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# Non-perturbative study of QCD on a 2 + 2 anisotropic lattice

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We present preliminary results for the non-perturbative determination of the parameters in a 2+2 anisotropic action. The parameters are required to restore Lorentz invariance in long-distance correlators, using the static inter-quark potential. Comparison with analytical results is made and further applications are discussed.

## 1. INTRODUCTION

QCD on 3+1 anisotropic lattices [1,2,3] has proved to be a successful tool for the non-perturbative investigation of interesting physical phenomema including the glueball spectrum, $b\bar{b}$ states, gluon strings, the spectral density. The method is successful for processes where the hadronic final state is static in the center of mass frame but it is perhaps less advantageous for processes in which the hadron momentum in the final state is non-negligible compared to its mass. In such cases, independent fine spatial directions for each momentum degree of freedom are needed to describe the system precisely. This increases dramatically the computational cost and, when considering an anisotropic approach, the difficulties linked with tuning the parameters.

For processes where the final state hadron is strongly peaked at a momentum approximately equal to its mass, a 2+2 lattice discretization may provide a solution. It offers the possibility to extend the range of final state momenta accessible to lattice calculations and enhance the overlap with experimental data. In addition, the study of high-momentum glueball form factors could benefit from such a program.

A 2+2 discretisation of the gauge action was proposed in Refs [4,5] and the parameters in the action were determined at the one-loop level in perturbation theory. In this talk a non-perturbative determination of these parameters is discussed. A 2+2 Wilson-like fermion action is discussed in Ref [7].

## 2. YANG-MILLS THEORY ON A 2+2 LATTICE

We start from the simple $1\times 1$ plaquette action

$$S_{\rm g} = \beta \sum_{x,\mu,\nu} c_{\mu\nu}[1 - \frac{1}{2N_c}{\rm Tr}\,(P_{\mu\nu}(x) + P^{\dagger}_{\mu\nu}(x))], \quad (1)$$

defined in a fixed hypercubic physical volume $V = L^4$. The ultraviolet cutoff is larger in the $z, t$ directions and $\xi = a_c/a_f$ is the asymmetry ratio. In the following, the coarse and fine lattice spacings are denoted $a_c$ and $a_f$ respectively. The coefficients $c_{\mu\nu}$ in Eq. 1 must be fixed by tuning their values so as to recover the correct continuum theory. This can be done either perturbatively [5] or non-perturbatively [6]. The perturbative expansion is written

$$c_{\mu\nu} = \begin{cases} \xi^2(1+\eta^{(1)}_{ff}g^2 + O(g^4)) & \text{f-f;} \\ 1+\eta^{(1)}_{cf}g^2 + O(g^4) & \text{c-f;} \\ \frac{1}{\xi^2}(1+\eta^{(1)}_{cc}g^2 + O(g^4)) & \text{c-c;} \end{cases} \quad (2)$$

and the calculation of the one-loop coefficients, $\eta^{(1)}_{\mu\nu}$ was the subject of Ref [5]. We note in passing that the tree-level values of the $c_{\mu\nu}$ are

$$c_{cc} = 1/\xi^2,\ c_{cf} = 1,\ \text{and}\ c_{ff} = \xi^2. \quad (3)$$

A non-perturbative determination of these coefficients is the focus of this talk.

*Talk given by Giuseppe Burgio

We choose to fix the coefficients to restore rotational invariance for the static inter-quark potential. The procedure is, in principle, the same as for a 3+1 discretisation. However, for 3+1 any choice of the parameters leads to a well-defined, albeit unknown, scale ratio $\xi$. In this case, a non-perturbative determination of $c_{\mu\nu}$ may be unnecessary if, as is true for some applications, one is not interested in the physical mass-scale ratio between the coarse and fine directions. If such a determination is necessary, both the coarse-fine and coarse-coarse Wilson loops must be measured. The same is true for the 2+2 case, where the potential between spatially separated color sources, measured using a fine temporal axis determines just one of the two free parameters in the action. The only drawback for the 2+2 discretisation compared to the 3+1 is that such a determination must be done *a priori*. Rotational invariance of the static inter-quark potential must be imposed for all combinations of source separation and transfer matrix axes. Alternatively, the slope of dispersion relations of physical (color singlet) states can be measured. This requirement can be observed in perturbation theory by examining the static potential, given by

$$V(R) = \lim_{T\to\infty} -\frac{1}{T} \log < W(R,T) > . \tag{4}$$

Expanding Eq. 4 at order $g^4$ and handling the resulting Bessel functions for large $T$, it is easy to check that only the coefficients $\eta^{(1)}_{\mu\nu}$ carrying one time index survive. A fine temporal axis will therefore give no information about $\eta^{(1)}_{cc}$, since constraining potential ratios in different directions leads to a constraint only on $c_{cf}/c_{ff}$ for 2+2 and no constraint for 3+1. This constraint degeneracy is illustrated in Fig. 1. The solid and dashed lines represent the constraints on the spatial potential: firstly that $V(n,0,0) = V(0,0,\xi n)$ and secondly that $V(n,n,0) = V(n,0,\xi n)$. The outer solid and dotted lines are the one sigma errors from a fit to these constraints. Within the errors shown the lines are degenerate and cannot provide a non-perturbative determination of the $c_{\mu\nu}$. Conversely, considering a coarse temporal axis results in no dependence on $\eta^{(1)}_{ff}$. For 2+2, only the ratio, $c_{cc}/c_{cf}$ will be constrained. But

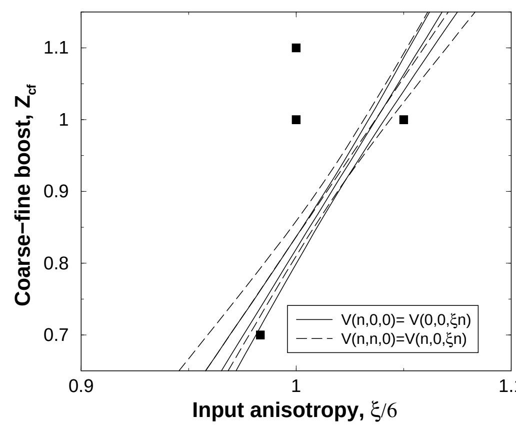


Figure 1. Degenerate constraint curves from the $SU(3)$ spatial potential.

now, this condition combined with the previous determination fixes the parameters uniquely. For 3+1 this is the only meaningful condition. With a coarse temporal axis obtaining good fits can be more challenging. However, we find that with reasonable statistics good $\chi^2/d.o.f.$ is achieved. This non-perturbative determination of the $c_{\mu\nu}$ is work in progress.

## 3. PRELIMINARY RESULTS

As a first step we show that the tuning described in the previous section is actually not as onerous as it might sound. Fig. 2 shows the potential in different directions, calculated using the tree-level coefficients. Even at the tree-level there is clearly no dramatic breaking of Lorentz invariance. This indicates that the tuning of parameters is mild and the scheme can be of practical use.

To demonstrate how powerful this scheme can be, Fig. 3 shows the $SU(3)$ potential calculated for $\xi = 6$ on a $8^2 \times 48^2$ volume, in the range $R = 0.04-1.6 fm$. These results were obtained after two days of workstation CPU time. The data show that the short distance term can already be calculated with very high precision. Longer runs and bigger volumes could easily allow one to reach

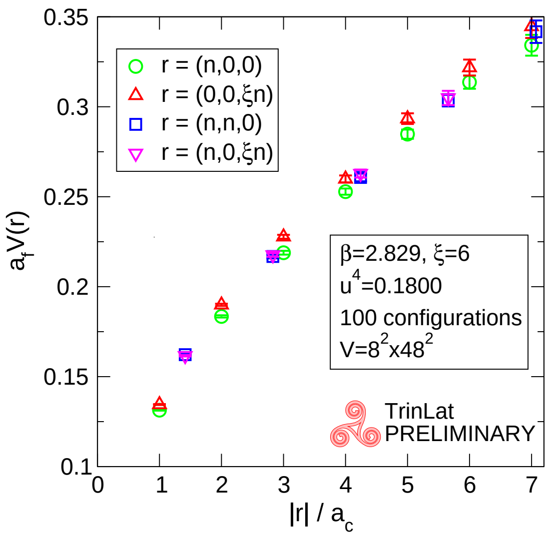


Figure 2. The $SU(3)$ static potential for $\xi = 6$ with tree-level coefficients $c_{\mu\nu}$, given in Eq. 3.

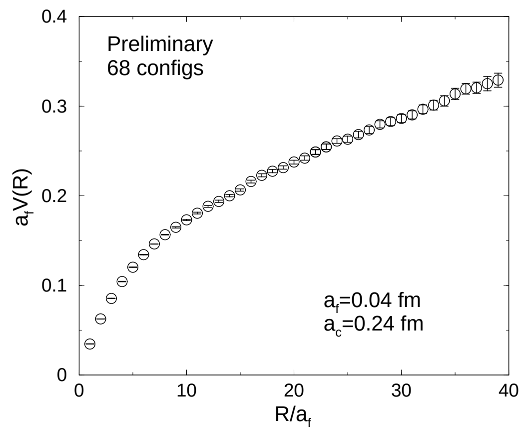


Figure 3. The $SU(3)$ static potential determined with the one-loop values of the coefficients, $c_{\mu\nu}$ [5]. The potential is measured over the range $R = 0.04 - 1.6fm$, for $\xi = 6$. The errors on the points shown are smaller than the symbol size.

even higher precision. The method is therefore immediately applicable to a very high resolution determination of the short-range inter-quark potential, which is of phenomenological interest.

## 4. OUTLOOK AND DEVELOPMENTS

We have shown how 2+2 lattice QCD can be relevant to useful physics. Although parameter tuning is necessary, both the perturbative [5] and the non-perturbative determinations are feasible. The results for the static inter-quark potential demonstrate that this scheme can be a useful tool for exploring energy ranges higher than those currently available to lattice simulations. In the gauge sector the scheme can naturally be applied to a calculation of glueball form factors, and this is under investigation. An extension to the quark action allows further interesting phenomenological applications and is described in Ref [7].

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## REFERENCES

1. M. Alford, T. Klassen, G. P. Lepage, Nucl. Phys. B 496 (1997) 377-407.
2. C. Morningstar and M. Peardon, Phys. Rev. D 56 (1997) 4043-4061.
3. C. Morningstar and M. Peardon, Phys. Rev. D 60 (1999 034509.
4. Trinlat Collaboration (Giuseppe Burgio et al.), Nucl. Phys. B Proc. Suppl. 119 (2003) 458-460.
5. Trinlat Collaboration (Giuseppe Burgio et al.), Phys. Rev. D 67 (2003) 114502.
6. M. Alford et al, Phys. Rev. D 63 (2001) 074501.
7. A. Feo, G. Burgio, M. Peardon, S.M. Ryan, these proceedings.