School of Mathematics

Course 428 — Prime Numbers 2005–06
(Optional JS & SS Mathematics, SS Two-subject Moderatorship)

Lecturer: Dr. T.G. Murphy

Requirements/prerequisites:

Duration: 21 weeks

Number of lectures per week: 3

Assessment:

End-of-year Examination:

Description: This course will be in three parts, corresponding roughly to the three terms.

After establishing the *Fundamental Theorem of Arithmetic* (unique factorisation into primes), the main thrust of the first part, will be to examine criteria for the primality of large numbers, including the new "Indian algorithm".

The second part is an introduction to analytic number theory, that is, the use of complex functions to study the distribution of primes. Our main aim here is to prove the *Prime Number Theorem*, that

$$\pi(x) \cong \frac{x}{\log x},$$

where $\pi(x)$ is the number of primes $p \leq x$. Our principal tool is Riemann's zeta function

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots,$$

which connects to prime number theory through *Euler's Product Formula*

$$\zeta(s) = \prod_p \left(1 - \frac{1}{p^s}\right)^{-1},$$

where the product extends through all primes $p = 2, 3, 5, \ldots$.

As a bye-product of this technique we are able to prove *Dirichlet's Theorem*, that there are an infinity of primes in any arithmetic sequence

$$c + dn$$

with $\gcd(c, d) = 1$.

The third part is an introduction to *algebraic number theory*, that is, the factorisation theory of algebraic integers, such as $3 - 2\sqrt{5}$. Dedekind showed that while the Fundamental Theorem of Arithmetic in general fails for algebraic integers, the Theorem can be rescued by introducing 'ideal numbers', or *ideals*.

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