School of Mathematics

Course 321 - Functional Analysis 2006-07
(Optional JS & SS Mathematics, SS Two-subject Moderatorship)

Lecturer: Dr. D. Zaitsev

Requirements/prerequisites: 212 is helpful but not essential

Duration: 21 weeks

Number of lectures per week: 3

Assessment: Regular assignments.

End-of-year Examination: One 3-hour examination

Description:

- *Fundamental Concepts*: Partial order, Zorn's lemma as an axiom, application to bases of vector spaces; cardinal numbers; ordinal numbers.

- *General Topology*: Neighbourhoods, first countable, inadequacy of sequences, second-countable, (relationship to separability), continuity of functions at points, product topology (weak topology for continuous projections).

  Nets, advantages over sequences, subnets; Hausdorff separation axiom, Urysohn's lemma, Tietze extension. Compactness via nets, Tychonoff's theorem (compactness of products), compactification (Stone-Cech and universal properties, one-point), local compactness, completions of metric spaces, Baire category theorem.

- *Functional Analysis*:

  Banach spaces: definitions and examples ($C(X)$, ℓ^p, Hölder and Minkowski inequalities, closed subspaces, c_0, $L^p(\mathbb{R})$, $L^p[0,1]$).

  Linear operators: examples of continuous inclusions among ℓ^p and $L^p[0,1]$ spaces, n-dimensional normed spaces isomorphic. Open mapping and closed graph theorems. Uniform boundedness principle.

  Dual spaces: Hahn-Banach theorem, canonical isometric embedding in double dual, reflexivity.

  Hilbert space: orthonormal bases (existence, countable if and only if separable), orthogonal complements, Hilbert space direct sums, bounded linear operators on a Hilbert space as a C*-algebra. Completely bounded and completely positive operators.

  Applications Fourier series in $L^2[0,2\pi]$.

There is a web site `http://www.maths.tcd.ie/~zaitsev/321.html` for the course, see also `http://www.maths.tcd.ie/~richardt/321` for the last year.

Objectives: This course aims to introduce general techniques used widely in analysis (and other branches of mathematics) and to treat a few topics that are active areas of research.

Textbooks:

Kolmogorov, A. N.; Fomin, S. V. Introductory real analysis. Translated from the second Russian edition and edited by Richard A. Silverman. Corrected reprinting. Dover Publications, Inc., New York, 1975. xii+403 pp.

Yosida, K. Functional analysis. Reprint of the sixth (1980) edition. Classics in Mathematics. Springer-Verlag, Berlin, 1995. xii+501 pp. ISBN: 3-540-58654-7

Lax, P. D. Functional analysis. Pure and Applied Mathematics (New York). Wiley-Interscience [John Wiley & Sons], New York, 2002. xx+580 pp. ISBN: 0-471-55604-1

Reed, M.; Simon, B. Methods of modern mathematical physics. I. Functional analysis. Second edition. Academic Press, Inc. [Harcourt Brace Jovanovich, Publishers], New York, 1980. xv+400 pp. ISBN: 0-12-585050-6

Dunford, N.; Schwartz, J. T. Linear operators. Part I. General theory. With the assistance of William G. Bade and Robert G. Bartle. Reprint of the 1958 original. Wiley Classics Library. A Wiley-Interscience Publication. John Wiley & Sons, Inc., New York, 1988. xiv+858 pp. ISBN: 0-471-60848-3

Kirillov, A. A.; Gvishiani, A. D. Theorems and problems in functional analysis. Translated from the Russian by Harold H. McFaden. Problem Books in Mathematics. Springer-Verlag, New York-Berlin, 1982. ix+347 pp. ISBN: 0-387-90638-X

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