School of Mathematics

Module MA1212 — Linear algebra II 2010-11
(JF Mathematics, JF Theoretical Physics)

Lecturer: Dr. Vladimir Dotsenko

Requirements/prerequisites: prerequisite: MA1111

Duration: Hilary term, 11 weeks

Number of lectures per week: 3 lectures including tutorials per week

Assessment: 100%*final exam mark or 70%*final exam mark + 15%*home assignments result + 15% of the midterm test result, whichever is higher.

ECTS credits: 5

End-of-year Examination: 2 hour end of year examination.

Description:

1. Kernels and images. Ranks. Dimension formulas.
2. Characteristic polynomials. Eigenvalues and eigenvectors. Diagonalisation in the case when all eigenvalues are distinct.
3. Cayley–Hamilton theorem. Minimal polynomial of a linear operator. Examples (operators with $A^2 = A$).
4. Invariant subspaces. An application: two commuting linear operators have a common eigenvector. Direct sums.
5. Normal form of a nilpotent operator. Jordan normal form (Jordan Decomposition Theorem). Applications: closed expressions for Fibonacci numbers and other recursively defined sequences.
6. Orthonormal bases; Gram–Schmidt orthogonalisation. Orthogonal complements and orthogonal direct sums. Bessel's inequality.
7. Bilinear and quadratic forms. Sylvester's criterion. The law of inertia. Spectral Theorem for symmetric operators.

Homeworks

Homework assignments will be handed out in class every week. Besides just obtaining answers to questions, you are supposed to justify your answers (in particular, every "yes/no" question also assumes the "why" question). Homeworks are due to hand in after Tuesday's classes; on the same evening solutions shall be posted on the course webpage, so late assignments are not accepted.

Assessment

An in-class midterm test plus an exam in the end of the year plus the continuous assessment.

The final mark is 100%*final exam mark or 70%*final exam mark + 15%*continous assessment mark + 15% of the midterm test mark, whichever is higher.

Web page

Homework assignments, selected solutions, various handouts and announcements will be posted on the course web page

`http://www.maths.tcd.ie/~vdots/indexLinearAlgebra.html`

Learning Outcomes: On sucessful completion of this module, students will be able to:

- compute the rank of a given linear operator, and use proofs of theoretical results on ranks explained in the course to derive similar theoretical results;
- compute the dimension and determine a basis for the intersection and the sum of two subspaces of a given space, determine a basis of a given vector space relative to a given subspace;
- calculate the basis consisting of eigenvectors for a given matrix with different eigenvalues, and, more generally, calculate the Jordan normal form and a Jordan basis for a given matrix;
- apply Gram–Schmidt orthogonalisation to obtain an orthonormal basis of a given Euclidean space;
- apply various methods (completing the squares, Sylvester's criterion, eigenvalues) to determine the signature of a given symmetric bilinear form;
- identify the above linear algebra problems in various settings (e.g. in the case of the vector space of polynomials, or the vector space of matrices of given size), and apply methods of the course to solve those problems.

July 27, 2011