School of Mathematics

Module MA3431 — Classical field theory 2011-12
(JS Theoretical Physics)

Lecturer: Dr. Nigel Buttimore

Requirements/prerequisites: prerequisite: MA2342

Duration: Michaelmas term, 11 weeks

Number of lectures per week: 3 lectures including tutorials per week

Assessment: Assignments will not contribute formally to the final result except in borderline cases.

ECTS credits: 5

End-of-year Examination: This module will be examined jointly with MA3432 in a 3-hour examination in Trinity term, except that those taking just one of the two modules will have a 2 hour examination. However there will be separate results for MA3431 and MA3432.

Description: *Rationale and Aims* The purpose of Module MA3431 is to outline the properties of a classical field theory that relate in particular to scalar and vector fields, to point out features of the tensor calculus suitable for the description of relativistic non-quantum field theories, and to indicate the importance of symmetry and invariance principles in the development of conservation laws for energy, momentum and other conserved quantities.

The module is mandatory for third year undergraduate students of theoretical physics but may optionally be taken by third or fourth year undergraduate students of mathematics. Postgraduate students from other institutions have taken the module in the past. The module forms an element of the undergraduate programme in theoretical physics being built upon prerequisite first and second year courses in classical dynamics and mathematics and leading to courses in the fourth and final year including quantum field theory.

From a teaching point of view, the intention of the lecturer is to indicate how powerful analytical and formal methods can be invoked to understand and solve many problems in mathematical physics. A further intention is to provide a sense of the important rôle played by field theories, particularly electrodynamics, in the development of theoretical physics and its applications.
Content

- Classical Lagrangian for a discrete system, Lagrangian density for a field
- Hamilton's variational principle, Lorentz invariance, time & space reversal
- Covariant field theory, tensors, scalar fields and the four-vector potential
- The Lorentz force, charged particle interaction, antisymmetric field tensor
- Euler-Lagrange equation for an interacting vector field; Maxwell equations

- Gauge invariance, the Lorenz gauge, charge conservation & four-currents
- Canonical stress tensor; conserved, traceless and symmetric stress tensor
- Particle & field energy-momentum and angular momentum conservation

Indicative textbooks:

1. Classical Electrodynamics, J. David Jackson, John Wiley (3rd edition) 1998
2. Classical Theory of Fields, L. D. Landau & E. M. Lifshitz, Heinemann, 1972
3. Classical Field Theory, Francis E. Low, J. Wiley and Sons (1st edition) 1997
4. Module website: `http://www.maths.tcd.ie/~nhb/sjscft.php`

Learning Outcomes: On sucessful completion of this module, students will be able to:

- describe how to find the equation of motion for a scalar field using a given Lagrangian density
- calculate the stress tensor and evaluate its four divergence, relating it to a conservation law
- employ a variational principle to find the relativistic dynamics of a charged particle interacting with an electromagnetic potential
- use the Euler-Lagrange equation to show how a Lorentz scalar Lagrangian density with an interaction term leads to the Maxwell equations
- explain the concepts of gauge invariance and tracelessness in the context of the stress tensor of a vector field
- demonstrate how the divergence of the symmetric stress tensor is related to the four current density of an external source.

October 11, 2011