School of Mathematics

Course 442 - Differential Geometry and General Relativity 2008-09
(SS Theoretical Physics, JS & SS Mathematics)

Lecturer: Dr. Calin Lazariou

Requirements/prerequisites: Analytical mechanics (241) and Classical Electrodynamics (432); Differential Geometry (manifolds, vector and tensor fields, differential forms and vector bundles)

Duration: 19 weeks.

Number of lectures per week: 3

Assessment:

End-of-year Examination: One 3-hour examination

Description: The course is an introduction to general relativity. See also `http://www.maths.tcd.ie/~calin/teaching/442.html`

Course content:
Differential Geometry

- basic theory of abstract manifolds (chart, atlas, differentiable structure, tangent space, smooth map, differential of a smooth map at a point)
- algebra of smooth real-valued functions on a manifold; vector fields on a manifold; Lie bracket of vector fields
- basic theory of vector bundles; sections of vector bundles; the tangent bundle; the differential of a map; natural operations on vector bundles (direct sum, tensor product, dual, symmetric and antisymmetric product)
- reminder on modules over commutative rings and their basic operations
- the module of sections of a vector bundle; its behavior under the natural operations on bundles
- natural vector bundles on a manifold; tensor fields, polyvector fields and differential forms on manifolds; symmetric and antisymmetric tensor fields
- connections in a vector bundle; curvature of a connection; induced connections
- pull-back of a vector bundle; pull-back connection
- parallel transport of a connection
- affine connections and their torsion tensor; covariant derivation of tensor fields; autoparallel curves with respect to an affine connection

- pseudo-Riemannian manifolds; signature; Riemannian, Minkowskian and Minkowsky manifolds.
- the Levi-Civita connection, geodesics, variational principle for geodesics.
- Riemann curvature tensor, Ricci tensor and Einstein tensor.
- orientability, volume form determined by a metric
- natural differential operators on a pseudo-Riemannian manifold

General Relativity

- Minkowsky manifolds, light cones and time orientability; space-times; timelike, lightlike and spacelike geodesics;
- observers, proper time
- The axioms of general relativity; action principle for gravity coupled to mater.
- Matter stress-energy tensor, Einstein equations, positivity and causality conditions
- matter models: free falling particle, dust, real scalar field, electromagnetic field
- the weak field limit, recovering Newtonian gravity, gravitational waves
- gravitational red shift
- basic special solutions of Einstein's equations (Schwarzschild, Robertson-Walker)
- motion in a central field; applications to gravitational lensing and precession of perihelia

Advanced topics (chosen at the interests of the class and as time allows): black hole physics; Kerr, Reisner-Nordstrom solutions; basic cosmology.

Core textbook:
R.K. Sachs, General Relativity for Mathematicians, Graduate Texts in Mathematics vol 48, Springer, 1977

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