School of Mathematics

MA3415 — Introduction to Lie Algebras 2011-12
(JS & SS Mathematics
SS Two-Subject Moderatorship)

Lecturer: Prof. R. Tange

Requirements/prerequisites: MA2215, MA2322 desirable, concurrent registration for MA3411 also desirable.

Duration: 11 weeks (Michaelmas Term)

Number of lectures per week: 3 including tutorials

Assessment:

ECTS credits: 5

End-of-year Examination: 2 hour exam in Trinity Term (April/May)

Description:
Lie algebras should be thought of as the infinitesimal analogue of groups. Lie theory is an important and very active branch of mathematics with many links to other areas: geometry, representation theory, mathematical physics amongst others.
Topics:

- Definitions, small examples, classical Lie algebras, subalgebras, ideals and homomorphisms.
- Nilpotent Lie algebras, Engel's theorem.
- Solvable Lie algebras, Radical and semisimplicity, Lie's theorem, the Killing form and Cartan's criterion.
- Representations of Lie algebras, Representation theory of the Lie algebra $sl(2)$.
- The structure of semisimple Lie algebras, Jordan-Chevalley decomposition.
- The classification of semisimple Lie algebras, possibly with some details: toral subalgebras, root systems ...

Refer to `http://www.maths.tcd.ie/~rtange/teaching/lie_algebras/Lie_algebras.html` for further details.

Learning Outcomes: On sucessful completion of this module, students will be able to:

- Give the definitions of: Lie algebra, homomorphism of Lie algebras, subalgebra, ideal, derivation, centre, representation of a Lie algebra, submodule, irreducible module, homomorphism of g-modules, the Killing form of a Lie algebra and the trace form of a classical Lie algebra, the derived and descending central series of a Lie algebra, nilpotent Lie algebra, solvable Lie algebra, solvable radical, semisimple and simple Lie algebra, maximal toral subalgebra, root system, irreducible root system.

- Give the definitions of and calculate with the classical Lie algebras.
- Describe the construction of the irreducible representations of sl_2.
- State Engel's Theorem, Lie's Theorem and Cartan's Criterion.
- Describe the direct sum decomposition into simple ideals and the Jordan-Chevalley decomposition for semisimple Lie algebras.
- Indicate how root systems correspond to semisimple Lie algebras and give the root space decomposition and root system of sl_n.

November 2, 2011