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NEWS AND ANNOUNCEMENTS

ADVANCES IN LINEAR PROGRAMMING

Linear programming is perhaps the most important mathematical technique in use today, at least if importance is judged by any economic or utilitarian measure. By some estimates nearly one-fourth of the scientific computation time of all the computers in the world is devoted to solving linear programming problems. Efficient solutions to these problems can save industry millions of pounds each month. Modern economics and management science depend very much on solutions to linear programming problems.

The first method of solving these problems was devised during World War II by George Danzig (now of Stanford University) in an attempt to resolve the logistical problems of maintaining steady supplies to distant troops subject to the constraints of wartime scarcities. This method is called the simplex method and since its development, enormous economic benefits resulted from its use. In the mid-1950s, the Exxon Corporation used it to improve the blending of petroleum products and saved 2% to 3% of the cost of its blending operations. The application soon spread within the petroleum industry and at the same time other industries began to adopt the method. Today, "packages" of computer programs based on the simplex algorithm are offered commercially to customers who pay sizeable fees for their use.

The algorithm relies on two key ideas: that the solution must be one of the vertices of the polytope of feasible points, and that the sure way to find it is to climb steadily uphill (or downhill) along the edges. The number of vertices is finite, but even in a routine problem, it can be enormous. It is estimated that for the problem of allocating 2,000 limited resources to 2,000 products, the number of vertices of the polytope is of the order of 2^{2000} or 10^{602}. Yet the simplex algorithm can generally find the optimum solution by examining only about 6,000 of the vertices. In theory, the simplex algorithm is what computer scientists call a "hard" problem, that is, the computations required grow at a rate which depends exponentially on the number of constraints. However, practitioners have found that a good rule of thumb is that the number of calculations increases as a linear function of the constraints.

In the late 1970s L.G. Khachian, working at the computer centre of the Soviet Academy of Sciences, developed an algorithm which proved that linear programming is not really a "hard" problem and provided an alternative computation that can be used in those cases where the simplex algorithm proves too slow. But there were difficulties with this method. Since matrix inverses need to be calculated in the Khachian algorithm, roundoff errors are introduced at an alarming rate, and it is possible that the intrinsic computer error will grow so rapidly that the algorithm will not converge, yielding nothing but nonsense in the end.

Stephen Smale of the University of California at Berkeley demonstrated in 1981 that there is an upper bound to the expected number of vertices that must be checked by the simplex algorithm. His bound is a function of the "size" of the problem, where the "size" is defined as the sum of the number of constraints and the number of items being manufactured/allocated. As the problems become larger Smale's upper bound grows more slowly. Hence, although he has not fully explained why the simplex algorithm has performed as well as it has, he has shown that for extremely large problems, the expected number of vertices investigated is even smaller than the number given by a rule of thumb widely used by mathematicians. His bound is not a guarantee that the simplex algorithm will always work rapidly: the problem is a probabilistic one, although it should apply to the great majority of cases. For an account of Smale's work see [2].

When Danzig invented his algorithm, he reportedly told his colleagues not to worry about its slowness as he believed a more efficient method of solving resource-allocation problems would soon be forthcoming. Recently his hopes were realized. A new algorithm for solving linear programming problems has been developed and it is apparently much faster than the simplex method [1]. The algorithm is due to a 28 year-old Indian-born mathematician, Narendra Karmakar, who works for Bell Laboratories in New Jersey. Unlike the simplex method, Karmakar's algorithm deforms the polytope of feasible solutions as it proceeds, but unfortunately no technical details are available yet in the research literature. Mathematicians are keen to see and analyse the details of the algorithm and several large corporations such as Exxon, American Airlines and A. T. & T. wish to use it in their scheduling and allocation problems.

REFERENCES

1. ANGIER, N.
"Folding the Perfect Corner", *Time*, December 3rd, 1984, Science Column, page 55.

2. GAINES, F.
"Recent Developments in Linear Programming", *Irish Mathematical Society Newsletter*, No. 7 (March 1983), 29-35.

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