School of Mathematics

MA2201 — Competition Problem Solving 2011-12
(SF Mathematics)

Lecturer: Timothy Murphy

Requirements/prerequisites:

Duration: 12 weeks

Number of lectures per week: 3 including discussion classes

Assessment: Continuous assessment.

ECTS credits: 5

End-of-year Examination: No exam, result based on assessments..

Description:

Draw a line through the centre of a cube so that the sum of the squares of its distances from the vertices is a) minimal, b) maximal.

The aim of this one-semester course is to train the student to tackle problems like the one above. The problems will be chosen from all branches of mathematics: number theory, linear algebra, real and complex function theory, differential equations, etc.

The course is experimental, and will rely to a large extent on input from those taking the course.

The format of the course will be this: Each week some 15 problems will be presented, of different orders of difficulty. The simplest problem (like the one above) will rely on standard tools. The harder problems will contain a trick or quirk, but will not go outside the student's knowledge. And each week there will be one difficult problem (which will probably not have been solved by the lecturer).

The lectures will hopefully reduce to discussions of the way in which the problems have been or might be solved.

In addition, as an assignment during the Reading Week, each student will be given a problem which almost certainly does go outside his or her knowledge, and which will require some reading to understand it. This assignment will count towards the final mark for the course.

Here is an example: *How many handles has the Riemann surface of the function* $w = \sqrt{1+z^n}$*?*

The weekly problems will appear (before the start of the week) in the folder `http://www.maths.tcd.ie/pub/Maths/Courseware/ProblemSolving/` on the Maths web-site. You may also find some relevant reading matter in the folder, as well as a list of books and online problem-solving sites.

In addition there will be a mailing list (or forum), where I hope students will put forward their thoughts on the problems, and some of the ideas they have tried. I will try to answer there any queries that arise.

A few words about the thinking behind the course.

There is an annual International Maths Competition (IMC), usually held in Eastern Europe,

which continues at university level the IMO (International Mathematical Olympiad) for schools. The School of Maths has taken part in the IMC on a number of occasions (largely due to the enthusiasm of Josh Tobin), and has not disgraced itself. There are also similar competitions in Computing and in Logic. This course might be described as a training ground for such competitions.

But it also reflects a movement in mathematical teaching (exemplified to some extent by the Project Maths Leaving Cert syllabus) away from what V.I. Arnold, one of the exponents of problem-solving, calls the definition-theorem-proof approach to mathematics. (You can read Arnold's rather wild article on Teaching Mathematics in the folder mentioned above on the departmental web-site.)

According to Arnold: *Mathematics is a part of physics. Physics is an experimental science, a part of natural science. Mathematics is the part of physics where experiments are cheap.*

Learning Outcomes: On successful completion of this module, students will be able to:

- Tackle competition problems with more confidence;
- Have a small collection of low-level resources at hand, eg generating functions, the pigeonhole principle, the principle of inclusion/exclusion, Fermat's Little Theorem and its extension, the Lagrange multiplier method;
- Have some possible strategical tools available, eg the Penultimate Step technique, where one asks what the last step might be if a problem were to be solved;
- Present a "micro project" involving a problem in an unfamiliar branch of mathematics.

November 5, 2011