

## School of Mathematics

### Course 419 — Partial Differential Equations

2006-07

(JS &amp; SS Mathematics

SS Theoretical Physics )

**Lecturer:** Dr. John Stalker

**Requirements/prerequisites:**

**Duration:** 21 weeks

**Number of lectures per week:** 3

**Assessment:**

**End-of-year Examination:** Annual examination in May/June.

#### Description:

More or less everything worth describing can be described by partial differential equations. While there is not much useful that one can say about partial differential equations in general, one can say a fair amount about broad classes of partial differential equations, and a great deal about some particular partial differential equations. Luckily these include the differential equation most relevant to physics.

The course will cover standard introductory material from the theory of partial differential equations. The approach will be fairly standard, except that the role of symmetry groups will be emphasized somewhat more than is usual.

A preliminary outline of course follows, but this is subject to change. Please look at <http://www.maths.tcd.ie/~stalker/419> for updates.

- Introduction
  - Examples
  - Terminology
    - \* Order of an Equation
    - \* Ordinary vs. Partial Differential Equations
    - \* Linear vs. Non-linear Equations
    - \* Scalar Equations vs. Systems
  - First Order Equations
    - \* Method of Characteristics
    - \* Examples
  - Simple Physical Examples
    - \* Transport
    - \* Diffusion
    - \* Vibration
  - Initial and Boundary Value Problems

- Well Posed Problems
  - \* Existence
  - \* Uniqueness
  - \* Stability
- The Wave Equation in 1+1 Dimensions
  - Method of Characteristics
    - \* Explicit Solution
    - \* Existence
  - Causality and Energy
    - \* Domain of Dependence
    - \* Domain of Influence
    - \* Uniqueness
    - \* Stability
  - Weak Solutions
- Diffusion in 1+1 dimensions
  - Maximum Principle
    - \* Uniqueness
    - \* Stability
  - Symmetries
  - Fundamental Solution
    - \* Existence
- Boundary Problems and Reflection
  - Problems on the Half Line
    - \* Diffusion
      - Dirichlet Condition
      - Neumann Condition
      - Robin Condition
    - \* Wave Equation
      - Dirichlet Condition
      - Neumann Condition
      - Robin Condition
  - Problems on Finite Intervals
    - \* Dirichlet at Both Ends
    - \* Neumann at Both Ends

\* Dirichlet and Neumann

- Inhomogeneous Problems
  - Diffusion
  - Wave Equation
- Separation of Variables
  - Harmonic Oscillator on the Real Line
  - Diffusion on a Finite Interval
  - Wave Equation on a Finite Interval
  - Diffusion on the Real Line
  - Wave Equation on the Real Line
- Harmonic Functions
  - Laplace Equation in 2 Dimensions
  - Maximum Principle
    - \* Uniqueness
    - \* Stability
  - Rectangular Domain
  - Circular Domain
  - Conformal Symmetry Group
  - Laplace Equation in 2 Dimensions
- Wave Equation in Higher Dimensions
  - 1+3 dimensions
  - 1+2 dimensions
  - Rays, Characteristics
  - Full Symmetry Group of the Wave Equation
  - Conserved Quantities
- Scattering Theory
- Burger's Equation

**Textbooks:** The text for the course is Partial Differential Equations, An Introduction by Walter Strauss. You don't need to buy it immediately, but you should do so eventually.

September 25, 2006