

Baker's argument takes the simpler result $f(n) \geq \frac{1}{2}$ instead of (2) and then uses $1 + \log n < 2 \log n$ for $n > 1$ to deduce $\phi(n) > \frac{1}{4}$. But (3) clearly gives the asymptotic result $\phi(n) > \frac{1}{2} - \epsilon$, for large enough n . Explicit calculation of the ratio of $n/\log n$ to $\frac{1}{2}n(1+n^{-\frac{1}{2}})(1+\log n)^{-1}$ gives a ratio of 0 for $n = 1$, of .349 for $n = 2$ and a ratio $\geq .412$ for $n > 2$, so this elementary method yields $\phi(n) > 2/5$ for $n > 2$. Further calculation, based on our result, shows that we actually have $\phi(n) \geq (\log 6)/3 = .59725$ for $n > 2$.

REFERENCE

1. BAKER, A.
'A Concise Introduction to the Theory of Numbers', (Cambridge University Press, Cambridge, 1984).

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MATHEMATICAL EDUCATION

REPORT ON THE BASIC MATHEMATICAL SKILLS TEST OF FIRST YEAR STUDENTS IN CORK RTC IN 1984

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1. INTRODUCTION

A test was given to all first year students in October 1984 to assess their basic mathematical competency. The results of this test show that our student intake have fundamental deficiencies in their basic mathematics.

While the direct remedy of this situation is outside our control, it is hoped that those involved in the teaching and drawing up of mathematical syllabi at primary and secondary level will consider the implications of this report.

2. THE TEST: ITS CONSTRUCTION AND PURPOSE

A copy of the test paper is given in Appendix A. It consists of 20 questions which the students had to attempt without the aid of tables or calculators in the allotted time of one hour. The aim of the questions and acceptable answers are also given in Appendix A.

After careful consideration as to what basic mathematical skills students should have after completing their Leaving Certificate (L/C) the pass level for this test was set at 15 or more correct answers.

As well as the answers to the test questions, the sex and best L/C mathematics grade of the students were recorded.

The test was administered in the fourth week of term. Students were told in advance about the test and what sort of questions to expect, but sample papers were not made available to them. They were also advised that the results of the test could be taken into account in assessing their end-of-year grade.

3. THE STUDENTS WHO TOOK THE TEST

All first year full-time students who were taking 3rd level courses or their equivalent were required to take this test. The total number involved was 682 all of whom had taken the L/C examination in mathematics, the majority with 1984 Leaving Certificate, repeat and mature students sitting in 1983 and earlier.

It was anticipated that students undertaking degree courses would differ in their capabilities from students undertaking certificate, diploma or professional business qualifications. This was confirmed by the data and the subsequent analysis will distinguish between the 83 degree students and what we term the 599 non-degree students.

4. OVERALL RESULTS

An overall pass rate of 27%, Table 4.1, confirmed that there are basic deficiencies in the mathematical skills of first year students.

TABLE 4.1

Results for All Students

Pass (%)	Total
187 (27)	682

However, when these figures are broken down by degree and non-degree students, it becomes clear that the overall pass rate

of 27% is misleading (see Table 4.2).

TABLE 4.2

Results for All Students by Degree and Non-Degree

Degree		Non-Degree	
Pass (%)	Total	Pass (%)	Total
66 (80)	83	121 (20)	599

The radical difference in pass rates of degree and non-degree students is related to the better L/C mathematics grade of the former. This point is considered in Section 7.

5. THE EXTENT OF THE PROBLEM

With a pass in the test set conservatively at 15 or more correct answers, the extent of success and failure by the students is of interest. Did all those who failed get 13 or 14 correct answers? and all those who passed 20 correct? The graphs in Fig. 5.1 answer these questions.

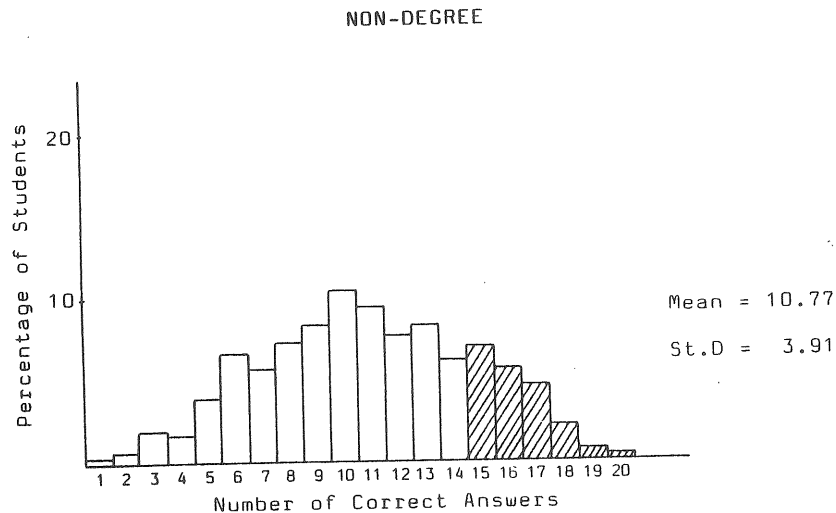
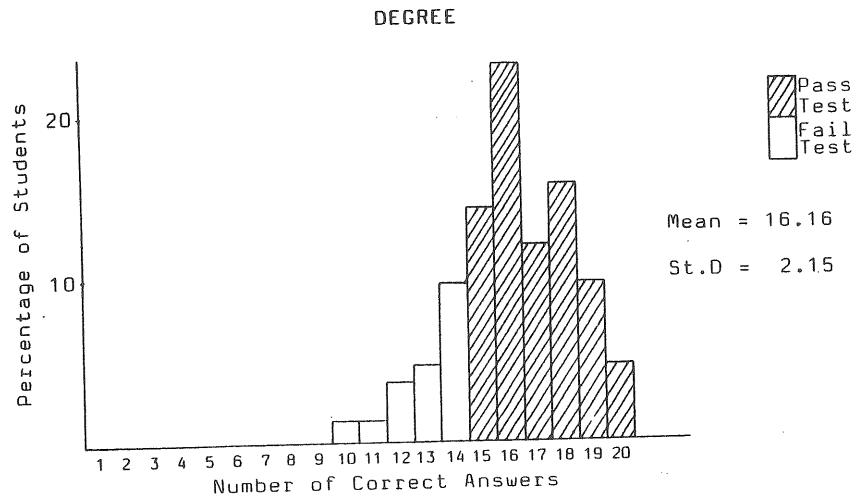
For degree students, 20% failed with 14% of students getting 13 or 14 correct answers. However our concern about the 80% of degree students who passed is caused by the fact that only 14% of students managed 19 or 20 correct answers.

For non-degree students the extent of the failures is alarming, not only did 80% of them fail, 48% of certificate students could only get 10 or less correct answers. Only 1% of non-degree students managed 19 or 20 correct answers.

Areas of difficulty for the student may be seen in the table of Appendix B where a question by question analysis is given.

FIGURE 5.1

Percentage of Students by Number of Correct Answers



The Data for these Graphs are given in Appendix C

6. STUDENT RESULTS BY SEX

Did the sex of students affect their test performance? The details are given in Table 6.1.

TABLE 6.1

Student Results by Course Type and Sex

Sex	Pass (%)	Total (%)	Sex	Pass (%)	Total (%)
Male	49 (80)	61 (73)	Male	94 (21)	444 (74)
Female	17 (77)	22 (27)	Female	27 (17)	155 (26)
Total (%)	66 (80)	83	Total (%)	121 (20)	599

Given the results of statistical tests of hypothesis and the awkward theoretic problems raised by the data (e.g. are they a random sample?), we consider that the sex of students is not a significant factor in test performance.

In Table 6.1 we see that the ratio of male to female students in both degree and non-degree courses is 3:1. This ratio does not apply to individual courses as some are exclusively male while in others females predominate.

7. RESULTS BY LEAVING CERTIFICATE MATHEMATICS GRADE OF STUDENT

The test results classified by L/C mathematics grade of all students is given in Table 7.1. In Table 7.2 we distinguish between degree and non-degree students. One mature non-degree student whose L/C predates the present grading system was omitted in both tables as being atypical.

TABLE 7.1

Results of All Students by L/C Mathematics Grade

L/C Grade		No. of Students	No. Passed	Pass Rate \pm 3S.E.*
Higher Course	A	1	1	1.000
	B	32	30	0.938 \pm .129
	C	68	55	0.809 \pm .143
	D	52	31	0.596 \pm .204
	E	5	1	0.200
Lower Course	A	37	22	0.595 \pm .241
	B	204	40	0.196 \pm .084
	C	197	6	0.030 \pm .036
	D	85	1	0.012 \pm .035
		681	187	

* S.E. = Standard Error, three standard errors are used to allow multiple comparisons.

From Table 7.1 we see that test performance is closely associated with L/C mathematics grade obtained, with significant difference between students with grade A and those with grades B, C or D on the lower course. The differences between students with grade B and those with grades C or D on the lower course are also significant.

It is tempting from Table 7.1 to order the L/C mathematics grades as follows:

<u>Higher Course</u>	<u>Lower Course</u>
A	
B	
C	
D	A
E	B
	C
	D

but this is only a tentative ordering since in some instances (grades A and E, higher course) there are too few observations and in others (grades B, C and D, higher course) the distinction is not clear.

TABLE 7.2

Results of Degree and Non-Degree Students by L/C Mathematics Grade

L/C Grade		DEGREE			NON-DEGREE		
		No. of Students	No. Passed	Pass Rate	No. of Students	No. Passed	Pass Rate
Higher Course	A	-	-	-	1	1	1.000
	B	19	18	0.947	13	12	0.923
	C	38	31	0.816	30	24	0.800
	D	16	12	0.750	36	19	0.528
	E	-	-	-	5	1	0.200
Lower Course	A	6	4	0.667	31	18	0.581
	B	4	1	0.250	200	39	0.195
	C	-	-	-	197	6	0.030
	D	-	-	-	85	1	0.012
		83	66		598	121	

From Table 7.2 we see that the better performance of degree to non-degree students is indeed related to their superior L/C grade. We also note that non-degree students with similar L/C grade to degree students (viz. grades B and C, higher course) are able to perform as well.

Finally in Table 7.2 we see that 81% of non-degree students intake had grade B, C or D on the lower course mathematics paper. Their subsequent poor test performance indicates a lack of basic mathematical skills in the majority of non-degree first year students.

CORK REGIONAL TECHNICAL COLLEGE

BASIC MATHEMATICAL SKILLS TEST 1

NAME: _____ CLASS: _____

TIME: 1 Hour

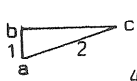
Instructions: Answer ALL questions. All questions carry equal marks.
Write answers clearly in boxes provided.
Use of calculators and mathematical tables not allowed.
Roughwork paper provided

APPENDIX A

By the term "basic mathematical skills" we mean a facility in handling the topics given in the following list:

- (i) addition, subtraction, multiplication, division, use of brackets,
- (ii) indices and logs,
- (iii) ratio and proportion, percentages,
- (iv) approximations,
- (v) units,
- (vi) factorisations,
- (vii) transposition and evaluation of formulae,
- (viii) simple equations,
- (ix) elementary geometry,
- (x) elementary trigonometry.

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- | | |
|---|--|
| 1. Evaluate $\frac{2}{3} - \frac{1}{5}$. | 1. <u>7/13</u> |
| 2. What value would you assign to $31 + 47 \times 10 - 3$? | 2. <u>498</u> |
| 3. Find the value of $\sqrt{6.4 \times 10^5}$. | 3. <u>8×10^2</u> |
| 4. Evaluate $3.21 \times 10^{-2} - 6.71 \times 10^{-4}$. | 4. <u>3.1429×10^{-2}</u> |
| 5. If $\log x = 2$, what is $\log(x^3)$? | 5. <u>6</u> |
| 6. Solve for x: $\log 3 + \log 9 - \log 12 = \log x$. | 6. <u>2.25</u> |
| 7. Express $4^4 \div 4^{-2}$ in the form 4^a . | 7. <u>4^7</u> |
| 8. If $5^x = \frac{1}{25^2}$, find x. | 8. <u>-4</u> |
| 9. The price of an article is £32.50 including 25% VAT. What is the price excluding VAT? | 9. <u>£26</u> |
| 10. Divide 72 in the ratio 1:5. | 10. <u>12:60</u> |
| 11. The approximate value of $\frac{0.077 \times \sqrt{120}}{(0.38)^3 \times (2.19)^2}$ is (a) 32, (b) 3.2, (c) 320, (d) 21.3, (e) none of these. | 11. <u>(b) or 3.2</u> |
| 12. Express 0.01 m^3 in cm^3 . | 12. <u>10^4 cm^3</u> |
| 13. By using factors or otherwise, find the value of $221^2 - 220^2$. | 13. <u>441</u> |
| 14. Factorise $2x^2 - x - 3$. | 14. <u>$(x+1)(2x-3)$</u> |
| 15. Re-arrange the formula $x = y(1+at)$ to give t in terms of the other quantities. | 15. <u>$t = \frac{x}{ay} - \frac{1}{a}$</u>
<u>or equivalent</u> |

16. Evaluate s in the formula $s = ut + \frac{1}{2}at^2$ when $u = 20$, $a = \frac{1}{4}$, $t = 8$. 16. 168
17. Find x if $3(x-2) = 12 + (5x-7)$. 17. $-\frac{11}{2}$
18. Find the solutions of the equation $(x-1)^2 - 4 = 12$. 18. 5, -3
19.  Δabc is a right angled triangle. Find $|bc|$. 19. $\sqrt{3}$
20. If $\cos A = \frac{4}{5}$, find the value of $1 - \sin^2 A$. 20. $\frac{16}{25}$

APPENDIX B

Question by Question Test Performance

TABLE B.1

Number and Percentage of Students Who Answered Each Question Correctly

Question No.	Degree 83 (%)	Non-Degree 599 (%)	All Students 682 (%)
1	79 (95)	439 (73)	518 (76)
2	39 (47)	189 (32)	228 (33)
3	73 (88)	358 (60)	431 (63)
4	64 (77)	272 (45)	336 (49)
5	67 (81)	234 (39)	301 (44)
6	44 (53)	125 (21)	169 (25)
7	73 (88)	307 (51)	380 (56)
8	63 (76)	153 (26)	316 (46)
9	74 (89)	317 (53)	391 (57)
10	41 (49)	521 (87)	598 (88)
11	41 (49)	243 (41)	284 (42)
12	36 (43)	116 (19)	152 (22)
13	74 (89)	466 (78)	540 (79)
14	68 (82)	397 (66)	465 (68)
15	79 (95)	338 (56)	417 (61)
16	81 (98)	417 (70)	498 (73)
17	78 (94)	412 (69)	490 (72)
18	72 (87)	322 (54)	394 (58)
19	78 (94)	468 (78)	546 (80)
20	83 (100)	211 (35)	294 (43)

Spearman's rank correlation between degree and non-degree students is .692. This correlation is significant at the 1% level and indicates that the areas of difficulty are common to both categories of student.

APPENDIX C

TABLE C.1

Number of Correct Answers by Number and Percentage of Students

DEGREE

No. of Answers Correct	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
No. of Students	0	0	0	0	0	0	0	0	0	1	1	3	4	8	12	19	10	13	8	4	83
% of Students	0	0	0	0	0	0	0	0	0	1.2	1.2	3.6	4.8	9.6	14.5	22.9	12.0	15.7	9.6	4.8	100

NON-DEGREE

No. of Answers Correct	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
No. of Students	3	5	12	11	25	41	34	43	51	52	57	47	49	38	41	34	28	13	4	1	599
% of Students	0.5	0.8	2.0	1.8	4.2	6.8	5.7	7.2	8.5	10.4	9.5	7.8	8.2	6.3	6.8	5.7	4.7	2.2	0.7	0.2	100

OVERALL

No. of Answers Correct	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
No. of Students	3	5	12	11	25	41	34	43	51	63	58	50	53	46	53	53	38	26	12	5	682
% of Students	0.4	0.7	1.8	1.6	3.7	6.0	5.0	6.3	7.5	9.2	8.5	7.3	7.8	6.7	7.8	7.8	5.6	3.8	1.7	0.7	100