School of Mathematics

Course 345 — Introduction to Solitons 2007-08
(SSTP, JS & SS Mathemaatics)

Lecturer: Dr. Yakov Shnir

Requirements/prerequisites:

Duration: 10 weeks (2nd semester)

Number of lectures per week: 3 (including tutorials)

Assessment:

End-of-year Examination: 2 hour exam in June

Description: This course is intended as an introduction to the theory of solitons.

In the last thirty years important progress was made in the understanding of properties of certain non-linear differential equations which arise in many different areas of physics, e.g., physics of plasma, solid state physics, biophysics, field theory etc. For these equations, the most prominent of which is the Korteweg-de Vries (KdV) equation, it was possible to find a general method of solution. A common interesting feature is the occurrence of solitons, i.e. stable, non-dissipative and localized configurations behaving in many ways like particles. In the analysis of these equations many interesting mathematical structures were discovered which surprisingly also appear in quantum mechanics and quantum field theory. From a pragmatic point of view these completely soluble non-linear equations are a substantial extension of the 'tool kit' of a physicist which otherwise is mainly restricted to solving linear systems. They also serve as valuable source for intuition about the behavior of non-linear systems.

This course aims at giving a self-contained introduction in this field: give an idea what the subject is about, introduce and explain in simple examples the new physical concepts and mathematical ideas which where developed in that context.

Syllabus:

1. Introduction to history of solitons.
2. Linear waves. D'Alembert's solution. Dispersion and dispersion relations, wave speed, wavelength, wave number. Linear superposition.
3. Introduction to nonlinear wave phenomena.
4. Burger's equation. The Korteweg and de Vries equation.
5. The scattering and inverse scattering problems.
6. The nonlinear Schrodinger equation, its soliton solutions.
7. Two solitons solutions and their interactions.
8. Further integrable non-linear differential equations, the Lax formulation.
9. Hirota's method and Bäcklund transformations

10. The Fermi-Pasta-Ulam problem.
11. Models for dislocations in crystals. The sine-Gordon equation.
12. Kink solution in 1+1 dimensional ϕ^4 model. Domain walls.
13. Solitons in field theory. Topological classification.
14. Ginzburg-Landau model. Abrikosov-Nielsen-Olesen vortex solution.

January 8, 2008