It seems clear that funding for such a project would reap as benefit an improvement in our teaching so that the country would produce, for example, better engineering students and more capable scientists. At any rate the increasing abundance of available computer time is still something that we are only just beginning to appreciate and there is now a need for programmers as "laboratory technicians", not just in computer science departments but also in mathematics departments.

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* While the author is on leave of absence at the University of East Anglia, Norwich, copies of the programs are available from Dr J.B. Quigley, Mathematics Department, U.C.D., at a cost of £30 to cover the magnetic tape, packing and postage.

Minor modifications will be necessary if DEC Basic is not available.



## BOOK REVIEWS

### "SINGLE-VARIABLE CALCULUS"

By *Robert A. Adams*

Published by *Addison-Wesley*, 1983, £19.95 (sterling), 590 pp.

It must be a daunting task to set about writing a 600-page text book. It could be that the author wishes to introduce to a wider audience material not available in book form: however, in the case of a calculus text, a new author must search for new approaches, which will bring a greater unity or clarity to the subject matter. In the past twenty years, calculus texts have grown considerably in size (satisfying logistic growth rather than exponential growth, I hope), the increase in size being partly due to increase in page size for clarity in reading, but also partly due to additions of appendices to make each new edition more comprehensive. In the preface of Single-Variable Calculus, Professor Adams claims to have produced a book which "is not as massive or bulky in appearance as many other books available in recent years." Although this is certainly true, measuring $7\frac{1}{2}$"x $9\frac{1}{2}$"x $1\frac{1}{2}$" and weighing $2\frac{1}{2}$ lbs it is in no way a pocket calculus. The author is able to keep the number of pages below 600 because he regards calculus of several variables as suitable for a separate text (perhaps, he is writing a sequel himself).

Single-Variable Calculus is primarily designed for a two-semester course for science and engineering students. In Chapter 1, Functions, Limits, Continuity, the author introduces the $\varepsilon$- $\delta$ definition of a limit but relegates the proofs of the results about limits and continuous functions to an appendix. The brief treatment of inequalities I found unsatisfactory but the inverse of a 1-1 function is introduced without the confusing terminology of injections and surjections.

Some new ideas appear in Chapter 2, Differentiation: Definition, interpretation and techniques. A worked example brings out the idea of a cusp and the modulus function $|x|$ is differentiated as sgn x. Both these ideas could give rise to interesting new problems in later chapters but unfortunately the opportunity is not taken. In this chapter, the author also introduces antiderivatives, indefinite integrals, differential equations and initial value problems. This is certainly a change from the traditional treatment. It appears to succeed but for those who teach differentiation then teach integration it may be difficult to change habits of a lifetime.

Chapter 3 is devoted to teaching The Elementary Transcendental Functions. This is done at this stage so that these functions can be utilised in the next chapter. Interestingly, the author gives the pronounciation of ln as 'lawn' but gives no guide as to how to pronounce sinh, tanh etc.

Chapter 4, Various applications of Differentiation deals fully with the search for local maximum and minimum points by noting that these can occur at critical points, end points and singular points. Systematic procedures are given for solving Optimisation and Related Rates problems which end with the sound piece of advice "Make a concluding statement answering the question asked." My only quibble with this chapter is that in treating Newton's method it is good practice to show how the calculations can be laid out neatly in tabular form. This is not done.

The smallest chapter, Curves in the Plane, consisting of optional material, is followed by the longest on integration. A list of 20 integrals is given that the reader is told to memorize, undoubtedly another piece of sound advice, although certainly frowned upon by most Irish secondary school teachers. One of the handiest rules of integration is

$$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + C.$$



No mention is made of this: has it gone out of fashion?

The standard applications of integration are covered in Chapter 7, viz. Volumes, arc length, surface area, center of gravity all within the limitations of one variable calculus.

We must wait until Chapter 8 on Infinite Series to meet those dreaded words "the proof is left as an exercise." Apart from an unusual way of proving $\lim x^n = 0$ if $|x| < 1$, the treatment of infinite series is good with a section on "Estimating the sum of a series" which other calculus authors should consider adding to their next editions. The book concludes with a short chapter on Power Series representation of functions.

Interspersed throughout the text are more than 2000 problems: the vast majority provide drill in basic techniques but there are also a number of more interesting and harder asterisked problems to challenge the better student. Solutions of odd-numbered problems appear at the end of the book.

Overall, the presentation and layout of the material is excellent, up to the high standard one has come to expect from Americal text books. With a first edition, there is always the problem of errors and misprints. Although the author does suggest that these have all been removed a number of mistakes remain. These are mainly of a minor nature although the statement of Theorem 2 of Chapter 9 and a subsequent statement are false, and there is a mistake in a worked problem on L'Hôpital's rule.

A modern trend, of which I wholeheartedly approve, is to give biographical details of those whose names appear in the text (one book recently even had photographs and half-page biographies of the principal workers in the area). I certainly feel that such (trivial?) details make a text book much more friendly. Perhaps, in America the history of mathematics is sufficiently covered in other courses but here in Ireland we seem to ignore these matters. I would certainly welcome such additions to this text.

In the Preface, the author argues for the need for a separate text on calculus of several variables. Personally, I am not convinced, for economic as well as other reasons, by this argument. However, for those who feel the need for a text on Single-Variable Calculus I can certainly recommend this book.

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"TOPOLOGY AND GEOMETRY FOR PHYSICISTS"

By *Charles Nash, St. Patrick's College, Maynooth, and Siddartha Sen, Trinity College, Dublin*

Published by *Academic Press, London,* London, 1983, Stg. £31.50,

ISBN 0-12-514080-0

MATHEMATICAL PHYSICS YOUR MOTHER NEVER TAUGHT YOU

Mathematical Physics as a discipline has been defined and dominated by a single book in a way that no other field of science has been. This book is, of course, Methods of Mathematical Physics by Richard Courant and David Hilbert. Courant and Hilbert first appeared in Germany in 1924, and has been continuously available in a sequence of different forms ever since. It is still in print, the two volumes costing well over £100.



The mathematics in Courant and Hilbert, despite some modern touches, has a curiously nineteenth century flavour to it. The book is focused on differential equations and so naturally deals with continuous functions. The way it links up with the discontinuous nature of much of modern physics, especially quantum mechanics is via the eigenfunction/eigenvalue approach where each individual eigenfunction is a solution to a differential equation and so inherits its differentiability properties from it, but the eigenvalues themselves tend to be discrete.

The only significant branch of Mathematical Physics which stood apart from the Courant-Hilbert approach was group theory, which used the discreteness of the group elements to model nature. Thus, ten years ago, if one had a good grounding in both Courant/Hilbert and some group theory, all one needed was a smattering of physics and one could hold one's head up as a mathematical physicist in the fanciest of company.

This golden age has completely vanished. In the last decade there has been a flood of new ideas flowing into physics from mathematics especially in geometry and topology. For example, a unit cell in a crystal is a three-manifold without boundary. This means that it is trivial to show that the total charge in each cell must be zero. Of course, this is not a new result, but it is a very simple example of how even elementary topology can and should be used.

The book under review is an attempt to codify and make available to the ordinary physicist the key ideas of modern geometry and topology. It assumes no previous knowledge, and so starts off with two introductory chapters, one on general topology and one on differential geometry. Then come four chapters on homotopy, homology and cohomology. The last of the mathematical chapters is a long (80 page) chapter called "fibre bundles and further differential geometry". The last three chapters apply the previously developed tools to a range of physical problems. Two of these are fairly short and deal respectively with Morse theory, which is applied to phase tran-