School of Mathematics

Course 414 — Complex Analysis 2007–08
(Optional JS & SS Mathematics, SS Two-subject Moderatorship)

Lecturer: Dr. Dmitri Zaitsev

Requirements/prerequisites: 221, 214

Duration: 18 weeks.

Number of lectures per week: 3

Assessment: Regular assignments.

End-of-year Examination: One 3-hour examination

Description:

1. Elementary functions of one complex variable: polynomials, exponential, logarithmic and trigonometric functions, their inverses. Real and complex differentiability. Holomorphic functions. Conformal mappings.

2. Piecewise smooth and rectifiable paths and curves. Complex integration along curves. Antiderivatives. Cauchy's theorem: Goursat's version for a triangle, for star-shaped regions and their unions, homotopy version. Elements of homology and homological version of Cauchy's theorem.

3. Cauchy's integral formula. Power series expansion of holomoprhic functions. Mean value property. Maximum modulus principle. Radius of convergence of power series. Cauchy-Hadamar formula. Differentiation of power series. Theorem of Morera. Cauchy's estimates. Liouville's theorem. Compact convergence and Weierstrass theorem.

4. Order of zeroes. The identity principle. Laurent series expansion in a ring. Isolated singularities. Removable singularities, poles, essential singularities. Riemann extension theorem. Meromorphic functions. Casorati-Weierstrass theorem.

5. Residues, their caculation. Residue theorem: for unions of star-shaped regions and the winding number version. Applications to calculation of integrals.

6. The argument principle. Rouché's theorem. Open mapping theorem. The univalence theorem (local injectivity criterion). Inverse function theorem. Branched covering structure theorem.

7. Spaces of holomorphic functions. Seminorms. Montel's theorem. Biholomphic maps between open sets. The Riemann mapping theorem.

8. Schwarz Lemma. Automorphisms of the disk. Homogeneity of the disk. Cayley transform. Automorphisms of the upper half-plane. Möbius transformations. Riemann sphere. Holomorphic and meromorphic functions on the Riemann sphere. Automorphisms of the Riemann sphere.

Further detailed information about the course will become available via the web site for the course at `http://www.maths.tcd.ie/~zaitsev/414.html`

Objectives: This course will build on material covered in 214. Initially it will cover some familiar material in greater detail and then continue on to cover basic material in complex analysis.

Textbooks:

[1] L. V. Ahlfors, *Complex Analysis*, Third Edition, McGraw-Hill, New York, 1978.

[2] J. B. Conway, *Functions of One Complex Variable,* Second Edition, Graduate Texts in Mathematics 11, Springer-Verlag, New York, 1978.

[3] R. Remmert, *Theory of Complex Functions,* Graduate Texts in Mathematics 122, Springer-Verlag, New York, 1991.

[4] R. V. Churchill, J. W. Brown, *Complex Variables and Applications*, Fourth edition. McGraw-Hill Book Co., New York, 1984.

[5] B. P. Palka, *An Introduction to Complex Function Theory*, Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1991.

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