontaneous baryogenesis part of $S_{H_1} - S_{H_2}$ from vanishing. In practice however it is small.

where

$$\begin{aligned} k_\pm^2 &= \frac{28\Gamma_{ss}D_h + 3(2\Gamma_{ss}+\Gamma_{hf})D_q \pm \sqrt{(28\Gamma_{ss}D_h + 3(2\Gamma_{ss}-\Gamma_{hf})D_q)^2 + 16\Gamma_{hf}\Gamma_{ss}D_q^2)}}{18D_q(D_h+D_q)} \\ \delta k_\pm &= \frac{3(3D_h+4D_q)k_\pm^2 - (3\Gamma_{hf}+34\Gamma_{ss})}{6D_q(6D_h+D_q)k_\pm^2 - 56\Gamma_{ss}D_h - 6(\Gamma_{hf}+2\Gamma_{ss})D_q} \end{aligned} \tag{17}$$

and $\mathcal{C}^{-1} = 3(6D_h + D_q)(k_+^2 - k_-^2)$, $\delta g_\pm = (2\delta k_\pm - 1/D_q)/k_\pm - 2k_\pm(\delta k_+ - \delta k_-)/(k_+^2 - k_-^2)$. It is important to go to first order in v_w here because of a cancellation that takes place at $O(v_w^0)$: the Green's function at this order is symmetric about the center of the wall, but the source is approximately antisymmetric.

The baryon asymmetry. Having the solution for the density of left-handed baryon number, it is simple to find the rate of baryon violation due to weak sphalerons in front of the wall: $\dot{n}_B = -9\Gamma_{ws}n_{q_L} = -9\Gamma_{ws}T^2B_L$. The time integral of this rate from $t = -\infty$ until the time the wall passes a given position, where the sphaleron interactions are assumed to switch off, can be converted to an integral of $B_L(z)$ in front of the wall, yielding the baryon density $n_B = (9\Gamma_{ws}T^2/v_w)\int_0^\infty B_L(z)dz$. The baryon-to-photon ratio is therefore

$$\eta \cong \frac{7n_B}{s} = \frac{2835\alpha_W^4}{2\pi^2 v_w g_*}\int_0^\infty B_L(z)\,dz \tag{18}$$

where g_* is the number of degrees of freedom at the phase transition temperature. To compute $\int B_L(z)dz$, we use eq. (14) and (16), where the z integral can be done analytically, leaving a single numerical integral over z'.

To obtain the final result we must still specify the shape and speed of the bubble wall and the values of diffusion constants and interaction rates. For the wall profile, we took the kink solution $v(z) = (v_c/2)\,(1 - \tanh(z/2w))$ with width $w = 1/(\sqrt{\lambda}v_c)$, corresponding to the Higgs potential at the critical temperature $V(v) = \lambda v^2(v - v_c)^2$. We explored a range of values of λ and v_c consistent with the analysis of the phase transition described in refs. [10], giving wall widths between $w = 0.07$ and $w = 0.08$ GeV^{-1}, which should be compared to the typical critical temperature of $T_c = 90$ GeV. The other parameters varied in the ranges $0.003 < \lambda < 0.014$ and 120 GeV $< v_c <$ 210 GeV. For the diffusion constants and decay rates, we take $\Gamma_{ss} = 6000\Gamma_{ws}$ [11], $\Gamma_{ws} = \alpha_w^4 T$ (it has recently been argued that parametrically $\Gamma_{ws} = C\alpha_w^5 T$ [12], but lattice measurements of the rate are consistent with a

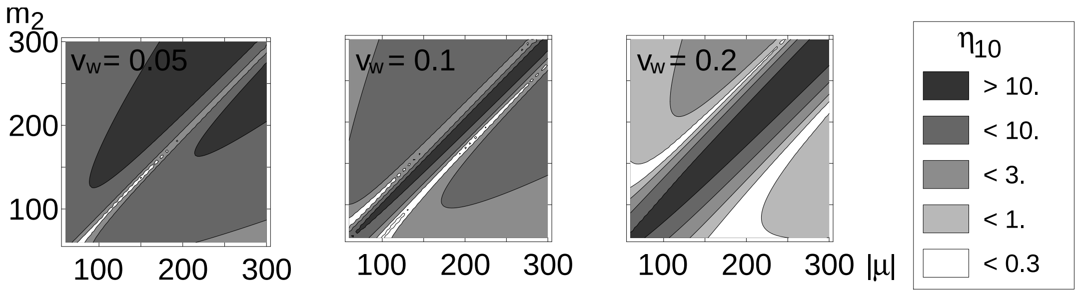

Figure 1: Contours of constant $\eta_{10} = 10^{10}\eta$ in the plane of m_2 (the Wino mass) and μ, in GeV, $v_{\rm wall} = 0.05$, 0.1 and 0.2, $\tan\beta = 2$, $\sin\delta = 1$ and wall width $2w = 14/T$.

value of $C \sim 1/\alpha_w$ [13] so this does not effect the numerical value of our estimate) and we have made the rough estimates $\Gamma_{hf} = 3y^2T/16\pi$, $D_h = 20/T$ and $D_q = 3/T$. Our results are not extremely sensitive to these values. For the wall velocity we took $v_w = 0.01 - 0.2$.

In general, we find that the baryon-to-photon ratio η is sufficiently large only if the CP-violating phase δ of the μ parameter is nearly maximal, $\delta = \pi/2$. In figure 1 we show the dependence of $\eta_{10} = \eta \times 10^{10}$ on the chargino mass parameters m_2 and μ for $\tan\beta = 2$ (favored by studies of the phase transition) $\lambda = 0.0065$, $v_c = 160$ GeV, and $w = 7/T_c$, and several values of the wall velocity. One finds certain parameter values, like $m_2 \sim \mu$ when $v_w \gtrsim 0.1$, where a smaller angle of $\delta \sim 0.1$ could be tolerated. It is usually assumed that such a large phase is excluded by the experimental limits on the neutron electric dipole moment, but this can be evaded if the top and bottom squarks are much lighter than their first generation counterparts, since these determine the size of the loop diagrams that generate the up and down quark EDM's that feed into that of the neutron. This is in some sense natural because a light top squark is also needed for a strong enough phase transition to satisfy the sphaleron bound.

Discussion. We conclude by pointing out some of the important differences between previous related work and ours, and why we consider the present analysis to be more satisfactory. The comparison can be divided into two parts: the derivation of the sourced diffusion equations, and their subsequent solution and estimation of the baryon asymmetry. With respect to the first point we have noted that in the present treatment the source terms and diffusion equations are consistently derived together. In several recent treatments [1, 3, 4], quantum mechanical contributions to various CP-odd currents induced by the wall are derived, and then converted into sources for diffusion equations using an ad hoc prescription introduced in [1]. If such a prescription could be rigorously derived, the derivation should start with a set of transport equations more general than those used here, incorporating quantum mechanical corrections. However any such consistent treatment should reduce to our equations in the classical limit, and this is not the case, as can be seen from the fact that the source terms of [1, 3, 4] are parametrically different from ours. Furthermore, insofar as the thick wall and on-shell quasi-particle approximations we have used are valid, such quantum mechanically derived source terms should be higher order corrections to those derived here.

One example of the different parametric dependence of our source term concerns the variation of $\tan\beta$ in the wall. The source terms discussed in refs. [1, 3, 5] are proportional to $v_1 v_2' - v_1' v_2$, which is zero if v_1/v_2 is constant in the bubble wall. As pointed out in ref. [3] and confirmed by the methods of ref. [10], v_1/v_2 varies at most by only a few percent over the wall in the MSSM. In ref. [5], on the other hand, it is simply assumed that v_1/v_2 changes by 100% in the wall, which gives an unjustifiably large estimate of the baryon asymmetry. We have concentrated on the chargino source because, in our derivation, it is apparently the sole exception to this rule in the MSSM. It escapes the suppression because the chargino is a Dirac particle, whose mass eigenvalues have a spatially varying phase when v_1/v_2 is constant, even though the eigenvalues of $M^\dagger M$ have no such phases.[2]

There are also important discrepancies between our calculation and those in [3, 4], aside from the differences in our respective source terms. First, in both papers strong sphaleron processes are put into equilibrium but the corresponding suppression of the baryon asymmetry is not observed, nor its absence explained. There are small effects related to the quark masses which will prevent B from relaxing to exactly zero in this limit, but there is no recourse to this in [3, 4]. Secondly we find that for $v_w > 0.01$ (so that electroweak sphalerons are still out of equilibrium on the wall passage time scale), the baryon asymmetry

[2]Actually the suppression would still occur if Winos and Higgsinos had identical interactions, since they come with opposite phases. It is the different interaction rates of the two species that undoes the suppression coming from constant v_1/v_2.

is $O(1)$ rather than $O(1/v_w)$ as found by refs. [1, 3, 4]. This can be traced to the fact that the Higgsino decay rate is taken to be zero in the symmetric phase, but non-zero inside the bubbles. In fact we believe it should be the other way around since Higgsinos have much more phase space to decay into massless quarks and light stops in the symmetric phase than in the broken phase where the top quark is heavy. But regardless of phase space, helicity-flipping scattering processes are still fast enough in the symmetric phase to give a significant rate of Higgsino damping.

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