School of Mathematics

Module MA1111 — Linear algebra I 2010-11
(JF Mathematics, JF Theoretical Physics & JF Two-subject Moderatorship)

Lecturer: Dr. Vladimir Dotsenko

Requirements/prerequisites:

Duration: Michaelmas term, 11 weeks

Number of lectures per week: 3 lectures including tutorials per week

Assessment: 100%*final exam mark or 80%*final exam mark + 20%*home assignments result, whichever is higher.

ECTS credits: 5

End-of-year Examination: 2 hour end of year examination.

Description:

1. Linear algebra in 2d and 3d. Vectors. Dot and cross products. Quaternions.
2. Systems of simultaneous linear equations. Gauss–Jordan elimination.
3. (Reduced) row echelon form for a rectangular matrix. Principal and free variables. Matrix product and row operations. Computing the inverse matrix using row operations.
4. Permutations. Odd and even permutations. Determinants. Row and column operations on determinants. $\det(A^T) = \det(A)$.
5. Minors. Cofactors. Adjoint matrix. Computing the inverse matrix using determinants. Cramer's rule for systems with the same number of equations and unknowns.
6. Fredholm's alternative. An application: the discrete Dirichlet's problem.
7. Coordinate vector space. Linear independence and completeness.
8. Fields: rationals, reals, and complex. Abstract vector spaces. Linear independence and completeness in abstract vector spaces. Bases and dimensions. Subspaces.
9. Linear operators. Matrix of a linear operator relative to given bases. Change of basis. Transition matrices. Similar matrices define the same linear operator in different bases. Example: a closed formula for Fibonacci numbers.

Homeworks

Homework assignments will be handed out in class every week. Besides just obtaining answers to questions, you are supposed to justify your answers (in particular, every "yes/no" question also assumes the "why" question). Homeworks are due to hand in after Tuesday's classes; on the same evening solutions shall be posted on the course webpage, so late assignments are not accepted.

Assessment

Exam in the end of the year plus the continuous assessment.

The final mark is 100%*final exam mark or 80%*final exam mark + 20%*continuous assessment mark, whichever is higher.

Web page

Homework assignments, selected solutions, various handouts and announcements will be posted on the course web page

`http://www.maths.tcd.ie/~vdots/indexLinearAlgebra.html`

Learning Outcomes: On sucessful completion of this module, students will be able to:

- operate with vectors in dimensions 2 and 3, and apply vectors to solve basic geometric problems;
- apply various standard methods (Gauss–Jordan elimination, inverse matrices, Cramer's rule) to solve systems of simultaneous linear equations;
- compute the sign of a given permutation, and apply theorems from the course to compute determinants of square matrices;
- demonstrate that a system of vectors forms a basis of the given vector space, compute coordinates of given vectors relative to the given basis, and calculate the matrix of a linear operator relative to the given bases;
- give examples of sets where some of the defining properties of vectors, matrices, vector spaces, subspaces, and linear operators fail;
- identify the above linear algebra problems in various settings (e.g. in the case of the vector space of polynomials, or the vector space of matrices of given size), and apply methods of the course to solve those problems.

July 27, 2011