

for  $f, g \in C(\mathbb{R}^+)$ ,  $f \otimes g$  is the function in  $C(\mathbb{R}^+)$  given by

$$f \otimes g(x) = \int_0^x f(t)g(x-t)dt.$$

By a theorem of Titchmarsh,  $C(\mathbb{R}^+)$  is an integral domain with this product, and Mikusinski was led to the quotient algebra  $Q(\mathbb{R}^+)$ . It turns out that  $Q(\mathbb{R}^+)$  contains the usual differential and integral operators, as well as many distributions. In particular, the Dirac  $\delta$  and the distributions are in  $Q(\mathbb{R}^+)$ , where  $s(f) = f' - f'(0)$  for  $C^1$  functions  $f$ . As a consequence, one can apply Laplace transform techniques, using  $s$ , to solve differential equations with constant coefficients, integral equations, etc.

The main body of the book is the last two chapters. Chapter 4 deals with convolution equations in spaces of distributions. It is here that the problem of finding fundamental solutions for various operators is addressed, with Kec examining the role of special spaces of distributions. The Cauchy initial value problem is considered, in a number of settings, and applications are made to the wave equation, heat equation, etc. Finally, in Chapter 5, the author applies the methods of the previous chapter to solve differential equations arising in electrical and mechanical engineering and in viscoelasticity.

The text is readable, although the English is not always idiomatic. It is evident that the translator has little mathematical experience. Thus, for example, we find ourselves considering the "body of real or complex numbers" and the open unit "bubble" of a normed space. A more substantive criticism can be made of the author's approach, from the mathematician's point of view. Routine results, such as properties of the convolution, are usually proved in full detail. On the other hand, the discussion is often incomplete in terms of (mathematically) more interesting results. For example, no attempt is made to discuss topological properties of the space of distributions, beyond some mention of the weak-\* topology. No mention is made of such beautiful results as the Titchmarsh-Lions

theorem on the support of convolutions, the Paley-Wiener theorems, etc. Indeed, the relation of analytic function theory to this subject seems to have been largely ignored. Unlike Schwartz's "Mathematics for the Physical Sciences", this book (which might be considered as "convolution equations for the engineering sciences") has no exercises.

These doubts having been raised, it must in fairness be mentioned that it seems remarkable that, as Kec shows, one can get many, apparently non-trivial, results in engineering mathematics using only the material developed in this volume. Thus, it may well be that the book serves the very useful purpose of introducing engineers to this fruitful area of mathematics.

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#### "MATHEMATICAL SNAPSHOTS"

By *H. Steinhaus*

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#### PREFACE TO THE GALAXY EDITION

By *Morris Kline, Professor Emeritus of Mathematics at the Courant Institute of Mathematical Sciences, New York University.*

This reprinting of the third, enlarged edition of Steinhaus' *Mathematical Snapshots* is more than welcome.

The book must be distinguished from numerous books on riddles, puzzles and paradoxes. Such books may be amusing but in almost all cases the mathematical content is minor

if not trivial. For example, many present false proofs and the reader is challenged to find the fallacies.

Professor Steinhaus is not concerned with such amusements. His snapshots deal with straightforward excerpts culled from various parts of elementary mathematics. The excerpts involve themes of sound mathematics which are not commonly found in texts or popular books. Many have application to real problems, and Steinhaus presents these applications. The great merit of his topics is that they are astonishing, intriguing and delightful. The variety of themes is large. Included are unusual constructions, games which involve significant mathematics, clever reasoning about triangles, squares, polyhedra, and circles, and other very novel topics. All of these are independent so that one can concentrate on those that attract one most. All are interesting and even engrossing.

Professor Steinhaus explains the mathematics and his fine figures and excellent photographs are immensely helpful in understanding what he has presented. He does raise some questions the answers to which may be within the scope of most readers but the reader is warned that some answers have thus far eluded the efforts of the greatest mathematicians. Mathematical proof demands more than intuition, inference based on special cases, or visual evidence.

This book should be and can be read by laymen interested in the surprises and challenges basic mathematics has to offer. Professor Steinhaus is mathematically distinguished, and, as evidenced by the very fact that he has undertaken to present unusual, though elementary, features, is seriously concerned with the spread of mathematical knowledge. The careful reader will derive pleasure from the material and at the same time learn some sound mathematics, which is as relevant today as when the original Polish edition was published in 1938.

## PROBLEMS

First the solutions to some previous problems.

1. A car park has spaces numbered  $1, 2, \dots, n$ . Any driver arriving with a ticket for space  $k$  parks at space  $k$  unless it is occupied, in which case he chooses the first vacant space from  $k+1, k+2, \dots, n$ . If these are occupied he leaves in disgust.

If  $n$  drivers arrive in turn, each with a ticket bearing a randomly chosen integer between 1 and  $n$ , prove that they can all park with probability  $(n+1)^{n-1}/n^n$ .

This problem appeared in Vol. 1, No. 1, of the Mathematical Intelligencer and the solution appeared in the next issue. Briefly, the idea is to consider a modified problem in which the tickets bear randomly chosen integers between 1 and  $n+1$ , and in which the car park has  $n+1$  spaces and is circular. The  $n$  drivers are able to park (since they can go round again) and there is always one space left at the end. The answer to the original problem is then clear because:

- (i) a successful outcome in the original problem corresponds to an outcome in the modified problem in which the space  $n+1$  is left vacant, and
- (ii) in the modified problem there are  $(n+1)^n$  sample points, exactly the same number of which leave any given space vacant (why?).

2. Ship A is moving due east at constant speed and, at a certain moment, ship B is moving due north at the same speed towards A. If B maintains this speed but continuously alters course towards A how closely can B approach A?

Let both ships have speed  $v$  and begin at a distance of  $d$  miles. Make the construction indicated in Fig. 1 overleaf.