School of Mathematics

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(JS & SS Mathematics JS & SS TSM)

Lecturer: Dr. Eleanor Denny

Requirements/prerequisites:

Duration: Michaelmas term, 11 weeks

Number of lectures per week: 3 lectures including tutorials per week

Assessment: There will be one term test in Michaelmas term which will be worth 10% of the final grade.

ECTS credits: 5

End-of-year Examination: This module will be examined jointly with MA3482 in a 3-hour examination in Trinity term, except that those taking just one of the two modules will have a 2 hour examination. However there will be separate results for MA3481 and MA3482.

Description: *Aims:* This module enables students to develop an in-depth understanding of the mathematics of financial instruments and introduces the economic applications of stochastic calculus.

Introduction

This course provides a detailed study of the financial, probabilistic, and statistical frameworks essential to understanding the modern theory and practice of financial options and derivatives pricing.

The first part of the module will be concerned with the basics of forwards and futures prices and will then move on to more complex methods of pricing derivative products. The basic principles and context of discrete and continuous time market theory will underpin the second section of the course, culminating in the pricing of various kinds of vanilla and exotic options. The remaining topics will be drawn from: an overview of the Brownian Motion process, including an elementary treatment of stochastic differential equations; modelling stock prices using geometric Brownian Motion; Ito's lemma; the Black-Scholes partial differential equation; and the Greek letters.

Course Structure and Assessment

This module consists of two lectures and one tutorial per week. The tutorial sessions will be used to work through examples and problem sheets.

There will be one term test in Michaelmas term which will be worth 10% of the final grade.

Learning Outcomes:

On successful completion of this module, students will be able to:

• Describe the characteristics and categorise the uses of a wide variety of financial options, futures and derivatives

- Prove and apply the CAPM and arbitrage theorem for the pricing of assets
- Demonstrate the precise mathematical detail of the definition and construction of the Ito integral and assess its uses
- Explain the Black-Scholes methodology, construct its PDE, and illustrate its application in deriving option prices in continuous time models.
- Explain and appraise the different measures for calculating the sensitivity of derivative prices to underlying conditions

Course Outline:

- 1. Futures markets
- 2. Interest rates
- 3. Determination of Forward and Futures prices
- 4. Interest rate Futures
- 5. Introduction to options
- 6. Binomial Trees
- 7. An introduction to stochastic calculus
 - a) Wiener Processes and Brownian Motion
 - b) How stochastic calculus differs from standard calculus
 - c) Taylor series expansion
- 8. Derivation of Ito's Lemma
- 9. Black Scholes Merton method of derivative pricing
- 10. The Greeks

Required Textbook

The required text book is John C. Hull's "Options, Futures and Other Derivatives", published by Pearson Prentice-Hall. Any edition will do but the seventh edition is preferable. There will be exercises set from this text book throughout the module so students will be at a significant disadvantage if they do not have a copy of this text.

There are numerous copies of this textbook in the Lecky Library. This is a fairly basic textbook but deals with the essentials well. It will be supplemented by lecture notes and handouts throughout the course.

Additional Reading

• Cvitanić, J & Zapatero, F. (2004), An Introduction to the Economics and Mathematics of Financial Markets, MIT press.

- Neftci, S., (2000), An Introduction to the Mathematics of Financial Derivatives, Academic Press.
- Steele, M.J. (2000), Stochastic Calculus and Financial Applications, Springer.
- Karatzas, I. and Shreve, S. (1991), Methods of Mathematical Finance, Springer.
- Dumas, B. & Allaz B. (1995), Financial Securities: Market Equilibrium and Pricing Methods, Chapman and Hall.
- Nielsen, L. T. (1999), Pricing and Hedging of Derivative Securities, Oxford University Press.
- Higham, D.J. (2004), An Introduction to Financial Option Valuation: Mathematics, Stochastics and Computation, Cambridge University Press.
- Brzezniak, Z & Zastawniak, T (1999), Basic Stochastic Processes: A Course Through Exercises, Springer.

Evaluation

Student feedback and evaluation of this course will be requested in both Michaelmas and Hilary terms and students are actively encouraged to participate in this process. Lecturer contact details Dr. Eleanor Denny Room 3016 Email: dennye@tcd.ie Office Hours: 11am to 1pm Mondays

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