School of Mathematics

MA2215 — Fields, rings and modules 2010-11
(SF Mathematics
SF Two-Subject Moderatorship)

Lecturer: Dr. R. Levene

Requirements/prerequisites: MA1214

Duration: Michaelmas Term (11 weeks)

Number of lectures per week: 3 including tutorials

Assessment: Regular assignments and tutorial work.

ECTS credits: 5

End-of-year Examination: 2 hour end of year examination in Trinity Term

Description:

For more details consult the website: `http://www.maths.tcd.ie/~levene/2215`

Last year, you met several algebraic structures: groups, fields, vector spaces and sets of matrices. In this course we'll start by studying rings, which come about when you consider addition and multiplication (but not division) from an abstract point of view. If we throw division into the mix, then we get the definition of a field. We'll look at how one field can be extended to get a larger field, and use this theory to solve some geometric problems that perplexed the Greeks and remained unsolved for 2,000 years. We'll also talk about modules over a ring, which generalise the idea of a vector space over a field.

Syllabus

1. Rings; examples, including polynomial rings and matrix rings. Subrings, homomorphisms, ideals, quotients and the isomorphism theorems.

2. Integral domains, unique factorisation domains, principal ideal domains, Euclidean domains. Gauss' lemma and Eisenstein's criterion.

3. Fields, the field of quotients, field extensions, the tower law, ruler and compass constructions, construction of finite fields.

4. Modules and examples. Direct sum decompositions and applications.

Textbooks:

- Peter J. Cameron, Introduction to Algebra (second edition) covers practically all of the course.

- John R. Durbin, Modern Algebra, An Introduction covers everything except modules.

Learning Outcomes: On sucessful completion of this module, students will be able to:

- State and apply the definition, and state and prove simple properties of, and give examples of: rings, subrings, zero elements, zero-divisors, the unity of a unital ring, division rings, invertible elements, commutative rings, integral domains, direct sums of two rings, ideals, quotient rings, integral domains, unique factorisation domains, principal ideal domains, Euclidean domains, fields, homomorphisms, isomorphisms, isomorphic rings.
- Describe, give simple properties of, and perform calculuations using the following examples: the ring of integers modulo n, the quaterions, the complex numbers, the Gaussian integers, the reals, the rationals, the ring of integers and its subrings, zero rings, the field of fractions of a ring, the ring M(n,R) where R is a ring, the ring R[x], field extensions F(a).
- State and apply the ring isomorphism theorems, and the correspondence theorem.
- Give the proof of the first isomorphism theorem.
- In an integral domain, state and apply the definition, and state and prove simple properties of, and give examples of:

  units, associates, divisibility, irreducible elements, principal ideals, greatest common divisors.
- State and apply Gauss' lemma.
- Prove that a principal ideal domain is a unique factorisation domain, and that a Euclidean domain is a principal ideal domain.
- Perform and apply the Euclidean algorithm in a Euclidean domain.
- State and apply the definition of the degree of a field extension.
- State and prove the tower law, and use it to prove the impossibility of some classical ruler and compass geometric constructions.
- Given a list of ring properties, give an example of a ring with those properties, or explain why no such example exists.
- State and apply the definition of a module over a ring.

September 29, 2011