School of Mathematics

Module MA1214 — Introduction to group theory 2011-12
(JF Mathematics, SF Theoretical Physics & JF Two-subject Moderatorship)

Lecturer: Prof. R. Tange

Requirements/prerequisites: prerequisite: MA1111

Duration: Hilary term, 11 weeks

Number of lectures per week: 3 lectures including tutorials per week

Assessment:

ECTS credits: 5

End-of-year Examination: 2 hour examination in Trinity term.

Description: See `http://www.maths.tcd.ie/~rtange/teaching/group_theory/group_theory.html` for more details.

The main source for this course is the book Modern Algebra: An Introduction, John Wiley & Sons by John R. Durbin.

Tentative syllabus: Sets and maps. Binary relations, equivalence relations, and partitions. Semigroups, monoids, and groups. Integer division; Z_d as an additive group and a multiplicative monoid. Remainder modulo n and integer division.

The symmetric group S_n. Parity and the alternating group. Generators for S_n.

Subgroups Matrix groups: GL_n, SL_n, O_n, SO_n, U_n, SU_n. The dihedral groups D_n and symmetries of the cube.

Cosets and Lagrange's Theorem. Additive subgroups of Z. Greatest common divisor.

Normal subgroups and quotient groups. Homomorphisms and the first isomorphism theorem for groups. Multiplicative group Z_n^*, Fermat's little theorem and the Chinese Remainder Theorem.

Group actions. A Sylow theorem. The classification of finite abelian groups.

Possible extra topic: The relation beween $SU(2)$ and quaternions.

Learning Outcomes: On successful completion of this module, students will be able to:

- Apply the notions: map/function, surjective/injective/bijective/invertible map, equivalence relation, partition.

  Give the definition of: group, abelian group, subgroup, normal subgroup, quotient group, direct product of groups, homomorphism, isomorphism, kernel of a homomorphism, cyclic group, order of a group element.

- Apply group theory to integer arithmetic: define what the greatest common divisor of two nonzero integers m and n is compute it and express it as a linear combination of n and m using the extended Euclidan algorithm; write down the Cayley table of a cyclic group $\mathbb{Z}_n$ or of the multiplicative group $(\mathbb{Z}_n)^\times$ for small n; determine the order of an element of such a group.

- Define what a group action is and be able to verify that something is a group action.

  Apply group theory to describe symmetry: know the three types of rotation symmetry axes of the cube (their "order" and how many there are of each type); describe the elements of symmetry group of the regular n-gon (the dihedral group D_{2n}) for small values of n and know how to multiply them.

- Compute with the symmetric group: determine disjoint cycle form, sign and order of a permutation; multiply two permutations.

- Know how to show that a subset of a group is a subgroup or a normal subgroup. State and apply Lagrange's theorem. State and prove the first isomorphism theorem.

April 1, 2012