School of Mathematics

Module MA22S1 — Multivariable calculus for science 2011-12
(SF Science
SF Human Genetics
SF Physics & Chemistry of Advanced Materials
SF Medicinal Chemistry
SF Chemistry with Molecular Modelling)

Lecturer: Prof. M. Vlasenko

Requirements/prerequisites: prerequisite: MA11S2

Duration: Michaelmas term, 11 weeks

Number of lectures per week: 3 lectures plus 1 tutorial per week

Assessment:

ECTS credits: 5

End-of-year Examination: 2 hour examination in Trinity term.

Description: As its name suggests, multivariable calculus is the extension of calculus to more than one variable. That is, in single variable calculus one studies functions of a single independent variable $y = f(x)$. In multivariable calculus we will study functions of two or more independent variables $z = f(x, y)$, $w = f(x, y, z)$, etc. These functions are essential for describing the physical world since many things depend on more than one independent variable. For example, in thermodynamics pressure depends on volume and temperature, in electricity and magnetism the magnetic and electric fields are functions of the three space variables (x, y, z) and one time variable t.

Multivariable calculus is a highly geometric subject. We will relate graphs of functions to derivatives and integrals and see that visualization of graphs is harder but more rewarding and useful in several geometric dimensions. By the end of the module you will know how to differentiate and integrate functions of several variables.

As the Newton-Leibniz rule relates derivatives to integrals in single variable calculus, in multivariable calculus this is done by the three major theorems (Green's, Stokes' and Gauss'). These are considered in MA22S2 in the second term.

Syllabus

- Areas and Lengths in Polar Coordinates
- Lines and Planes, Cylinders and Quadric Surfaces
- Vector-Valued Functions and Space Curves
- Functions of Several Variables, Partial Derivatives
- Tangent Planes and Linear Approximations
- Directional Derivatives and the Gradient Vector

- Maxima and Minima, Lagrange Multipliers
- Double Integrals Over Rectangles and over General Regions
- Double Integrals in Polar Coordinates
- Triple Integrals in Cylindrical and Spherical Coordinates
- Change of Variables, Jacobians
- Vector Fields and Line Integrals

See `http://www.maths.tcd.ie/~vlasenko/MA22S1.html`

Recommended reading: Calculus, H. Anton, I. Bivens, S. Davis.

Learning Outcomes: On successful completion of this module, students will be able to:

- write equations of planes, lines and quadric surfaces in 3-space
- determine the type of a conic section and write a change of coordinates turning a quadratic equation into its standard form
- use cylindrical and spherical coordinate systems
- write equations of a tangent line, compute unit tangent, normal and binormal vectors and curvature at a given point on a parametric curve; compute the length of a portion of a curve
- apply the above concepts to describe motion of a particle in the space
- calculate limits and partial derivatives of functions of several variables
- write local linear and quadratic approximations of a function of several variables, write the equation of the plane tangent to its graph at a given point
- compute directional derivatives and determine the direction of maximal growth of a function using its gradient vector
- use the method of Lagrange multipliers to find local maxima and minima of a function
- compute double and triple integrals by application of Fubini's theorem or use of change of variables
- use integrals to find quantities defined via integration (such as average, area, volume, mass).

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