School of Mathematics

Course 131 — Mathematical Methods 2005-06
(JF Mathematics, Theoretical Physics & Two-Subject Moderatorship)

Lecturer: Dr. C. Lazaroiu

Requirements/prerequisites: None

Duration: 22 weeks

Number of lectures per week: 3

Assessment: The combined score of all homeworks counts 60% of the final grade.

End-of-year Examination: 3 hour paper in June (counts 40%).

Description:

The course is meant as a complement for Linear Algebra and Analysis.

Its main purpose is to illustrate the concepts introduced in those classes and discuss certain supplementary topics, such as affine and Euclidean spaces and ordinary differential equations. The focus is on a deductive presentation of such applications, and not on mere computation.

(0) Review of basic math:

(0.1) Elements of mathematical logic: propositional and predicate calculus.

(0.2) Sets, operations on sets; Cartesian product and binary relations

(0.3) Functions; injective, surjective and bijective functions; finite, countable and uncountable sets.

(0.4) Equivalence relations; quotient of a set by an equivalence relation.

(1) Linear and affine spaces:

(1.1) Construction of the vector space R^n; vector addition; scalar multiplication.

(1.2) Linear subspaces and linear maps.

(1.3) Convex combinations and convex sets; intervals

(1.4) Linear changes of coordinates; matrices; linear bases; orientation

(1.5) The group $GL(n, R)$

(1.6) R^n as a Euclidean space: scalar product, norm and distance; basic properties.

(1.7) Orthogonal and orthonormal vectors; the Gramm-Schmidt algorithm

(1.8) Orthogonal transformations; the group $O(n, R)$.

(1.9) Volumes and determinants. The groups $SL(n, R)$ and $SO(n, R)$

(1.10) Cross products.

(1.3) The real affine space A^n; vectors in affine space.

(1.4) Affine subspaces; parallelism.

(1.5) Intervals; distance formulae

(1.6) Affine coordinates; translations and general affine transformations; the affine group.

(1.9) Euclidean coordinates; euclidean transformations; the Euclidean group.

(2) Matrix algebra and linear systems of equations

(2.1) Systems of linear equations; indeterminate, determinate and over-determinate systems

(2.2) The Cramer formula(s); space of solutions

(2.3) Determinant formulae; invertibility.

(2.4) Row reduction and similarity transformations

(2.5) Elementary transformations; Gauss elimination

(2.6) Symmetric matrices and their canonical forms

(2.7) Matrix exponentiation; induced group of transformations; connection wuth $GL(n, R)$ and $O(R)$

(2.8) Positive matrices, square roots, LR decomposition.

- ODEs:

(1) Linear and nonlinear ODEs; systems of ODEs

(2) Elementary ODEs; reduction to quadratures

(3) Basic theory of linear ODEs; space of solutions; wronskian; part solutions

(4) Inhomogeneous linear ODEs; basic theory

(5) ODEs with constant coefficients

(6) Initial value problems

(7) Separable and homogeneous equations; Pfaffian equlations; integrating factors.

(8) Advanced subjects

– Multivariable Calculus:

(1) Differential operators on functions and vectors in R^n

(2) Classical differential operators: gradient; divergence; curl and Laplacian ; basic properties

(3) Multiple integrals

(4) Other subjects

See `http://www.maths.tcd.ie/~calin/teaching/131.html` for more information.

References

Core textbook: Tom Apostol, Calculus, Vol. 2: Multi-Variable Calculus and Linear Algebra with Applications, Wiley; 2 edition (1969) ISBN 04710000

Recommended Reading: Even though this is a "methods" course, one can't get far without a good understanding of Linear Algebra and Analysis. It is very important to acquire a thorough conceptual background by studying some of the books listed below.

1. Linear Algebra:
   (a) P.R. Halmos, Finite-Dimensional Vector Spaces, Springer; 1 edition (1993) ISBN 0-387-90093-4;
   and its companion:
   Paul R. Halmos, William Watkins, Linear Algebra Problem Book , Docliani Mathematical Expositions, The Mathematical Association of America, 1996, ISBN 0-88385-322-1
   (b) Georgi E. Shilov, Linear Algebra, Dover Publications; Rev. English ed edition (1977) ISBN: 048663518X
   (c) S. Lang, Introduction to Linear Algebra (Undergraduate Texts in Mathematics) Springer; 2 edition (1997), ISBN: 0387962050
   (d) Peter D. Lax, Linear Algebra, Wiley-Interscience; 1 edition (1996), ISBN: 0471111112
2. Matrix Analysis:
   (a) Roger A. Horn, Charles R. Johnson, Matrix Analysis, Cambridge University Press; Reprint edition (1990) ISBN: 0521386322
   (b) Joel N. Franklin, Matrix Theory, Dover Publications (February 8, 2000) ISBN: 0486411796
   (c) Franz E. Hohn, Elementary Matrix Algebra Dover Publications; 3rd edition (January 27, 2003)ISBN: 0486425347
3. ODEs:
   (a) Earl A. Coddington, An Introduction to Ordinary Differential Equations, Dover Publications; Unabridged edition (1989) ISBN: 0486659429
   (b) V. I. Arnold, Roger Cooke (Translator), Ordinary Differential Equations, Springer; 3rd edition (1992)ISBN: 0387548130
4. Analysis:
   (a) Walter Rudin: Principles of Mathematical Analysis, International Series in Pure & Applied Mathematics,McGraw-Hill Science/Engineering/Math; 3rd edition, ISBN 0-07-054235-X
   (b) Tom Apostol, Mathematical Analysis, Addison Wesley Publishing Company; 2nd edition (1974) ISBN: 0201002884