School of Mathematics

Module MA3411 — Abstract Algebra I 2011-12
(JS & SS Mathematics, JS & SS Two-subject Moderatorship)

Lecturer: Dr. David Wilkins

Requirements/prerequisites: prerequisite: MA1214

Duration: Michaelmas term, 10 weeks

Number of lectures per week: 3 lectures per week

Assessment:

ECTS credits: 5

End-of-year Examination: This module will be examined jointly with MA3412 in a 3-hour examination in Trinity term, except that those taking just one of the two modules will have a 2 hour examination.

Description:

Further detailed information about the course:
`http://www.maths.tcd.ie/~dwilkins/Courses/MA3411/`

Basic principles of group theory.

Basic principles of ring theory.

Basic properties of polynomial rings with coefficients in a field. Gauss's Lemma concerning products of primitive polynomials. Eisenstein's irreducibility criterion for polynomials.

Basic properties of field extensions. The Tower Law. Basic properties of algebraic extensions. Proof that the degree of a simple algebraic extension is equal to the degree of the minimum polynomial of the adjoined element generating the extension.

Solvability and Insolvability of Ruler and Compass Constructions.

Splitting fields. Existence and isomorphism theorems for splitting fields. Normal extensions. Separability. Basic properties of finite fields. The Primitive Element Theorem. The Galois Group of a finite Field Extension. The Galois Correspondence.

Procedures for determining the roots of quadratic, cubic and quartic polynomials from the coefficients of such polynomials. Galois groups of polynomials of low degree.

The class equation of a finite group. Cauchy's theorem concerning the existence of elements of prime order in finite groups. Simple groups. Solvable groups.

Galois's Theorem concerning the solvability of polynomial equations. A quintic polynomial that is not solvable by radicals.

Textbooks:

1. Ian Stewart, Galois Theory, Third Edition, Chapman and Hall. London, 2004.

Learning Outcomes: On sucessful completion of this module, students will be able to:

- justify with reasoned logical argument basic properties of polynomial rings and finite field extensions
- specify and justify with reasoned logical argument aspects of the relationships between polynomials with coefficients in some ground field, finite extensions of that ground field, and the groups of automorphisms of those field extensions
- determine the Galois groups of appropriately-chosen polynomials of low degree

April 2, 2012