School of Mathematics

Module MA2224 — Lebesgue integral   2011-12
(SF Mathematics, SF Two-subject Moderatorship)

Lecturer: Professor Richard Timoney

Requirements/prerequisites: prerequisite: MA2223 (or 121)

Duration: Hilary term, 11 weeks

Number of lectures per week: 3 lectures including tutorials per week

Assessment: Tutorial work 15%.

ECTS credits: 5

End-of-year Examination: This module will be examined jointly with MA2223 in a 3-hour examination in Trinity term, except that those taking just one of the two modules will have a 2 hour examination. However there will be separate grades for MA2223 and MA2224.

Description: The basics of the theory of the Lebesgue integral and Lebesgue measure on the real line. Monotone and dominated convergence theorems.
   In more detail:

- Countable versus uncountable sets; inverse images; characteristic functions; boolean algebra for subsets.

- Algebras of subsets of the real line; length measure on the interval algebra; finite-additivity, subadditivity and countable-additivity; outer measure; Lebesgue measurable sets; extension to sigma algebra; Borel sigma algebra.

- Lebesgue measurable functions; simple functions; integrals for non-negative functions; limits of measurable functions and the monotone convergence therorem; Lebesgue integrable functions; generalisation of the Riemann integral (for continuous functions on finite closed intervals).

- Fatou's lemma; dominated convergence theorem; integrals depending on a parameter; almost everywhere.

   See `http://www.maths.tcd.ie/~richardt/MA2224` for additional information.

Learning Outcomes: On successful completion of this module, students will be able to:

- discuss countable sets, characteristic functions and bolean algebras;

- state and prove properties of length measure, outer measure and Lebesgue measure for subsets of the real line and establish measurability for a range of functions and sets;

- define the Lebesgue integral on the real line and apply basic results inclucing convergence theorems.

January 15, 2012