DRAWING THE LINE AT THE FOURTH DIMENSION*

There are three dimensions of space and one of time - that makes the world four dimensional, as every schoolchild knows. But two Austrian physicists have come up with an ingenious way of physically measuring the number of dimensions of space and time, and they calculate with something less than four: 3.99999947 to be exact.

Just an error? Well, not necessarily. Space-time *could* have a fractional dimension, and it could be something more or less than four. So perhaps Anton Zeilinger of the Vienna Atominstitut and Karl Svozil of the Institute for Theoretical Physics in the Technical University of Vienna are on to something.

What Zeilinger and Svozil have done is to apply the concept of "fractals" to space-time. Fractals, developed by French mathematician Benoit B. Mandelbrot, are objects like coastlines or mountain ranges whose exact lengths, or areas, or whatever, are impossible to measure in principle. Thus for example, a geographer might pace out the arc of a bay as 1,000 metres. But suppose he were to measure round every rock — the length would be greater - say 1,500 metres. And then around every grain -- greater still; and so on.

Similar effects occur in pure mathematics. For example a line can be defined from A to B in this way: first draw a zig-zag from A to B. Then on each straight portion of the zig-zag, replace the straight line by a miniature version of the zig-zag. Now you have a zig-zag zig-zag. Take each miniature straight portion of the zig-zag zig-zag and replace it by a tiny zig-zag. As this process is continued *ad infinitum*, the result - which is surely composed of lines, and so one-dimensional, begins to fill in a whole area between A and B. But areas are two-dimensional. So is the infinitely zig-zagged line "really" one- or two-dimensional?

To solve questions such as that, mathematicians sometimes redefine terms - and here an early 20th century mathematician, Felix Hausdorff, provided the redefinition. If you measure the "length" (the sum of little lengths in the line) of an infinitely zig-zagged line, Hausdorff argued, the answer will be infinity. And if you measure its "area" (the sum of little areas in the line), the answer will be zero - as mathematical lines are infinitely thin.

But what if you measure something in between length and area - a fractional power, between 1 and 2, of the distance between points in the line, Hausdorff asked. Now, Hausdorff showed, the answer comes out to be neither infinite nor zero, but finite, for one precise power of distance. It is this power that enables a measure of the "size" of the set of points (just as length and area measure the size of more familiar sets). The value of the power required to produce a finite result is now called the "Hausdorff dimension" of a set.

Fractals are then lines, surfaces or what have you that twist and turn so much their Hausdorff dimensional is fractional.

So Zeilinger and Svozil asked a simple but profound question: what really is the Hausdorff dimension of space-time? It could be more than four, if the collection of points which we can reach with our particles - accessible space-time - twists and turns in some other space in which we are embedded (just as the zig-zag line twists on the page). This is not unreasonable to suppose, for particle physicists are now being driven to consider 10 and 12-dimensional spaces, in order to explain the properties of the elementary particles, of which our familiar space-time is only a part.

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Or, the Hausdorff dimension of space-time could be less than four, if we are embedded in strictly 4-dimensional space-time, all the points of that space-time are not accessible to us. Werner Heisenberg, for example, had the idea that space-time might be granular - that there might be steps in space and time.

But how to measure the Hausdorff dimension of the world? Zeilinger and Svozil have an answer to that. In the world of high-energy particle physics, particles are spread out in the "wave-particles" of quantum mechanics - and as such they probe every cranny of space-time. By reinterpreting certain very precise measurements on the magnetic moment of the electron as estimates of the Hausdorff dimension, Zeilinger and Svozil come out with their value of 3.99999947.

This is so close to 4 that it may be questioned whether the result is real. But it may just indicate the beginnings of the detection of granularity in space-time. For the measurements are limited by the quantum "wavelength" of the particles used. Experimentally, the waves cannot penetrate "crannies" or discontinuities shorter than their wavelength. And wavelength decreases with energy. The energy of the magnetic moment experiment was low, and so was perhaps like the geographer pacing the coastline with a long step.

The requirement must be now for higher energy tests (though the higher the energy, the more difficult precise measurements become). Ideally, the tests should be done at energies approaching the levels where theorists expect the 10 or 12 dimensional twists and turns to appear. Sadly, however, such energy regions are astronomically high and probably inaccessible, and we may have to content ourselves with Zeilinger and Svozil's 3.99999947.

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EIGENVECTORS AND DIAGONALIZATION

P.D. Barry

Can we motivate or encounter naturally eigenvectors of matrices, especially as occurring in the diagonalization of a quadratic form, or must they be produced apparently out of the blue?

When deriving the equation of an ellipse E in standard form we select the axes of coordinates to be along the major and minor axes of the ellipse. Thus if

$$ax^2 + 2hxy + by^2 = 1 \qquad \text{(i)}$$

represents an ellipse with centre at the origin O, to convert it to standard form we need to map its major and minor axes onto the x' and y' axes. If $P_0 = (x_0, y_0) \neq 0$ is a point on either the major or the minor axis, we then seek a linear transformation

$$\begin{aligned} x &= a_{1,1}x' + a_{1,2}y' \\ y &= a_{2,1}x' + a_{2,2}y' \end{aligned} \qquad \text{(ii)}$$

which maps the line OP_0 to the x'-axis; in matrix form (ii) is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} . \qquad \text{(iii)}$$

As then

$$\begin{aligned} x_0 &= a_{1,1}x_0' \\ y_0 &= a_{2,1}x_0' \end{aligned}$$

for some $x_0' \neq 0$, we have

$$a_{1,1} = jx_0, \quad a_{2,1} = jy_0$$