School of Mathematics

Course 321 — Modern Analysis

(Optional JS & SS Mathematics, SS Two-subject Moderatorship)

Lecturer: Dr. R.M. Timoney

Requirements/prerequisites: 212 is almost essential

Duration: 21 weeks

Number of lectures per week: 3

Assessment: Regular assignments.

End-of-year Examination: One 3-hour examination

Description:

- *Fundamental Concepts*: Partial order, Zorn's lemma as an axiom, application to bases of vector spaces; cardinal numbers; ordinal numbers.
- *General Topology*: Neighbourhoods, first countable, inadequacy of sequences, secondcountable, (relationship to separability), continuity of functions at points, product topology (weak topology for continuous projections).

Nets, advantages over sequences, subnets; Hausdorff separation axiom, Urysohn's lemma, Tietze extension. Compactness via nets, Tychonoff's theorem (compactness of products), compactification (Stone-Čech and universal properties, one-point), local compactness, completions of metric spaces, Baire category theorem.

- Functional Analysis:
 - Banach spaces: definitions and examples $(C_0(X), \ell_{\infty}, C(K), \text{Hölder and Minkowski}$ inequalities, ℓ_p , closed subspaces, $c_0, L_p(\mathbb{R}), L_p[0, 1]$).
 - Linear operators: examples of continuous inclusions among ℓ_p and $L_p[0, 1]$ spaces, *n*-dimensional normed spaces isomorphic. Open mapping and closed graph theorems. Uniform boundedness principle.
 - Dual spaces: Hahn-Banach theorem, canonical isometric embedding in double dual, reflexivity.
 - Hilbert space: orthonormal bases (existence, countable if and only if separable), orthogonal complements, Hilbert space direct sums, bounded linear operators on a Hilbert space as a C*-algebra.
- Applications Fourier series in $L_2[0, 2\pi]$, Wavelet bases for $L^2(\mathbb{R})$.

There is a web site http://www.maths.tcd.ie/~richardt/321 for the course. Objectives: This course aims to introduce general techniques used widely in analysis (and other branches of mathematics) and to treat a few topics that are active areas of research.

2005 - 06