School of Mathematics

Course 214 — Complex Variable 2006-07
(SF Mathematics, SF Theoretical Physics, SF Two-subject Moderatorship with Economics)

Lecturer: Dr. D.R. Wilkins

Requirements/prerequisites:

Duration: 12 weeks

Number of lectures per week: 3

Assessment: Two assignments, providing 10% of the credit for the course

End-of-year Examination: One 2-hour examination

Description: See `http://www.maths.tcd.ie/~dwilkins/Courses/214/` for more detailed information.

Section 1: Complex Numbers and Euclidean Spaces. Basic theorems of real analysis; the complex plane; definition and basic properties of limits of infinite sequences of points in Euclidean spaces; basic definitions of limits and continuity for functions between subsets of Euclidean spaces; basic theorems concerning limits and continuity; open and closed sets in Euclidean spaces; properties of continuous functions on closed bounded subsets of Euclidean spaces; uniform continuity.

Section 2: Infinite Series. Definition of convergence for infinite series; the Comparison and Ratio Tests; absolute convergence; Cauchy products; uniform convergence; power series; the exponential function.

Section 3: Winding Numbers of Closed Paths in the Complex Plane. The Path Lifting Theorem; winding numbers; path-connected and simply-connected subsets of the complex plane; the Fundamental Theorem of Algebra.

Section 4: Path Integrals in the Complex Plane. The definition of the path integral; path integrals and boundaries.

Section 5: Holomorphic Functions. The definition of holomorphic functions and their derivatives; the Cauchy-Riemann equations; the Chain Rule for holomorphic functions; differentiation of power series.

Section 6: Cauchy's Theorem. Path integrals of polynomial functions; winding numbers and path integrals; Cauchy's Theorem for a triangle; Cauchy's Theorem for star-shaped domains; more general forms of Cauchy's Theorem; residues; Cauchy's Residue Theorem.

Section 7: Basic Properties of Holomorphic Functions. Taylor's Theorem for holomorphic functions; Liouville's Theorem; Laurent's Theorem; Morera's Theorem; meromorphic functions; the Maximum Modulus Principle; the Argument Principle.

Section 8: Examples of Contour Integration.

Section 9: The Gamma Function.

Section 10: Elliptic Functions.

October 3, 2007