School of Mathematics

MA342J — Introduction to Modular Forms 2011-12
(JS & SS Mathematics)

Lecturer: Prof. M. Vlasenko

Requirements/prerequisites:

Duration: Hilary Term (11 weeks)

Number of lectures per week: 3 including tutorials

Assessment: Maximum of (70% exam + 30% homework) and 100% exam.

ECTS credits: 5

End-of-year Examination: 2-hour exam in Trinity Term

Description:

Classical (or "elliptic") modular forms are functions in the complex upper half-plane which transform in a certain way under the action of a discrete subgroup of $SL(2, R)$ such as $SL(2, Z)$. There are two cardinal points about them which explain why modular forms are interesting. First of all, the space of modular forms of a given weight on a given group is finite dimensional and algorithmically computable, so that it is a mechanical procedure to prove any given identity among modular forms. Secondly, modular forms occur naturally in connection with problems arising in many areas of mathematics, from pure number theory and combinatorics to differential equations, geometry and physics. We start with the analytic base of the theory of modular forms, prove finiteness of dimensions and construct enough examples, such as Eisenstein series, theta series and eta-products. In the second part of the course we study families of elliptic curves, view modular curves as their moduli spaces and show how modular forms naturally arise in this context. At the end we discuss application of modular forms to Fermat's last theorem.

- Introduction: modular forms arising in elementary and advanced geometry, combinatorics and physics
- A supply of modular forms
  - Definitions and first examples: Eisenstein series
  - Further examples: the discriminant function and cusp forms
  - Computation of dimensions of the spaces of modular forms
  - Modular groups and modular forms of higher levels, modular curves, dimension formulas and Riemann-Roch Theorem
  - More examples: theta series
- Families of elliptic curves and modular forms
- Hecke operators and Hecke Eigenforms

- Discussion of Fermat's last theorem and modularity theorem for rational elliptic curves

There is a web site for this module at `http://www.maths.tcd.ie/~vlasenko/MA342J.html`

Literature:

1. J.-P. Serre, A Course in Arithmetic, Chapter VII
2. J.S. Milne, Modular Functions and Modular Forms
3. From Number Theory to Physics, Introduction to Modular Forms by D.Zagier
4. The 1-2-3 of Modular Forms, Elliptic Modular Forms and Their Applications by D.Zagier
5. F. Diamond, J. Shurman, A First Course in Modular Forms
6. Yu.I.Manin, A.A.Panchishkin, Introduction to Number Theory, Part II: Ideas and Theories, Chapters 6 and 7

Learning Outcomes: On successful completion of this module, students will be able to:

- give examples of modular forms, namely Eisenstein series, theta series and eta-products;
- use dimension formulas to prove various identities between modular forms;
- analyze families of elliptic curves with the help of modular forms.

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