IRISH MATHEMATICAL SOCIETY

NEWSLETTER

NUMBER 6 DECEMBER 1982

THE IRISH MATHEMATICAL SOCIETY

President: *J.J.H. Miller* School of Mathematics, 39 Trinity College, Dublin 2.

Vice-President: *F. Holland* Department of Mathematics, University College, Cork.

Secretary: *S. Dineen* Department of Mathematics, University College, Dublin 4.

Treasurer: *R. Ryan* Department of Mathematics, University College, Galway.

Membership and Correspondence

Applications for membership, notices of change of address or title or position, and other correspondence, except as noted below, should be sent to the Secretary. Subscriptions and correspondence related to accounts should be sent to the Treasurer.

Newsletter

The aim of the Newsletter is to inform members of the Society of the activities of the Society and also of items of general mathematical interest. Information on activities of interest to members is sought, as well as survey articles, problems and solutions.

The present address for correspondence relating to the Newsletter is

Irish Mathematical Society Newsletter,
Department of Mathematics,
University College, Cork.

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Editorial

Since the appearance of the last issue of the Newsletter, there have been two developments which will be of great assistance in consolidating and developing the production of the Newsletter.

Firstly, as you will be immediately aware on receiving this issue, the printing is a welcome improvement on the duplicated production of the last issue. The National Board of Science and Technology have very generously agreed to print the Newsletter for us, beginning with this issue. The Irish Mathematical Society appreciates very much this assistance from the N.B.S.T., and indeed other forms of support it is providing which you can read about in the Secretary's report. We are indebted to Sean Dineen, the Secretary of the I.M.S., who negotiated this deal with the N.B.S.T..

Secondly, Mathematical Reviews Editors wrote to me requesting copies of the Newsletter as they are interested in reviewing the publication. This is a reflection of the quality of contributions to the Newsletter in the past.

However, there is still much room for improvement. On the technical side, the layout needs to be improved so as to have a more visually attractive Newsletter. On the other hand, the range of articles needs to be broadened. One of the major criticisms of the last issue was that it was an almost exclusively Pure Mathematics publication. I have tried to rectify this by requesting articles from several Applied Mathematicians for inclusion in future issues. In this issue there is some material of particular interest to Statisticians which provides the kind of balance I wish to have between the Pure and Applied areas of Mathematics.

Again, I encourage all members to consider writing for some of the various sections of the Newsletter. For those who may have brief comments on any articles, I intend to introduce a Letters to the Editor section as soon as some material is at hand.

Finally, I take the opportunity to wish all members of the Society a happy Christmas and the best of luck for the New Year.

Donal Hurley

Dr. James J. McMahon

Obituary:

J.J. McMahon died in September 1981 after a brief illness. His unexpected passing was a shock to all who knew him, but most of all to his wife, Catherine.

J.J., as he was called, was born at Woodford, Co. Galway in 1924 and went to school at St. Joseph's College, Garbally Park, Ballinasloe. He then studied at St.Patrick's College, Maynooth where he received his B.Sc. degree in 1946 and his M.Sc. in 1948 and was, in fact, the first M.Sc. graduate of Maynooth (in Mathematics). Having worked under J.L. Synge at the D.I.A.S., he obtained his Ph.D. degree from N.U.I. in 1953, and was the first person ever to receive a Ph.D. in Mathematical Sciences from N.U.I.. He was ordained to the priesthood in 1950.

His early interests were in applied mathematics, studying electricity and magnetism, and quantum theory for his M.Sc..His Ph.D. thesis concerned the hypercircle method. During his stay at Stanford University (1952-54), where he worked with Polya and Szego, he developed an interest in pure mathematics - mainly differential geometry and algebra.

He returned to Maynooth as a lecturer in 1954 under Professor McConnell who was then head of the (single) department of Mathematics and Mathematical Physics. They both lectured the entire range of Mathematics courses (pass and honours). J.J. spent a sabbatical year (1959-60) at Fordham University and in 1960, he was appointed Professor of Mathematics in Maynooth. Another sabbatical year (1972-73) was spent at the University of Ottawa. While Professor, he supervised some master's theses.

In June 1974, J.J. resigned as Professor and became a layman. He lectured for a year at University College, Galway and then spent a term teaching at Worth School, Crawley, Sussex while awaiting a visa for Nigeria. In January 1976 he went to the University of Benin as Senior Lecturer and stayed there until 1979 when he returned to Ireland to take up a position at Thomond College of Education in Limerick.

He was an enthusiastic member of the Committee of the Irish Mathematical Society and was appointed Editor of the Newsletter in the year before his untimely death.

R. Timoney

Publications:

1. Lower bounds for the electrostatic capacity of a cube, Proc. Roy. Ir. Acad. Sec. A 55 (1953) 133-167 (MR 15 425).

2. Lower bounds for the Dirichlet integral in Euclidean n-space, Proc. Roy. Ir. Acad. Sec. A 58 (1956), 1-12 (MR 18 202).

3. Matrix proof of Pascal's theorem, Amer. Math. Monthly 65 (1958) 24-27 (MR 20 4804).

4. (With William Clifford), The rolling of one curve upon another, Amer. Math. Monthly 68 (1961) 338-341 (MR 23 A2134).

5. Group similar isometries, Proc. Roy. Ir. Acad. Sec. A 65 (1967) 51-61 (MR 35 4244).

6. Lower bounds for the Dirichlet integral, Studies in Numerical Analysis (Papers in honour of Cornelius Lanczos on the occasion of his 80th birthday), Academic Press, London (1974) 219-234 (MR 50 11814).

7. $SL(2,\mathbb{C})$ and the Lorentz group, Proc. Roy. Ir. Acad. Sec. A 75 (1975) 79-83 (MR 53 725).

Appreciations:

J.J., as he was universally known, was a man of great integrity and principle, a very hard worker, thorough and diligent, and anxious to help students. He could appear withdrawn, since he worked quite hard and had few close friends or confidants; however he had a very winning sense of hunour and this, allied with a noticeable twinkling of the eyes, soon dispelled any initial sense of awkwardness.

J.J. was a great outdoors man and regaled me once with stories of scuba diving off the West Coast of Ireland where he was amazed at the variety and colour of underwater life. He was also a keen swimmer, and I also recall a story he told me about being 'dive-bombed' by sea-gulls whilst climbing on the cliffs at Howth Head. That was one time he was really frightened, he said. I am sure he had many more such tales if I could but persuade him to talk about them. In addition I often saw him hurling in the sportsfield with the students and he liked to go down to the midland lakes in May for the 'dapping', when the trout rose to the mayfly.

He was a man with a rugged self-sufficiency, independent mind, and could speak bluntly and fearlessly when occasion demanded. He was very patient and helpful with students among whom he had a reputation for absent-mindedness. There is a story, perhaps apocryphal, that he met a student at the entrance gate after morning lectures one day and engaged in a long conversation with him at the end of which J.J. asked "Was I coming in or going out when you met me?" "Going out" was the reply. "Ah well, I must have had my dinner so" was J.J's conclusion.

A non-smoker, he drank abstemiously and watched his diet and his health carefully. It was therefore, a real shock to find that he died of cancer at the age of 55.

He also had an interest in astronomy and took me out one winter's night to look at the planets through his telescope.

He will be missed greatly by all those who appreciated his inquiring mind, his rugged individual nature, his loyalty to his friends and his unique sense of humour.

David Walsh

† † †

In January 1976, I first met Jim McMahon (or J.J. as he was known in Ireland) when he came out to the University of Benin to join a relatively new Mathematics Department about to produce its first mathematics graduates. After a few initial problems adjusting to the climate, Jim soon settled down to teaching in his new environment where his wide knowledge of mathematics enabled him to lecture on courses right across the mathematical spectrum. He played a major role in a revision of the mathematics courses which gave rise to degrees in mathematics, industrial mathematics and mathematics/education. Life at the University of Benin during his time there was never dull: the university was closed on two occasions and several scandals involving those in power brought the university front page headlines in the press. Jim performed his duties unobtrusively and had a quiet laugh at the goings on elsewhere. He got on well with both staff and students: it was thus with great regret that the Mathematics Department bid farewell to him after his decision to return to Ireland.

I was fortunate to meet up with him again in Ireland and was keenly looking forward to renewed mathematical contact as well as occasional encounters on the tennis courts when I moved to Limerick in September 1981. Unfortunately it was not to be.

As Sir Alexander Oppenheim, who had been our Head of Department in 1976, wrote on hearing of his death 'Jim was a man of rare and unusual quality, truly integer vitae, good for all around'. He had been a true friend, mathematically and socially. His death, a great loss to the Irish mathematics community, was also a great personal loss.

Gordon Lessells

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Dr. J.J. McMahon died on 28 September 1981. Only a few weeks previously I had learned with deep shock, almost with disbelief, of his terminal illness. I felt, and hoped, that a miracle would give a new lease of life to one who was still comparatively young and who had, in his own inimitable way, always lived life to the full.

I had known J.J. for almost thirty years, ever since his appointment to the Department of Mathematics in Maynooth in June 1954. Ten years earlier I had met him briefly when he was a second year Science student. It was his turn to act as sacristan-cum-Mass server, in an oratory, popularly known as the 'Synagogue'. Of all the students whom I encountered there J.J. was the only one who remained firmly fixed in my memory. There was something about him that attracted one's attention - a gentleness, a charming simplicity, a beguiling blend of friendliness and deference - that set him apart from other students. When he later became my colleague I remembered him immediately from that first brief encounter.

During his last years in Maynooth we became very close friends, often going on long walks together and having discussions on a variety of topics. During these walks he sometimes became expansive in a way that he never did in the College. He was rather reticent by nature, even secretive. It is true that he was most discreet, but never irritatingly so - a roguish chuckle always saw to that. Rarely, if at all, did he wear his heart on his sleeve. The last thing he ever wished to do was to bore his friends with his personal problems.

Others have paid tribute to J.J. McMahon's mathematical expertise. But he was much more than a mathematician. In St. Joseph's College, Garbally Park, he acquitted himself brilliantly in many other subjects - Irish, Latin and, in particular, Greek. He always maintained a keen interest in languages. He read and spoke French fluently. He mastered

Spanish while studying at Stanford University, California. German he later added to his linguistic repertoire. Shortly before he left Maynooth he was reading one of Thomas Mann's novels in the original. J.J. was a true philomath.

He also had many non-academic interests. He was amazingly versatile, an all-rounder, a man for all seasons, with more than a dash of Odyssean polutropia. As a student in Maynooth he was, I believe, captain of his class hurling team. He was a keen and experienced mountain climber and swimmer. During the year he spent in an English Benedictine school after leaving Maynooth, he taught himself archery and then taught it to some of his pupils. (I teased him later with being a cryptotoxophilite). Spectator sports were not for him.

His versatility extended even further. When leaving for England he hired a large furniture van in Dublin, drove it to Maynooth, collected all his belongings and then drove to Rosslare, crossed over to Fishguard and continued across Wales and England, getting meals in roadhouses at a reduced rate, (as he told me with a chuckle) just like any other trucker. Later he drove the empty van back to Dublin.

His appetite for knowledge was almost Aristotelian in its range and intensity. Shortly after his arrival in Benin City (Nigeria) to take up a post in the Mathematics Department of the University, he wrote me a long letter, much of which was devoted to en enthusiastic description of the local fauna and flora. He never lost his sense of wonder.

His decision to leave Maynooth and seek laicisation must have been an exceptionally agonising one. I think he was upset by some of the issues raised by the Second Vatican Council. He read extensively in the new theology without, apparently, finding there the reassurance which he so earnestly sought for his priestly ministry. A man of complete intellectual honesty, he was utterly incapable of even diplomatic double-think. He pursued truth with a Socratic singlemindedness. Plato's exhortation 'let us follow the argument whithersoever it leads', could well have been his motto.

I shall always treasure the memory of this lovable, self-effacing man, a most loyal friend who loved to do good by stealth and blushed to find it fame. To me he was the perfect embodiment of Tertullian's anima naturaliter christiana. May he rest in peace.

William Meany

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Ordinary Meeting

An Ordinary Meeting of the Society will be held on Tuesday 21st December from 12.30 to 1.30 p.m. at the Dublin Institute for Advanced Study, 10 Burlington Road, Dublin 4.

The Committee of the I.M.S. have proposed that the meeting consider the following change in Rule 5 of the Constitution of the Society. Rule 5 states: "The Committee shall consist of the President, Vice President, Secretary, Treasurer and six additional members. No person may serve as an additional member for more than three sessions consecutively". The proposed change is to increase the number of additional members from six to eight.

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Secretary's Report for period January 1, 1981 to 31st December 1982.

The basic philosophy of the Officers and Committee over the last two years was to maintain and stabilize the operations of the society (which up to now had been developed by the efforts of certain individuals - notably Tom Laffey and Finbarr Holland) and to ensure continuity. To this end, we examined in detail what the society was doing, what is should be doing and what it could be doing. Persistent effort and realistic goals enabled us to achieve a number of our aims.

1. Financial Developments

A society needs a steady source of income. Until 1981, the Society's only funding came from the £2 membership fees. The haphazard method of collecting fees was a great waste of time for our Treasurer and Committee. It also resulted in a very low income and this in turn limited our choices and did not help morale. On the other hand, certain established projects - the Newsletter, Mathematics Contest and Conferences, were almost exhausting our meagre funds and at the same time not getting the financial support they deserved. We decided to increase our income and lower our expenses.

We increased our income by raising the membership fee to £3.50 and by introducing the direct debit system of payment. We were thus assured

of a regular income at a fixed time and the nuisance of collecting fees was reduced to a manageable size. These changes on their own put the Society on a sound financial footing. In the summer of 1982 we introduced Institutional Membership at £35 per institution. This has also proved very successful (see the Treasurer's report) and should match pound for pound our ordinary membership income. We reduced our financial outgoings (see items 2,3,5 below) in a proportional manner without reducing our services in any way and the net result that we are now in a secure and stable financial position.

Finally, to rationalize our financial dealings we decided to plan our budget for the year at our Easter Committee meeting and it is hoped that any members who are seeking financial support for conferences, etc. will apply to the Committee to have their case discussed at that meeting.

2. Publications

The official publication of the Society is the Newsletter. We decided that it was desirable that the Newsletter should appear at regular intervals, that it should be as self-supporting as possible and that the presentation be improved. As a first step, we appointed D. Hurley as editor and P. Fitzpatrick as subeditor for 1981-1983. The Newsletter will now appear twice a year. Numerous discussions have taken place on the exact direction the Newsletter should take and we are still experimenting. Members will be happy to note the presentation of the current volume, to know that this quality will be maintained and to know that the practical assistance of the NBST allows us to present this improved format for our members with little drain on our finances.

In 1981, R. Timoney produced a comprehensive directory of Irish Mathematicians and this was distributed to all members. It is planned to update this directory triannually. Changes should be forward to the Secretary of the Society during the first half of 1984.

In 1981, we arranged with the R.I.A., that the Proceedings of the Academy (Series A) would be sold to members at a $33\frac{1}{3}$% discount. We also arranged that members can obtain DIAS publications at considerably reduced prices (see p.62 of this volume for further details). For various technical reasons, these schemes, which were only begun this year, were slow to get off the ground, but members are already benefitting from these arrangements and the future looks promising.

3. Mathematical Contest

Two successful mathematical contests were held in 1981 and 1982. These were organised jointly with the IMTA. Finbarr Holland and Tom Laffey organised on behalf of the IMS. Details of the contests, winners, etc. have already appeared in the Newsletter. This year, we obtained sponsorship for the running of the contest from the NBST and the Bank of Ireland. We also decided that our next priority is to send a team to the International Olympiad in Mathematics which is normally held in some European city. The team would consist of the top four students in the National Competition and a coach. If this could be achieved then we feel that the numbers entering the competition would double - the current figures are about 1500 - and perhaps inspire more young people to a career in Mathematics. We are currently exploring ways of realising this goal.

4. Liaisons

In 1981, we established formal contact with the NBST. As a result of this contact the NBST appointed Dr. Brian O'Donnell to liaise with mathematicians and all members of the Society, and also prospective members, should feel free to contact Dr. O'Donnell with any enquiries regarding research grants, etc.

As a result of our contact, the NBST has become an Institutional Member of the Society, and has given us considerable help with the Newsletter and the Math. Contest. We also have been kept informed on details of the drafts of the many international cultural and scientific exhange programmes which are currently being prepared by the NBST in order to make sure that the interests of the mathematical community are safeguarded. We hope to publish details of agreements of interest at regular intervals in the Newsletter. As a particular example, we would like to mention the NBST/CNRS (or Irish-French) scientific agreement. This agreement has been extremely useful, especially in the current recession, to mathematicians in Ireland. It was frequently used by mathematicians, and in fact mathematics was a priority area in the agreement. This made it easier for our members to avail of the agreement. In 1980 the CNRS replaced mathematics by some other area in the priority list. The reason was not any reflection on mathematics but due to the fact that there is a limit on the number of priority areas. The CNRS wanted a new priority area and chose at random to remove mathematics. After making representations to the NBST and some correspondence with Societe Math. de France, we are happy to say that mathematics was restored as a priority area in 1982. (Members interested in this scheme should contact Grainne Ni Uid at 01-68311.)

Finbarr Holland represented the Society to the European Math. Council (= U{European Math. Soc.}) and John Miller is on a Committee of this council which is looking at the crisis facing mathematical publishing. A short report of the work of this council has appeared and may be obtained from the secretary of the Society.

We are currently having informal discussions with the IMTA (Irish Math. Teachers Association) concerning more cooperation on matters of mutual interest and with the possibility of establishing reciprocal membership. Details will be presented at an ordinary meeting before any decisions are made on this matter.

Our Newsletter has been distributed to many national mathematical societies and has been quite useful in informing people abroad of the nature and extent of mathematical activities in Ireland.

For the past two years, we have endeavoured to secure the right to nominate two members to the National Committee for Mathematics - which up to now, represented Irish mathematicians to the world, and to the National Subcommission for Mathematical Instruction, rather than accept the current individual membership nominations. It appears that the constitutions of the National Committees are currently under review and perhaps there will be a change when this review is completed.

5. Conferences and Visiting Speakers

The Society has supported practically all mathematical conferences held in Ireland over the last two years. Our policy is to help financially as much as possible but to strongly encourage organisers to seek funds elsewhere. The fact that we can now guarantee reasonable support should give organisers confidence to make plans. At the same time, the knowledge that unused funding will be given to the next conference (organised perhaps by their own colleagues) is a strong motivation not to call in our guarantees.

The only speaker supported by the society recently was Steve Smale who gave an excellent lecture on the Simplex Method in Dublin in September 1982. This lecture was very well attended. More effort could perhaps have been made in this regard.

6. Prize for Mathematical Research

The Committee proposed to an ordinary meeting (December 1981) the motion that a prize for research in Mathematical Science be awarded by the

Society. This motion was passed and a Committee was set up to consider the most suitable way of implementing this motion. The Committee will report shortly on this matter to an ordinary meeting of the Society.

7. Humanitarian Support

The most controversial topic of the last two years discussed at Committee meetings (and also at ordinary meetings) was the topic of how much, and in what fashion, the Society should support mathematicians who are prevented from practising their profession because of racial, religious or political discrimination. At the December 1981 ordinary meeting of the Society, a motion was passed agreeing in principle to support such people (see the minutes of that meeting for an exact wording of the motion). At the April 1981 meeting, the ordinary members clarified the procedure that should be adopted by the Committee in dealing with such situations.

In September 1981, the Secretary, on the instructions of the Committee, wrote a letter of support for Jose Luis Massera (an Uruguayan mathematician). A complete dossier on the Massera case will be on public display at the next ordinary meeting of the Society. A voluntary collection will also be taken at that meeting and the proceeds sent to the Canadian Math. Society which is currently coordinating the campaign to obtain Massera's release.

8. Administration

To ensure greater efficiency in the running of the Society, the Committee organised and implemented certain changes in the last two years. These are as follows:

(a) All Officers and Committee members are now elected for a two-year period. Half the Committee is now elected every second year. This involved a change in the rules and the required changes were passed at the ordinary meeting in December 1981. Consequently, the incoming and the outgoing Committee have nonempty intersection and we have more continuity in the Committee. It also means that Committee members have more time to get familiar with the workings of the Society. At the December 1982 meeting, the Committee are also proposing that the Committee be increased from six to eight members.

(b) The general philosophy until 1980 on the composition of the Committee and Officers was that most major institutions and regions should be represented. While this is admirable in theory, it led to two major problems in practice. In the first place, it led to a

Committee which was very dispersed geographically and had difficulty meeting, and secondly, people who had no interest in the running of the Society were elected sometimes without their consent or knowledge.

To overcome these problems the Committee decided to approach certain people to go forward for election. This partial panel was to ensure that people were not put forward without their consent and that there was at least one conscious candidate for each important position. To have an efficient set of Officers and Committee, we felt it best that the chief Officers of the Society (the President and Secretary) plus two more Committee members should be located close to each other. Thus the main centre of the Society should be located in a certain area, but could move to other centres such as Cork, Galway, etc. whenever a group of people from these places were willing to run the Society for a few years. This plan would also eliminate any regional problems we might have.

(c) To further improve the problem of representation, we are currently investigating the question of appointing local representatives. These representatives would be appointed by the Committee and help in the distribution of the Newsletter and in the collection of membership fees. In this way, all institutions would have some representation, and communication between the Committee and the ordinary members would improve.

Sean Dineen

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TREASURER'S REPORT

The state of the Society's finances as of Nov. 1, 1982 is as follows:

INCOME	£	EXPENDITURE	£
Balance from 1981	48.35	Newsletter (Sept. issue)	94.66
Membership subscriptions	360.00	Direct. Of Irish Mathmns.	15.00
Institutional Memberships	140.00	Visit of S. Smale	11.00
Groups in Galway '82		Groups in Galway '82	
Conference fees	55.00	Expenses & Refreshments	215.00
Support from R.I.A.	100.00	Support for Algebra Conf.	
		at M.I.C.E., Limerick	50.00
		Balance in hand	317.69
	703.35		703.35

R.A. Ryan

FOREIGN MEMBERSHIP

The Irish Mathematical Society has decided to launch a membership recruitment drive amongst mathematicians based abroad. At present we have a small number of members in this category and we are confident that this can be greatly increased, especially by making the Society known to Irish people in the mathematics departments of overseas universities and colleges.

Foreign membership, which is available on the same basis as home membership, ensures regular contact with the Irish mathematical scene through the Newsletter.

Existing members, who know of colleagues abroad who may be interested in joining the Society, are requested to send names and addresses to me.

G.M. Enright
Mathematics Department,
Mary Immaculate College,
Limerick, Ireland.

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NEWS AND ANNOUNCEMENTS

EUROPEAN MATHEMATICAL COUNCIL

A meeting of the European Mathematical Council took place in Warsaw on August 10, 1982 at the Banach Centre, under the Chairmanship of Professor M.F. Atiyah. Delegates from nineteen European Mathematical Societies attended. The Irish Mathematical Society was represented by Finbarr Holland who prepared this report.

1. Chairman's address:

In his opening remarks, the Chairman recalled the origins of the European Mathematical Council, reminding those present that it was a loosely structured body of representatives of European Mathematical Societies working together for the mutual benefit of the entire European mathematical community, and reviewed its activities.

2. European Mathematical Newsletter

This is prepared and circulated every 3 months on request to mathematical societies by the Mathematisches Forschungsinstitut, Oberwolfach; it contains information about forthcoming mathematical conferences and other items of common interest.

Information for inclusion in the Newsletter should be sent, in the first instance, to

Professor M. Barner,
Mathematical Institute, Hebelstr. 29,
D-7800 Freiburg/Br.,
German Federal Republic.

3. European Directories:

It was announced that, by August 11th, 1982, eleven European Societies, including the Irish Mathematical Society, had compiled either national or regional directories of mathematicians working in their country or region.

Information about these directories is available from Professor A. Dold, Math. Institut Universität, Im Neuenheimer Feld 288, D-6900 Heidelberg 1.

4. Research Reports:

Professor P.C. Baayen outlined a proposal from the Librarian of the Mathematical Centre, Amsterdam, concerning the collection and distribution of information about such reports. The librarian has offered to include information about any research reports, including dissertations and preprints, which he receives free of charge from Mathematics Departments, in the acquisitions lists which are issued by the Centre, eight times a year. In return, copies of these acquisitions lists will be circulated free of charge to cooperating departments. Other interested departments may also be included on the mailing list to receive the acquisitions lists.

This news was welcomed and societies were asked to encourage mathematics departments to participate in the scheme.

The address to write to is:-
Stichting Mathematisch Centrum, Kruislaan 413
NL-1098 SJ Amsterdam.

5. Coordination of Conferences:

Council felt that it was a worthwhile exercise to try and avoid overlapping of Conferences dealing with the same topics and Professor László Márki, Mathematical Institute, Hungarian Academy of Sciences, H-1053 Budapest, Reáltanoda, N.13-15., offered to continue to act as a "match-maker" for people thinking of organising meetings. Organisers of future meetings are asked to clear their dates with Professor Marki as early as possible.

6. Publications Committee

A committee was formed to look into the economic, technological and storage problems, both long and short term, facing mathematical publications. The committee was also asked to examine the possibility of very fast publication of abstracts of recent work.

John J.H. Miller was nominated to serve on this committee.

7. Survey Articles:

Council set up a committee to (i) stimulate the writing of good quality survey articles, (ii) referee the incoming manuscripts and (iii) suggest suitable journals which would publish these surveys.

Finbarr Holland

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INTERNATIONAL MATHEMATICAL UNION

GENERAL ASSEMBLY

The Ninth General Assembly of the International Mathematical Union was held as planned in Warsaw, Poland, on Sunday and Monday, August 8 and 9, 1982. It was well attended by about 100 people, made up of members of the Executive Council and delegates from 36 of the 50 members of the Union. Finbarr Holland represented Ireland.

The President of the Polish Academy of Sciences welcomed the delegates prior to the opening session on Sunday morning and invited them to a reception in Radziejowice, a village about 40 km from Warsaw, at the conclusion of their business on Monday.

In his Presidential Address, Professor Lennart Carlesen, expressed thanks to the Polish Authorities for the opportunity of holding the meeting in Warsaw; reported that Berkeley, California would host the 1986 Congress; announced the winners of the Fields Medals and the new Nevanlinna Prize in Information Science (for details, see the last issue of the Newsletter); outlined some suggestions to counter the rising cost of mathematical journals and emphasised the need for closer cooperation between workers in different branches of mathematics.

For the guidance of the Executive Committee, Sunday afternoon was completely devoted to an explanation of the reasons why the 1982 Congress was postponed and an examination of the prospects for holding it in August 1983.

The Chairman of the Organising Committee for the 1982 Congress, Professor Czeslaw Olech, reviewed the work done in preparation for the Congress and reported that everything was going according to plan until December 13, 1981, when martial law was imposed. Thereafter, prices shot up, goods were rationed, currency was devalued, factors which militated against holding it in August 1982. He and many of his Polish colleagues felt nevertheless that it should have been held then or cancelled altogether and wondered what changes in the present situation prevailing in Poland would make it any easier to hold it in Warsaw next year.

Professor George Mostow, leader of the U.S. delegation, expressed concern that many of his American colleagues might not attend the 1983 Congress unless martial law was lifted and civil liberties were restored to all mathematicians and scientists currently in detention witout trial. This view was shared by the French delegation who circulated a document to the assembled delegates listing four conditions which would have to be met before they would encourage their fellow countrymen to attend.

On the other hand, members of delegations from Warsaw Pact countries were strongly of the opinion that it could and should be held in Warsaw next year. In particular, the members of the Polish delegation were all in favour, albeit for different reasons. (Professor Andrzej Schinzel, reminded his colleagues in other countries that for 2,000 years it was considered a charitable act to feed the poor and to visit the sick and the imprisoned!)

Almost without exception, every other delegation present, expressed the hope that it would take place in Warsaw next year. The Executive Committee will take a final decision on the matter in November.

It was decided that, in any case, the Proceedings of the Congress would be published.

The remaining business of the Assembly, conducted on Monday, included (i) the adoption of the financial reports for the years 1978-81 (ii) increasing the dues for the period 1983-1986 (iii) approving the budget for 1983-1986; (iv) the elections of members to the Executive Commitee of IMU (President, Jürgen Moser; Secretary, Olli Lehto), the Executive Commitee of ICMI (President, Jean Pierre Kahane; Secretary, A.G. Hanson) and the Commission on Development and Exchange (Chairman, H. Hogbe-Nlend); (v) passing a number of resolutions, one expressing "warm gratitude to the Organising Committee for its hospitable reception, excellent arrangements and admirable frankness".

Finbarr Holland
Mathematics Department,
University College, Cork.

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LIMERICK REPORTS

MARY IMMACULATE COLLEGE LIMERICK

Mary Immaculate is the Limerick College of Education for student teachers for the primary schools. It was founded in 1898 and became a Recognised College of the National University of Ireland in 1974. The new B.Ed. degree replaced the old-style N.T. qualification at that time. The Mathematics Department was founded in 1975.

All undergraduates follow a three-year course of studies for the B.Ed. degree, consisting of Education, theoretical and practical, and one elective subject chosen from the range; Irish, English, History, Geography, Mathematics, Philosophy, French, Music. Two additional subjects from this list are taken in first year.

The Bachelor of Education degree programme in mathematics, like most primary degree courses in mathematics, centres around the core areas of Algebra and Analysis. Several aspects of these subjects are explored, including the Theory of Groups and Rings, Vector Spaces and Matrix Theory, Real and Complex Analysis. Self-contained units on Computer Studies and Statistics are also included. Options, such as Geometry, Number Theory, History of Mathematics and Differential Equations, are sometimes available in third year. The College's newly established Computer Centre is used not only for Computer Programming courses but also as a computational aid in conjunction with tutorial classes in other areas.

The pass standard in B.Ed. degree mathematics is equivalent to pass B.A. while students who achieve honours are within one year of M.A. Qualifying or B.A. Honours level. A recent graduate of Mary Immaculate College has just been conferred with a first class honours M.A. degree in mathematics from University College Cork.

There is a staff of three in the Mathematics Department at Mary Immaculate College;

Gerard Enright	B.Sc.University College, Galway
(Head of Department)	M.Sc. University College, Galway
	Ph.D. Cambridge University.
Patrick O'Sullivan	B.Sc. St. Patrick's College, Maynooth
(Lecturer)	M.Sc. St. Patrick's College, Maynooth
	Ph.D. St. Patrick's College, Maynooth
Diarmuid O'Driscoll	B.Sc. University College, Cork
(Lecturer)	M.Sc. University College, Cork

THOMOND COLLEGE OF EDUCATION

INTRODUCTION.

Thomond College of Education incorporates the former National College of Physical Education. The Oireachtas conferred statutory authority on the College through the Thomond College of Education Act 1980. The College now provides degree level courses for teachers in Physical Education, Wood and Building Technology, Metalwork Technology and General and Rural Science. In addition the College offers a one year Graduate Diploma in Business Education.

DEGREE PROGRAMMES

The four year B.A. degree programme, unlike the other College programmes, is designed to bring students to teaching competency in two areas namely Physical Education and an Elective. Students on this programme must select one of five options from English Studies, Irish, Geography, Science Studies/Chemistry and Mathematical Studies. The student studies his 'elective' in each of the four years. There is a mathematics component in each of the other three degree programmes, Metalwork Technology being the most extensive.

MATHEMATICS DEPARTMENT

The role of mathematics in Thomond College has evolved over the years. Presently the mathematics department which comprises three full-time and one part-time member is charged with responsibility for:

1. Developing, teaching, examining the Mathematical Studies Elective in the B.A. programme.
2. Providing service programmes for each of the Woodwork and Building Technology, Metalwork Technology and General and Rural Science degree programmes.
3. Developing and administering computer facilities and courses.
4. Providing in-service courses for second level mathematics teachers.
5. Research.

From a mathematics point of view the Mathematical Studies elective is the most demanding. It is a 600 hour programme spread over four years and includes courses in Analysis, Modern Algebra, Transformation Geometry, Statistics, Computing, Graph theory and History of Mathematics. The service mathematics may be broadly described as engineering mathematics.

(a) COMPUTER FACILITIES

The department runs a fairly extensive computer facility which includes a micro-computer laboratory and 8 terminals linked to the NIHE (L) VAX II/780. Students on all degree programmes learn computing.

(b) CAMET (Ireland)

CAMET (Ireland), the Centre for Advancement of Mathematical Education in Technology established at Thomond College in 1979 is affiliated to CAMET, Loughborough University of Technology, U.K. and directed by the head of the mathematics department. Through this association CAMET (Ireland) offers opportunities for obtaining higher degrees by research (part-time) to selected experienced mathematics teachers in Irish schools. Currently three teachers are registered for an M.Phil. degree and one for a Ph.D.

STAFF

Currently there are three full-time staff members and one part-time. They are:

Jim Leahy (Lecturer)	B.Sc. (Extern) London University M.Sc. University College Cork.
Eoghan MacAogain (Lecturer)	B.A. Trinity College Dublin M.Sc. Warwick University Graduate Diploma in Electronics NIHE(L)
John O'Donoghue (Senior Lecturer/Head of Department)	B.S. St. John Fisher College, Rochester, N.Y. M.S. Rensselaer Polytechnic Institute, Troy, N.Y. Ph.D. Loughborough University of Technology.

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NATIONAL INSTITUTE FOR HIGHER EDUCATION

The mathematics group at NIHE, Limerick provides teaching in the areas of mathematics, statistics and numerical analysis for all programmes in the institute including business studies, humanities, engineering and applied science.

The group is also responsible for a degree programme in Applied Mathematics (Industrial and Management) with the objective of producing graduates capable of applying analytical skills in the planning and control of business and industrial activities. The programme involves elements of business studies and engineering science integrated with courses in mathematics, statistics, operations research, systems theory and computer studies. There are two periods of industrial placement, each of six months duration, included in the four year programme. There is a current enrolment of 7 students in fourth year, 15 in third year, 20 in second year and 25 in first year. The first graduates appear in 1983.

Post-graduate activity is beginning with the first students registered for masters degrees by thesis.

The Graduateship of the Institute of Mathematical Applications has been offered on a part-time basis at night. To date approximately ten students have successfully completed Part I; two have completed Part II and a group of six took the Part II examinations in September 1982.

The mathematical activity at NIHE, Limerick is mainly of an applied nature which is reflected in the expertise and interests of the mathematics staff who are:

Dr. P.F. Hodnett	(Senior Lecturer, Applied Mathematics; Fluid Mechanics)
Mr. M. Wallace	(Senior Lecturer, Statistics)
Dr. R. Critchley	(Lecturer, Applied Mathematics; Quantum Mechanics)
Dr. M. Burke	(Lecturer, Applied Mathematics; Control Theory)
Mr. J. Buckley	(Lecturer, Statistics)
Mr. D. Tocher	(Lecturer, Applied Mathematics; Operations Research)
Dr. J. Kinsella	(Assistant Lecturer, Applied Mathematics; Numerical Analysis)
Mr. G. Lessells	(Assistant Lecturer, Applied Mathematics; Functional Analysis)
Mr. A. Hegarty	(Assistant Lecturer, Applied Mathematics; Numerical Analysis)
Mr. B. Kelly	(Teaching Assistant, Mathematics)

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PERSONAL ITEMS

Dr. Richard Aron has returned to the Mathematics Department at T.C.D. after a sabbatical year at Kent State University, Ohio.

Prof. Sean Dineen has been appointed Head of the Department of Mathematics, U.C.D.

Dr. Desmond Fanning has been appointed to a lecturing position at the Mathematics Department, U.C.C. Dr. Fanning works on Block Designs; is a graduate of U.C.G. and received his Ph.D. at Westfield College, London.

Dr. Padraic Houston will take up a Postdoctoral Fellowship at T.C.D. in January 1983. He is presently at Uni. Paris-Sud and was at CERN until recently.

Dr. Dennis O'Brien is spending this academic year at the Mathematical Physics Department U.C.D. He spent the past few years at Q.U.B.

Dr. Johannes Siemons has been appointed to a position at the Mathematics Department, U.C.D. Dr. Siemons spent the past two years at U.C.C.

Dr. Richard Ward of T.C.D. will take up an appointment at Durham University in January 1983.

Travelling Studentship in Mathematical Science 1982

Eugene Curtin, B.Sc. (U.C.D.)

Prizes were awarded to:-

Clare C. Cunningham, B.Ed., M.A. (U.C.C.)
Eugene G. Gath, B.Sc. (U.C.D.)
Kevin M. Hutchinson, B.A. (U.C.D.)
Mary A. MacDonough, B.Sc. (U.C.G.)
Fiacre A O Cairbre, B.Sc. (Maynooth)

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ON THE DEFINITION OF MEASURABILITY FOR VECTOR VALUED FUNCTIONS

J.J.M. Chadwick

Various conditions for measurability of a real-valued function are examined in the more general case of a vector valued function.

1. Introduction

In the book "Vector Measures" by Diestel and Uhl, a function $f:\Omega \to X$ (where X is a Banach Space and (Ω,Σ,μ) is a measure space) is called measurable if f is a pointwise almost everywhere limit of measurable simple functions. No mention at all is made of the standard definition in terms of open sets i.e. f is measurable provided $f^{-1}(G)$ is measurable for every open set G.

It is well known that the two definitions are equivalent in the case of real-valued functions. This is established in Section 2 below. Naturally, one asks whether the two conditions are also equivalent in the more general case of vector valued functions. An attempt is made in Section 3 to generalise the standard arguments for the real-valued case to those functions which take their values in a Banach Space.

The generalisation of well-known arguments is not merely a formal exercise. In the process insight is gained into these arguments. For example, the separability of the reals is vital to many of the standard proofs. The standard proof that a measurable real-valued function is a pointwise limit of measurable simple functions depends on the fact that any bounded set can be covered by finitely many translates of a given open interval. This means that the proofs will only generalise to Montel spaces (i.e. locally convex spaces in which bounded sets are totally bounded).

The question of why Diestel and Uhl use the definition in terms of simple functions, rather than the topological definition in terms of open sets, will occur to anyone who reads the book. The answer is presumably known and in the literature somewhere. I have not looked for it. Moreover, I have not answered the question fully here. However, the partial results obtained here may be of some interest and perhaps a member of the Irish Mathematical Society will find the enthusiasm to establish the converse of Theorem 3.10 or, alternatively, to find reasonable necessary and sufficient conditions for the existence of a nonmeasurable function which is topologically measurable.

The notation (Ω,Σ,μ) stands for a finite measure space i.e. a non-empty set Ω , a σ-algebra Σ of subsets of Ω and a finite, positive, countably additive measure μ on Σ . The reader is assumed to be familiar with the terminology in the preceding sentence and should have some acquaintance with elementary measure theory and functional analysis.

2. Real-valued measurable functions.

The various equivalent conditions for measurability of a real-valued function on a measure space (Ω,Σ,μ) are presented here. The proofs are well-known. I give them partly as motivation for section 3 and partly in order to see how they might be generalised to vector-valued functions. There is little point in going for full generality here. It is assumed that the measure μ is finite (i.e. $\mu(\Omega) < \infty$) and that the functions are real-valued. Since the notion of an extended-real-valued function does not readily generalise, I do not consider it at all.

2.1 **Definition.** A function $f:\Omega \to \mathbb{R}$ is called measurable if $f^{-1}((\alpha,\infty)) = \{\omega \in \Omega: f(\omega) > \alpha\}$ is measurable for each real α.

It is hopeless to consider generalising this definition as it stands. If the set $\mathbb{R}$ of real numbers is replaced by an arbitrary topological space, or even a Banach Space, the ordering is lost and the definition makes no sense. One must look for equivalent conditions which involve the topology of $\mathbb{R}$ in some way.

2.2 **Lemma.** If $f:\Omega \to \mathbb{R}$ is measurable then the set $\{\omega \in \Omega: f(\omega) \geq \alpha\}$ is measurable for every real α.

Proof. I can write $\{\omega \in \Omega: f(\omega) \geq \alpha\} = \bigcap_{n=1}^{\infty} \{\omega \in \Omega: f(\omega) > \alpha - 1/n\}$. Since f is measurable, each of the sets $\{\omega \in \Omega: f(\omega) > \alpha - 1/n\}$ is measurable. Since the σ-algebra is closed under countable intersections the set $\{\omega \in \Omega: f(\omega) \geq \alpha\}$ is measurable.

2.3 **Corollary.** If $f:\Omega \to \mathbb{R}$ is measurable then $f^{-1}(I)$ is measurable for every bounded open interval I.

Proof. Set $I = (\alpha,\beta)$ and note that $f^{-1}(I) = \{\omega \in \Omega: f(\omega) > \alpha\} \cap \{\omega \in \Omega: f(\omega) < \beta\}$ $= \{\omega \in \Omega: f(\omega) > \alpha\} \cap [\Omega - \{\omega \in \Omega: f(\omega) \geq \beta\}]$. Lemma 2.2 together with the properties of the σ-algebra now guarantees that $f^{-1}(I)$ is measurable.

The first equivalent condition for measurability can now be established.

2.4 **Theorem.** A function $f:\Omega \to \mathbb{R}$ is measurable if and only if $f^{-1}(G)$ is measurable for every open subset G of $\mathbb{R}$.

Proof. Suppose that f is measurable. The bounded open intervals with rational endpoints form a countable base for the topology on $\mathbb{R}$. Therefore, if G is open, I can find a sequence (I_n) of bounded open intervals such that $G = \bigcup_{n=1}^{\infty} I_n$. Then $f^{-1}(G) = \bigcup_{n=1}^{\infty} f^{-1}(I_n)$. Since each $f^{-1}(I_n)$ is measurable by the above Corollary, $f^{-1}(G)$ is also measurable. The converse is clear since (α,∞) is open.

Notice how the proof depends on the fact that the topology has a countable base. There would be no way of proceeding if we had to write G as an uncountable union of intervals.

2.5 **Definition.** A simple function $S:\Omega \to \mathbb{R}$ is a function with finite range. If $S(\Omega) = \{\alpha_1, \ldots, \alpha_n\}$ then we can write $S = \Sigma_1^n \alpha_i \chi_{E_i}$ where $E_i = \{\omega \in \Omega: S(\omega) = \alpha_i\}$ and, for any subset A of Ω, $\chi_A(\omega) = \begin{cases} 1, & \omega \in A \\ 0, & \omega \notin A \end{cases}$

It is easy to see that $S = \Sigma_1^n \alpha_i \chi_{E_i}$ is measurable if and only if each E_i is measurable.

2.6 **Theorem.** If $f:\Omega \to \mathbb{R}$ is measurable, then there exists a sequence (S_n) of mesurable simple functions such that $S_n(\omega) \to f(\omega)\ (n \to \infty)$ for every ω in Ω.

Proof. For each n and for $-n2^n+1 \leq k \leq n2^n$ write $E_{n_k} = \{\omega \in \Omega: (k-1)2^{-n} < f(\omega) \leq k2^{-n}\}$. Then each E_{n_k} is measurable, since it is the inverse image of a half-open interval. The sets E_{n_k} are disjoint. Write $S_n = \Sigma\, k2^{-n}\chi_{E_{n_k}}$ the sum being over all values of k from $-n2^n+1$ to $n2^n$. Now S_n is a measurable simple function. For any $\omega \in \Omega$, select N such that $|f(\omega)| < N$. If $n > N$ then $|f(\omega)| < n$ and so $-n < f(\omega) < n$. There exists k with $-n2^{-n}+1 \leq k \leq n2^{-n}$ such that $(k-1)2^{-n} < f(\omega) \leq k2^{-n}$. Then $\omega \in E_{n_k}$ and we get $S_n(\omega) = k2^{-n}$. Since $(k-1)2^{-n} < f(\omega) \leq k2^{-n}$ we have $|S_n(\omega) - f(\omega)| < 2^{-n}$. It follows that $S_n(\omega) \to f(\omega)\ (n \to \infty)$.

I mention here that, of all the proofs in this section, the one I have just given is the most difficult to generalise. Indeed, it is because of this that the various equivalent conditions for measurability in this section do not remain equivalent in the more general setting of section 3. The above proof seems to depend ultimately on the fact that bounded sets are totally bounded. This is not true for the norm topology on an infinite-dimensional Banach Space. (It is true for the weak topology, and one should be able to find a generalisation of the above result to that case, with a bit of effort.)

2.7′ Theorem. If there exists a sequence (S_n) of measurable simple functions such that $S_n \to f$ pointwise on Ω, then f is measurable.

Proof. The fact that the S_n are simple functions is not needed at all. Let α be real. I claim that

$$f^{-1}((\alpha,\infty)) = \bigcup_{\substack{q>\alpha \\ q \text{ rational}}} \bigcup_{N=1}^{\infty} \bigcap_{n>N} S_n^{-1}((q,\infty)).$$

This is reasoned out as follows. Suppose that $\omega \in f^{-1}((\alpha,\infty))$ i.e. $f(\omega) \in \alpha$. There exists a rational $q > \alpha$ such that $f(\omega) > q$. Since $S_n(\omega) \to f(\omega)$ $(n \to \infty)$ there exists N such that $S_n(\omega) > q$ for all $n > N$. But this all means that

$$\omega \in \bigcup_{\substack{q>\alpha \\ q \text{ rational}}} \bigcup_{N=1}^{\infty} \bigcap_{n=N+1}^{\infty} S_n^{-1}((q,\infty))$$

The argument is essentially reversible: if ω is in the latter set there exists $q > \alpha$ such that $\omega \in \bigcup_{N=1}^{\infty} \bigcap_{n=N+1}^{\infty} S_n^{-1}((q,\infty))$. Then there exists N such that $\omega \in S_n^{-1}((q,\infty))$ for all $n > N$ i.e. $S_n(\omega) > q$ for all $n > N$. Since $S_n(\omega) \to f(\omega)$ $(n \to \infty)$ we then get $f(\omega) \geq q > \alpha$. Consequently $\omega \in f^{-1}((\alpha,\infty))$. Therefore the set equality holds. Since each S_n is measurable, the sets $S_n^{-1}((q,\infty))$ are all measurable. Since the unions and intersection are countable, the set

$$\bigcup_{\substack{q>\alpha \\ q \text{ rational}}} \bigcup_{N=1}^{\infty} \bigcap_{n=N+1}^{\infty} S_n^{-1}((q,\infty))$$

is measurable i.e. $f^{-1}((\alpha,\infty))$ is measurable, as required.

Observe that the set of rationals plays a fundamental role in the proof. Any generalisation of this type will involve separability.

3 Vector-valued functions.

We can now look at the case where the function f with domain Ω takes its values in a Banach Space X. In view of the results on real-valued functions, there are two reasonable definitions of measurability available in the more general case. The first is purely topological in nature and does not mention the measure μ at all.

3.1 Definition. Let X be a Banach Space and let $f:\Omega \to X$. Say that f is topologically measurable if $f^{-1}(G)$ is measurable for every open subset G of X.

The second definition is based on Theorems 2.6 and 2.7 but the pointwise convergence requirement is relaxed somewhat. This definition involves the measure μ, because sets of measure zero are important here. The theory of vector measures is based on this approach and depends completely on it. A simple function $S:\Omega \to X$ is a function with finite range. If $S(\Omega) = \{x_1 \dots x_n\}$ we can write $S = \Sigma_1^n x_i \chi_{E_i}$ where $E_i = \{\omega \in \Omega : S(\omega) = x_i\}$. Such a simple function is called measurable if each E_i is measurable.

3.2 Definition. Say that a function $f:\Omega \to X$ is measurable provided there is a sequence (S_n) of measurable simple functions such that $S_n \to f$ a.e. on Ω, in the sense that $\|S_n(\omega) - f(\omega)\| \to 0 (n \to \infty)$ for almost all ω.

The objective is to investigate the equivalence of measurability and topological measurability so let us first dispose of the case of simple functions.

3.3 Proposition. A simple function $S:\Omega \to X$ is measurable if and only if it is topologically measurable.

Proof. Let $S = \Sigma_{k=1}^n x_i \chi_{E_i}$ be measurable. Let G be an open subset of X. If $S^{-1}(G) = \phi$ then $S^{-1}(G)$ is measurable. Otherwise $S^{-1}(G) = \bigcup_{x_i \in G} E_i$

which is measurable, since $\{x_i : x_i \in G\}$ is finite. Conversely, if $S^{-1}(G)$ is open for every open set G, I can show that each E_i is measurable as follows. I have $E_i = S^{-1}(\{x_i\}) = \Omega - S^{-1}(X - \{x_i\})$. Since $X - \{x_i\}$ is open, $S^{-1}(X - \{x_i\})$ is measurable and therefore so is $E_i = \Omega - S^{-1}(X - \{x_i\})$.

3.4 Proposition. If $f:\Omega \to X$ is measurable then f is "essentially separably valued" in the sense that there is a subset A of Ω such that $\mu(\Omega - A) = 0$ and $f(A)$ is separable.

Proof. Write $A = \{\omega \in \Omega : S_n(\omega) \to f(\omega)\}$ where (S_n) is a sequence of measureable simple functions with $S_n \to f$ a.e. Then $\mu(\Omega - A) = 0$. Now $\bigcup_{n=1}^{\infty} S_n(\Omega)$ is countable since each $S_n(\Omega)$ is finite. Moreover, if $S_n(\omega) \to f(\omega)$ then $f(\omega) \in \overline{\bigcup_{n=1}^{\infty} S_n(\Omega)}$. Consequently $f(A) \subseteq \overline{\bigcup_{n=1}^{\infty} S_n(\Omega)}$ which is separable. This means that $f(A)$ is separable.

3.5 Lemma. Let f be measurable and let $B = B(x,r) = \{y : \|x-y\| < r\}$ be any open ball. Then $f^{-1}(B)$ is measurable.

Proof. There is a sequence (S_n) of measurable simple functions such that $S_n \to f$ a.e. on Ω. Write $A = \{\omega \in \Omega : S_n(\omega) \to f(\omega)\}$ so that $\mu(\Omega - A) = 0$. For any $t > 0$ put $B_t = \{y : \|x-y\| < t\}$. First of all,
$f^{-1}(B) = [f^{-1}(B) \cap (\Omega - A)] \cup [f^{-1}(B) \cap A]$ so it suffices to show that the sets $f^{-1}(B) \cap (\Omega - A)$ and $f^{-1}(B) \cap A$ are both measurable. The set $f^{-1}(B) \cap (\Omega - A)$ is certainly measurable since it is a subset of $\Omega - A$ which has measure zero. In order to show that $f^{-1}(B) \cap A$ is measurable we show first that

$$f^{-1}(B) \cap A = \bigcup_{\substack{q < r \\ q \text{ rational}}} \bigcup_{N=1}^{\infty} \bigcap_{n=N+1}^{\infty} [S_n^{-1}(B_q) \cap A] \qquad (*)$$

Let $\omega \in f^{-1}(B) \cap A$. Then $\|f(\omega) - x\| < r$. Select a rational $q < r$ such that $\|f(\omega) - x\| < q$. Since $\omega \in A$, I have $S_n(\omega) \to f(\omega)$ $(n \to \infty)$. Therefore there is an N such that $\|S_n(\omega) - x\| < q$ for all $n > N$. All of this shows that ω is in the right hand side of (*). If ω is in the right

hand side of (*) there exists a rational $q < r$ such that for some N, $\omega \in S_n^{-1}(B_q) \cap A$ for all $n > N$. Thus $\omega \in A$ and $\|S_n(\omega) - x\| < q$ for all $n > N$. This gives $\|f(\omega) - x\| \leq q < r$ and so $\omega \in f^{-1}(B) \cap A$. The equality (*) is established and the fact that $f^{-1}(B) \cap A$ is measurable now follows from Proposition 3.3 and the properties of the σ-algebra.

3.6 Theorem. If f is measurable then f is topologically measurable.

Proof. Let (S_n) and A be as in the proof of Lemma 3.5. Let G be any open subset of X. Since $f^{-1}(G) = [f^{-1}(G) \cap (\Omega - A)] \cup [f^{-1}(G) \cap A]$ it is enough to show that the sets $f^{-1}(G) \cap (\Omega - A)$ and $f^{-1}(G) \cap A$ are measureable. The first of these is measurable since it is a subset of the set $\Omega - A$ which has measure zero. Observe that $f^{-1}(G) \cap A = f^{-1}(G \cap f(A)) \cap A$. By Proposition 3.4 $f(A)$ is separable. This means that the relative topology on $f(A)$ has a countable base of open balls. Consequently I can find a sequence (B_n) of open balls in X with $G \cap f(A) = \bigcup_{n=1}^{\infty} B_n \cap f(A)$

Now we get

$$f^{-1}(G) \cap A = f^{-1}(G \cap f(A)) \cap A = f^{-1}\left(\bigcup_{n=1}^{\infty} B_n \cap f(A)\right) \cap A$$

$$= \bigcup_{n=1}^{\infty} f^{-1}(B_n \cap f(A)) \cap A =$$

$$= \bigcup_{n=1}^{\infty} f^{-1}(B_n) \cap f^{-1}(f(A)) \cap A = \bigcup_{n=1}^{\infty} f^{-1}(B_n) \cap A,$$

since $f^{-1}(f(A)) \cap A = A$.

But now each of the sets $f^{-1}(B_n)$ is measurable by Lemma 3.5. It follows that $f^{-1}(B_n) \cap A$ is measurable for each n and therefore so is $f^{-1}(G) \cap A$. This completes the proof.

At this point the investigation is half over. It is established that measurability implies topological measurability. It would be nice if the converse could be established, but we run into problems. However, a partial converse can be found, and it will be convenient to use the following known result from the theory of vector-valued measurable functions. (see Diestel and Uhl).

3.7 Pettis Measurability Theorem. Let $f:\Omega \to X$ where X is a Banach Space. Then f is measurable if and only if f is essentially separably valued and weakly measurable (in the sense that the real-valued function $\varphi o f$ is measurable for every φ in the dual X^* of X)

3.8 Theorem. If f is topologically measurable and essentially separably valued then f is measurable.

Proof. By the Pettis Measurability Theorem, it is enough to show that if $\varphi \in X^*$ then $\varphi o f$ is measurable (as a real-valued function on Ω). By section 2, it suffices to show that $\varphi o f$ is topologically measurable. Let G be an open subset of $\mathbb{R}$. Then $(\varphi o f)^{-1}(G) = f^{-1}(\varphi^{-1}(G))$. Now $\varphi^{-1}(G)$ is open since φ is continuous. Then $f^{-1}(\varphi^{-1}(G))$ is measurable since f is topologically measurable. It follows that $\phi o f$ is measurable.

The only remaining question is whether the condition that f is essentially separably valued can be dropped from Theorem 3.8. In order to obtain more insight, we can adopt the following device which simplifies the problem conceptually.

3.9 Proposition. The following are equivalent

(a) There exists a measure space (Ω,Σ,μ) and a topologically measurable function $f:\Omega \to X$ which is not measurable.

(b) There exists a Borel measure λ on X such that the identity on X is not λ-measurable.

(c) There exists a Borel measure λ on X which is not concentrated on any separable subset of X i.e. such that there is no separable subset P of X with $\lambda(X-P) = 0$.

Proof (a) $\Rightarrow$ (b). Suppose that (a) holds. The collection $\mathcal{M} = \{E \subseteq X : f^{-1}(E) \text{ is measurable}\}$ is a σ-algebra on X, as is easily verified. Since f is topologically measurable $\mathcal{M}$ contains the open sets and therefore also contains the Borel sets of X. Define $\lambda(E) = \mu(f^{-1}(E))$ for every Borel set E. Then λ is a Borel measure on X. Suppose that the identity is λ-measurable. There exists a sequence $S_n:X \to X$ of λ-measurable simple functions with $S_n(x) \to x$ a.e. on X. Let $E_o = \{x:S_n(x) \to x\}$ so that $\lambda(X-E_o) = 0$. Define $t_n:\Omega \to X$ by $t_n = S_n o f$. Then t_n has finite range. Moreover, if $G \subseteq X$ is open then $t_n^{-1}(G) = f^{-1}(S_n^{-1}(G))$. But $S_n^{-1}(G)$ is λ-measurable and so $f^{-1}(S_n^{-1}(G))$ is (μ-)measurable. This means that t_n is topologically measurable and therefore measurable by Proposition 3.3. If $\omega \in f^{-1}(E_o)$ then $f(\omega) \in E_o$ and so $S_n(f(\omega)) \to f(\omega)$ $(n \to \infty)$. This gives $t_n \to f$ on $f^{-1}(E_o)$. But $\Omega - f^{-1}(E_o) = f^{-1}(X - E_o)$ and $\mu(f^{-1}(X-E_o)) = \lambda(X-E_o) = 0$. Hence $t_n \to f$ a.e. on Ω and f is measureable. This contradicts the assumption that (a) holds and therefore the identity on X cannot be λ-measurable.

(b) $\Rightarrow$ (c). If (b) holds then the identity on X is topologically measurable but not measurable. By Theorem 3.8 the identity cannot be essentially separably valued. This establishes (c) at once.

(c) $\Rightarrow$ (a). Let $\Omega = X$. Let Σ be the Borel algebra on X and take f as the identity on X. Then f is topologically measurable. However, for the given measure λ there is no sequence (S_n) of measurable simple functions converging almost everywhere to f. If there were such a sequence then f would be essentially separately valued, which is impossible, since λ is not concentrated on any separable subset of X.

The problem now reduces to finding an answer to the following question: is it possible to have a Banach Space X and a finite Borel measure λ on X which is not concentrated on any separable subset of X?

The question is related to Ulam's Measure Problem: does there exist a set D and measure ν such that every subset of D is measurable, $0 < \nu(0) < \infty$ and every countable subset of D has measure zero?

The answer to this question is somewhere in the foundations.

3.10 Theorem. If the answer to Ulam's Measure Problem is 'Yes' then there exists a topologically measurable function which is not measurable.

Proof. Let D be a set with measure ν satisfying Ulam's criteria. Let X be the Hilbert space $\ell_2(D)$ consisting of all functions $\phi : D \to \mathbb{R}$ for which $\{\delta \in D : \varphi(\delta) \neq 0\}$ is countable and $\Sigma_{\delta \in D} |f(\delta)|^2$ is finite. The inner product on $\ell_2(D)$ is given by $< \varphi\ \psi > = \Sigma_{\delta \in D} \varphi(\delta) \psi(\delta)$. The measure space is $(D, 2^D, \nu)$ and we define $f : D \to X$ by $f(\delta) = \ell_\delta$ where $\ell_\delta(y) = \begin{cases} 0 & y \neq \delta \\ 1 & y = \delta \end{cases}$.

As is well-known, the set $\{\ell_\delta : \delta \in D\}$ is an orthonormal basis for $\ell_2(D)$ and so $||\ell_\delta - \ell_y|| = \sqrt{2}$ for $\delta \neq y$. The set f(D) therefore has no limit points and so every subset of f(D) is closed. This means that a subset of f(D) is separable if and only if it is countable.

If f is measurable then it is essentially separably valued and so there is a subset A of D with $\nu(D-A) = 0$ and f(A) separable. But then f(A) is countable and hence so is A which gives $\nu(A) = 0$. Now we have $\nu(D) = \nu(A) + \nu(D-A) = 0$, a contradiction. This means that f is not measurable.

It would be nice to complete the work by proving the converse of Theorem 3.10. However, I have not succeeded in establishing it. My numerous attempts have left me with the impression that the converse holds. On the other hand, the fact that those attempts failed leaves room for doubt. It is left to the reader to pursue this question.

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FORE OR FIVE? - THE INDEXING OF A GOLF COURSE

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1. Introduction

In every golf club, for reasons which relate mainly to the rules and regulations of handicap competition, it is necessary to rank the 18 holes of the course in order of difficulty so as to provide a strokes index for each hole ranging from 1 (most difficult) to 18 (least difficult). This is normally done on a subjective basis, and is consequently a frequent topic of controversy in many golf clubs. In this note, we propose a more objective method for performing this indexing, and we apply it to a particular case (i.e. our own golf course).

The par score for a hole is the score which a top-class (scratch) player would be expected to have at that hole. An obvious measure of the difficulty of a hole is the average amount by which players exceed the par score for that hole. It was decided, however, not to use this particular measure because of the distortive effect on the mean of outlier data points; such scores could occur quite frequently at particular holes (e.g. those with psychologically intimidating features such as out of bounds close to the tee, water hazards close to the green, etc., etc.) thereby inflating the mean score disproportionately to the intrinsic difficulty of the hole. A related measure which overcomes this problem is the percentage or proportion of players who equalled or bettered par at the hole. This is the measure which we have adopted, and which is used in this paper.

Some features of this measure are worthy of note, as they motivate the analysis described below. Firstly, the percentage who better or equal par at any hole can be readily estimated from the records of golf competitions which are available in all golf clubs. In fact, the authors of this note have the distinction as applied statisticians of not only having collected their own data but also through their golf, actually contributing to it. Secondly, it is clear that this measure can be evaluated for players of different calibres (i.e. handicaps) and that different percentages (or probabilities) can be expected for the various categories of golfer: by definition, the average percentage who equal or better par at any hole will be higher for good golfers than for poor golfers. Finally, it is intuitively clear, and from (bitter!) experience glaringly obvious, that the relative difficulty of any hole is very dependent on weather conditions, and in particular, on wind speed and direction.

In this paper, we postulate that the probability of equalling or bettering par at any hole depends on the variables handicap, wind speed and wind direction via a logistic function; we estimate the parameters of this function from a large data-set of golf scores; test the goodness of fit of the model and, having accepted the model, calculate the expected value of this criterion, with respect to the variables handicap, wind speed and wind direction, for each hole and thus provide a ranking of the 18 holes.

2. The Data

The data consisted of scores for a total of 575 players, spread over five different competitions (i.e. days). An initial analysis confirmed that the probability of at least equalling par was heavily dependent on both the handicap of the player and weather conditions (i.e. wind speed and direction). Data were also available for the actual speed and direction on each day (at six-hourly intervals); the particular competitions whose scores were used in this analysis were chosen specifically on the basis of minimum variability of the six-hourly readings, and the wind speed and direction (taken as the average of the two day-time readings) were then assumed constant throughout the day.

For illustration, we provide in table 1, a typical data set - i.e. that for hole 5. This set highlights many of the points already made. In particular, on any given day the probabilities of at least equalling par are markedly different for the two handicaps classes. Furthermore, this probability, for either handicap class, varies widely from day to day. For example, in the case of competition 1, for category 1 golfers this probability was 0.20, whereas for competition 5 it was virtually doubled (to 0.39). An explanation of this variation is provided by the fact that the wind direction on day 1 was 330 (virtually directly against the hole which faces 5 degrees east of North) whereas on day 5 the wind direction was 200, almost a directly following wind.

3. Fitting the Model

We have now established that the probability of at least equalling

HOLE 5

COMPETITION	W	θ	H	# LE PAR	# GT PAR	OBSERVED PROPORTION ≤ PAR
1	8	330	1	9	35	.20
	8	330	2	1	49	.02
2	10	80	1	12	30	.29
	10	80	2	1	26	.04
3	4	210	1	37	751	.34
	4	210	2	12	115	.09
4	18	290	1	11	29	.28
5	7	200	1	53	84	.39

TABLE 1

par at a hole appears to depend on a number of factors. These are represented by the variables:

W = Wind Force

θ = Wind Direction

H = Handicap class of player (1 if player's handicap ≤ 10)
(2 if player's handicap > 10)

We postulate that there is a functional dependence of P_i, the probability of at least equalling par at hole i, on these variables, i.e. that:

(1) $P_i = g_i\ (W,\theta,H) \quad (i = 1,2, \ldots , 18)$.

We further postulate that the logistic model represents an appropriate class of functional forms to describe the relationships (1). The logistic model is given by

(2) P_i = prob {Score ≤ par $X_1, X_2, \ldots ,$}

$$= \frac{\exp\left(a_0 + \sum a_j X_j\right)}{1 + \exp\left(a_0 + \sum a_j X_j\right)}$$

where $X_1, X_2, \ldots ,$ are independent variables. The independent variables used in our analysis were

X_1 : W

X_2 : W cos $(\theta - \alpha_i)$, where α_i = direction of hole i

X_3 : H

the inclusion of each of which can be justified – *a priori*, on heuristic grounds, and *a posteriori* on the basis of their explanatory capacity.

The model (2) was then fitted to the data (using the software package BMDP) to provide estimates of the co-efficients a_0, a_1, a_2, a_3 for each hole, t-values for these co-efficients and appropriate goodness-of-fit statistics.

For illustrative purposes, we describe here the fitting of the logistic model to the data for hole 5 (presented in table 1). Similar analyses were performed for each hole.

The parameters estimated for equation (2) for hole 5 were (t-values in brackets)

a_o = 0.767 (1.39)

a_1 = 0.015 (0.43)

a_2 = −0.071 (−.255)

a_3 = −1.759 (−5.55)

The goodness of fit chisquare (1.639) has p-value 0.802, and does not lead to rejection of the model. A more intuitive presentation to highlight the adequacy of the model is to use the estimated model to predict the expected or theoretical probabilities of at least equalling par for each of the five days (i.e. W, θ combinations) and for each handicap category - i.e. a predicted probability corresponding to each row of table 1. Table 2 presents these theoretical probabilities for hole 5, together with the observed probabilities as already given in table 1. The extent of the agreement is remarkably good.

A similar analysis performed for each hole produces an estimated logistic function formulation of the functional dependence of the probability P_i on the various independent variables, which in all cases produces very good agreement between observed and predicted probabilities. In only one case does the chisquare goodness of fit statistic lead to rejection of the model at the 5% significance level, and this is just about what we would expect if the model were appropriate. We have, therefore, now established a relationship of the form $P_i = g_i(W,\theta,H)$ for each hole for any given combination of the variables W,θ,H.

To obtain an overall "average" index it is necessary to establish the joint distribution of W,θ,H. H is clearly independent of W,θ, and had for our data the very simple probability distribution

$$p(H = 1) = p(H = 2) = \tfrac{1}{2}$$

Denoting the joint distribution of W,θ by f(W,θ), the expected value of P_i as formulated by us will therefore be given by

$$\bar{P}_i = \sum_{H=1}^{2} \frac{1}{2} \iint g_i(W,\theta,H)\, f(W,\theta)\, dW\, d\theta \qquad (3)$$

HOLE 5

COMPETITION	H	OBSERVED PROPORTION ≤ PAR	PREDICTED PROBABILITY ≤ PAR
1	1	.20	.21
	2	.02	.04
2	1	.29	.26
	2	.04	.06
3	1	.34	.34
	2	.09	.08
4	1	.28	.26
5	1	.39	.40

TABLE 2

In sections 4 and 5 we describe the empirical determination of $f(W,\theta)$, the resultant derivation of $\bar{P}_i$, and the corresponding indices 1 to 18.

4. Joint Distribution of Wind Velocity and Direction

The probability distribution of wind speed is commonly assumed to be a member of the Weibull family of distributions. This is an empirically based assumption which usually gives a good fit to data from sites which exhibit a prevailing wind direction - when there is no prevailing wind direction, the Rayleigh distribution is generally found to provide a good fit. Since the Weibull model does not provide information on wind direction, it is inappropriate in our case. In a 1979 paper, MacWilliams, Newmann and Sprevak presented a simple theory for modelling the joint distribution of wind speed and direction. In a later (1980) paper, MacWilliams and Sprevak showed that the model provided a very good fit to wind data from the 14 sites in the Republic of Ireland and 5 in Northern Ireland for which hourly data were available. Our model and estimation procedure are effectively those introduced by MacWilliams et. al.

Let W = wind velocity

W_x = Component of wind velocity along the prevailing wind direction

W_y = component of velocity perpendicular to the prevailing direction

θ = Radian measure of the prevailing wind direction.

We assume that

(i) $W_x \sim N(\mu,\sigma^2)$; $W_y \sim N(0,\sigma^2)$

(ii) W_x, W_y are stochastically independent; hence the joint p.d.f. of W_x, W_y is given by

$$g(W_x, W_y) = \frac{1}{2\pi\sigma^2} \exp\left\{-\frac{1}{2\sigma^2}\left[W_y^2+(W_x-\mu)^2\right]\right\}$$

By making the transformation

$W_x = W\cos\theta$, $W_y = W\sin\theta$

we obtain the joint p.d.f. of wind velocity and direction

$$f(W,\theta) = \frac{W}{2\pi\sigma^2} \exp\left(\frac{-\mu^2}{2\sigma^2}\right) \exp\left\{-\frac{1}{2\sigma^2}\left[W^2-2\mu W\cos\theta\right]\right\},$$

$$0 \leq \theta < 2\pi, \quad W \geq 0$$

The marginal distributions of wind velocity and direction are obtained by integrating $f(W,\theta)$ over θ and W respectively giving

$$h_W(\omega) = \frac{W}{\sigma^2} \exp\left(\frac{-\mu^2}{2\sigma^2}\right) \exp\left(\frac{-\omega^2}{2\sigma^2}\right) I_0\left(\frac{\mu}{\sigma^2}W\right), \quad \omega \geq 0$$

$$h_\theta(\theta) = \frac{1}{2\pi} \exp\left(\frac{-\mu^2}{2\sigma^2}\right)\left\{1+\frac{\mu}{\sigma}\sqrt{2\pi}\cos\theta \exp\left(\frac{\mu^2\cos^2\theta}{2\sigma^2}\right)\right\}\Phi\left(\frac{\mu\cos\theta}{\sigma}\right),$$

$$0 \leq \theta < 2\pi$$

where $I_o(.)$ is the modified Bessel function of the first kind and order zero and $\Phi(.)$ is the standard normal distribution function. It is worth noting that $h_w(w)$ and $h_\theta(\theta)$ became the Rayleigh density function and $^1/2\pi$ respectively when μ is zero.

5. Derivation of Expected Probabilities

Before applying $f(W,\theta)$ to compute P_i : 1, 2, ... , 18 we must first supply estimates for μ, σ^2. The data used in the estimation were recorded (4 times daily) by the Galway Meteorological Station over the 12 month period September, 1980 - August, 1981. Of the data available, only those values relating to daylight hours were used. The data consisted of wind speeds which were presented in integral units of the Beaufort scale and wind direction - specified in sectors of 10° width so that for example, a recorded direction of 9 means a wind blowing from a point 85° - 95° measured clockwise from North. The parameters specifying the function $f(W,\theta)$ were estimated by resolving the observed data into the two orthogonal directions after the prevailing direction had been obtained. The prevailing direction was taken as being the centre of the sector having the largest frequency of occurrence.(sector 24).

The estimates obtained were as follows:

(i) Prevailing Direction: 240°

Hole No.	Expected Probabilities of $\leq$ par		Average with respect to handicap	Corresponding Indices
	Handicap Class 1	Handicap Class 2		
1	.36929	.21128	.29029	5
2	.41420	.22830	.32125	7
3	.61590	.41086	.50888	13
4	.58107	.35697	.46902	11
5	.34095	.08877	.21486	1
6	.29515	.17937	.23726	3
7	.69868	.42739	.56304	15
8	.72849	.43354	.58102	17
9	.38985	.29714	.34350	9
10	. 8524	.16725	.22625	2
11	.51290	.21006	.36148	10
12	.61996	.49026	.55511	18
13	.44748	.18975	.31862	4
14	.44264	.27441	.35853	8
15	.51062	.35130	.43096	14
16	.50353	.22195	.36274	12
17	.59382	.27348	.43365	16
18	.42472	.22587	.32530	6

TABLE 3

(ii) $\bar{W}_x = 3.215, \; S_x^2 = 58.996$

$\bar{W}_y = -0.037, S_y^2 = 60.611$

$\bar{W}_x, \bar{W}_y, S_x, S_y$ represent the average wind velocities and standard deviations in the velocities along the prevailing and perpendicular directions. The values obtained were consistent with the assumptions made and 3.215 and 59.0 were then used as estimates for μ, σ^2 respectively in the model. The integrals.

$$\int\int g_i \, (W,\theta,H) f(W,\theta) dW d\theta \; : \; i = 1, 2, 3 \ldots , 18$$

were evaluated using the NAG library routine DOIFCF. The results together with the suggested indices are given in Table 3.

6. Summary

The dependence of the probability of at least equalling par at any given hole on handicap, wind speed and direction is shown to be adequately described by the logistic function. The expected value of this probability with respect to the empirically derived joint distribution of the independent variables is evaluated to provide a strokes index for a golf course.

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1. MacWilliams, B., Newmann, M.M., Sprevak, D., (1979) The Probability distribution of Wind Velocity and Direction". Wind Engineering 3., 264-269.

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A Final Remark on Infinite Exponentials

P. J. Rippon

The last issue of the Newsletter contained an article on infinite exponentials in which the following problem (dating back to 1907) was stated.

For any complex number a let $a_1 = a$ and

$$a_{n+1} = \exp[a_n \log a], \quad n = 1, 2, \ldots ,$$

where the principal branch of log a is taken.

Is the sequence a_n convergent whenever a lies in

$$R_C = \{e^{\zeta e^{-\zeta}} : |\zeta| \leq 1\} \ ?$$

This problem has now been given a more-or-less complete solution by Dr. I.N. Baker of Imperial College, London, who uses the classical theory of iteration as developed by Fatou and Julia. In fact a_n is convergent when a is an interior point of R_C and when a is of the form $e^{\zeta e^{-\zeta}}$, ζ being a root of unity. However for most points of ∂R_C the sequence a_n is divergent.

Details should appear.

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School Mathematics - Knocking on Heaven's Door

M. Brennan

You who are reading this most likely know what's wrong with secondary school mathematics in Ireland [1]. The only question you need to have answered now is: "When is something going to be done about it?"

At the time of writing (October 1982) the Department of Education Mathematics Syllabus Committee is supposed to be in session. The agenda, as far as I know, does not mention Syllabus change but the Irish Maths. Teachers Association representative on the Committee will be asking that the IMTA's Draft Syllabus submitted last year be considered as a basis for a new scheme. For some IMTA members this session of the Syllabus Committee is tantamount to a sitting of the Delphic Oracle. Years of deliberation and consultation of members have gone into preparing the IMTA's case. Their brief is now in the hands of God. Or rather, it has been for about 18 months.

The IMTA's case is for a reduction by about one-third in the Higher Leaving Cert. courses - which should result in large percentages of examinees scoring high marks (why not?) in their exams. The IMTA are firmly behind an anti-abstract groundswell among teachers.

The groundswell has been there for years but with a fall-off of interest by Maths. teachers in their subject (as evidenced by enrolments in their Association) and the onset of the micro-computer, the Maths. teachers dummytit, there is no more hope of the groundswell coming to anything than there is of the Exchequer financing radical change.

This last consideration might prove to be a decisive one. Proponents, for example, of three Leaving Cert courses (lower, middle and higher) are almost certain to have their ideas frozen in the chill of public cut-backs. Indeed it may even be that any change which requires expenditure (e.g. on teacher retraining courses) which run into five (would you believe four?) figures will die a lonely death in the corridors of Marlborough Street where Department of Education finances are controlled. But that's only speculation.

Yet how dire is the need for change? Pretty dire, even viewed from the outside. That dirty word "fail" has not yet been erased from the minds of this country (surely an object lesson in how to brain-wash a whole people if ever there was one) even though no such category exists officially. One cannot fail the Leaving Certificate. It is an official impossibility.

Unofficially, and outside in a brain-washed world, if you don't get 40% in Maths.you've got nothing: if you don't get 65% in Maths you've only "passed" (another illegitimate word). There are even things called "red honours" in popular lore: a "red honour" is 55% on a Lower Leaving Cert paper.

Unofficially there is concern in high places. Higher Course Maths numbers fell dramatically during the last 8 years or so as children began to make the sensible decision that the Higher Course wasn't worth the time it took up, even for extra University points. The Department responded (or was it coincidence?) by making the Higher Course papers easier - and by propping up lower course grades at marking-time whenever there was a danger of catastrophic numbers falling into the "fail" category or of a sensational divergence from the usual proportions in the other marks-categories.

At the heart of the matter one suspects that the make-or-break importance of the Leaving Cert which draws huge numbers into Maths classes (e.g. all aspirants for nursing, of which there is a never-ending supply) has placed a strain on the syllabus which it was never intended to take. A syllabus after all should be no more than a coatstand on which we hang our coats, a steel-mesh onto which we pour our concrete, a line on which we hang out our washing. Far from these humble stations, the secondary school syllabus has become the Word of God. Teachers teach the syllabus now. There is little time left to teach mathematics. If there are chinks in the syllabus, there are chinks in the Word of God and it's bound to show. Any starryeyed nurse-aspirant will tell you, her mind filled with pictures of bedside manners, emergency drips, tucking in the children, switching off the lights. The word is <u>Irrelevance</u>.

Further back, at Inter Cert, all is not well either. In editions of the IMTA Newsletter in May and September of this year I detailed faults in the present Geometry course as well as faults in proofs of theorems given in standard commercial textbooks. Within its own rationale the Geometry course is a chaotic bag of difficult (to me) jumble. It lacks order and elegance and to this day I am not satisfied with answers given to key questions such as "why is it important for pupils to prove that the image of a line under a translation or under a reflection in a point is a line, although they are asked to take no notice of the fact that the image of a line under reflection in a line is a line?" (They are encouraged to use this latter result, without comment. Worse, they have already had it motivated to them that a translation and a reflection in a point are composi -

tions of reflections in lines!!).

Axioms are sloppy and there are other faults - not least being the thinking that methods which were found good in the older School-Euclid Geometry should be exhumed and inserted into the present course. Was our changeover to "Papy's" Geometry no more than a classical native solution to having to make a choice?: "We'll make it look as though we have Papy but in reality we'll have Euclid". For example, some teachers have taken the step of teaching proofs of congruence of triangles before beginning the Geometry proper and maintain that inter alia the Side-Angle-Side (SAS) and SSS and ASA results give more machinery to the pupils for solving problems [2].

For my part, I will not present a course of Geometry to pupils which purports to be based upon a transformation approach but depends for its life upon a course of Geometry which it replaced. For one thing I don't agree with a deus ex machina group of congruence theorems to ease the present difficulties: for I couldn't make it work in class, another, the doublethink behind this state of things - which has lasted now for 14 years - is bad for our integrity as Maths, teachers.

Indeed, all our present country's problems are not economical. If one allows that the changed Geometry was first moved around 1968, consolidated in 1973 and will continue to be examined at least until 1984 (what else!), then there are a quarter-million people walking around this country who have been brought up on a diet of bad Geometry - a Geometry, worse in quality than that fed to their parents.

Of course some of the material is so difficult that luckily perhaps 80% of the students have not digested it. Mr. Fred Holland writing in 1977 said "Try as I will I cannot convince my first and second year students of the necessity for equipollence in the definitions of the transformations of the plane. Their minds turn to and run along their preconceived knowledge of length. They appear to accept equipollence just to humour me. If it were in reverse and equipollence followed equality of length, everything would be fine. But to them I am putting the cart before the horse" [3].

How far all of this is from what one Maths. Professor said at an IMTA Conference recently: "We do not require of entrants to our courses that they have smatterings of this, that, or the other topic. The main requirement is a clear mind, an ability to reason and beyond the basics we'll teach them the rest. In fact, we'll even teach them the basics".

Dark ages indeed. The IMTA has campaigned publicly (well before the present hubbub) about the need for syllabus reform; it has set up its commitees, held meeting upon meeting, poured out its sweat, fleshed out its ideas and after four years work submitted its Proposals for new Maths. Syllabi.

Here outside the Oracle the weather is as bleak as ever. We could do with good news to cheer us up.

References:

[1] Over 5 years ago John Kelly of Wicklow Vocational School listed the main faults as follows: (Irish Maths Teachers Association, Newsletter, No. 31, January 1977).

(a) Inadequate teacher training; (b) Demotion of traditional skills (mental arithmetic, graphs, estimation, logs(!), algebra);
(c) Distancing from Science curricula; (d) Over-abstraction (irrelevance to life of laws of associativity, etc., Axiomatics and Symbolism too soon); (e) Superficiality, e.g. treatment of groups;
(f) Unsuitability, general orientation towards University Mathematics.

[2] Article by Gerard Coogan, IMTA Newsletter, No. 30, October 1976. See also "Computer Mathematics 1". G. Coogan. Folens 1982.

[3] IMTA Newsletter, No. 31, January 1977.

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MATHEMATICS HORSES FOR ELEMENTARY PHYSICS COURSES

Niall Ó Murchú and Colm O'Sullivan

Recent publicity about the 1982 summer examinations at U.C.C. has served to draw attention to the fact that for a number of years significant numbers of first year students have been having difficulties in their first university Physics course. While this problem is not confined to U.C.C., nor indeed to Irish Universities, it does seem to exist in first science classes in U.C.C. in a particularly severe form. Given that the first year Physics course in U.C.C. is taught at a low level (significantly lower than U.K. A-levels and certainly not at any higher standard than prevails in most North American universities) it is important that we attempt to ident-

ify the reasons for this problem.

We reject totally the argument that the course is intrinsically too hard for the students. We feel that it is a counsel of despair to suggest that Irish university students are less able than their counterparts abroad. Further, those students who attempt either the exam or the course a second time have a high success rate. Thus one must conclude that the primary reasons for the student's failures must lie in their background and preparation.

One contributory factor is that more than 40% of the first science Physics class in U.C.C. have not done Physics at Leaving Certificate level. We accommodate this by teaching a course which assumes no previous knowledge of the subject. The course is very compact (only about half the Leaving course is covered) and nothing is included that is not on the Leaving Certificate syllabus. We believe, however, that an absence of a background in Physics is not the primary source of the difficulties. The failure rate among students who did Physics but obtained a poor grade (e.g. D,E or F on the higher paper) in the Leaving Certificate is significantly worse than the failure rate among students starting the subject from scratch.

We have become convinced that a major contribution to the difficulties that students are having comes from their grasp, or rather lack, of the fundamentals of Mathematics. We recognize that Physics is Physics and Mathematics is Mathematics in that each discipline has its own ethos and understanding. On the other hand it is clear that Mathematics provides the language of Physics. In all Physics courses at this level the insights and ideas involved are conveyed by mathematical relationships which both codify and illuminate the physical processes. In turn, we expect students to manipulate these formulae and be able to extract numbers from them.

Both the Science Faculty at U.C.C. as a whole and the Department of Physics have carried out a number of detailed surveys into the background and skills of incoming students during the past five or six years. Dianostic tests in basic mathematics have been a regular part of such investigations. We have discovered an appalling lack of the most elementary mathematical preparation among the first year science students (the problem is much less severe among the pre-medical, pre-dental and first engineering classes). The problem is not an absence of knowledge but rather a total lack of facility with even the simplest operations. It is not possible here to list all skills found lacking, but a few examples might help to illustrate the problem to some extent.

(i) Reassembling of terms done routinely in Physics classes such as

$$(a-b) - (c-d) = (a-c) - (b-d)$$

is seen by a significant number of students as a black art, designed to confuse.

(ii) While students are not expected to know any values of Sinθ except for θ=0 and θ= 90°, if sin0 = 0 or Sin90° = 1 are used to simplify an expression as much as 20% of the class may be lost.

(iii) The following question was asked on one of the diagnotic tests; over two-thirds of the class gave a wrong answer

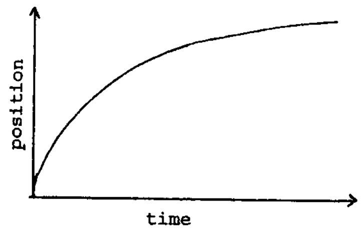
This a rough position versus time graph for a moving car. Is the car

(1) increasing speed

(2) moving with constant speed

(3) slowing down

(4) going backwards



The sort of mathematics required for an elementary course in Physics is far from sophisticated as can be seen from the examples, but rather the material is that which should be covered in the junior cycle in secondary school. We can get by if students do not know how to differentiate sinθ, but we do expect them to know the general shape of the trigonometric functions. In these days of calculators, we do not expect students to be able to multiply, however we do expect them to be at least able to estimate an answer. A significant fraction of the class is not convinced that the inverse of a number less than one is greater than one and, when dealing with exponents, up to 30% will get division wrong.

There seems to be continuing tension in the second-level schools where some teachers feel that the courses are 'too academic'. This is often understood to be a result of having the courses 'geared to the needs of third-level'. The basis for the latter view is unclear. We, as third-level teachers, feel that many of the criticisms of the 'anti-academic' camp are valid and ought to be considered seriously. We see too many students with 'good' Leaving results coming out lacking basic survival mathematics. They are unable to interpret graphs, they do not understand scaling (double something in the numerator and the quantity doubles,

double the denominator and the quantity halves). They are unable to perform the simplest of manipulations (A=BC → B=A/C); all variables are either x or y, everything else is a constant; they do not understand sine or cosine. They are unable to transfer knowledge; they seem to have been drilled to solve a particular sort of problem when it is posed in a set way, but are unable to recognise an equivalent problem in a different context.

We feel sure that none of our students have passed through secondary school without meeting the ideas mentioned above. We expect that all students knew at one time that sin 90° = 1. What seems to be missing from the learning experience is any attempt to reinforce the elementary concepts. One of the most undesirable, and we presume unintended, side effects of the course reforms in second level education in the last decade or so has been an increase in the strategy of leaving out large sections of the syllabus for the purposes of examinations. This seems to be coupled with an increased reliance on rote memory as an alternative to understanding. Of course, these problems are by no means unique to Mathematics, they pervade the entire system. One obvious response to this problem is to reduce the quantity of new material introduced at each stage. This would enable the basic hard-core material to be covered in more depth and would allow more time for students to achieve mastery of the basic skills involved.

We hope that it will be understood by all that these criticisms are intended to be constructive and are not an attempt to distract attention from our own shortcomings. Physics in school or in university is an immediate user of mathematics. Like any course which depends on simple mathematics, it will prove to be a difficult subject for those students who lack the basic skills. We readily accept that a large number of students leave second level well motivated and well equipped in basic mathematics. The quality of Mathematics teaching in some Irish schools compares favourably, in our view, with the very best in other countries. Unfortunately we have found that a large number of students also emerge from secondary school very badly equipped indeed. Given that we are talking here about minimal mathematics (the sort of mathematics that is used to justify the importance of the subject in the schools) this is deeply disturbing and calls for some response.

Much of the basic training in Mathematics required for success in an introductory course in Physics is the same as that required for survival in any situation. There seems to be no conflict here between the needs of third-level education and 'education for life'. Indeed we would argue that the simple mathematical skills we have isolated as being required for elementary physics are also precisely those required for survival in our modern technological society. A serious lack of these skills in an adult population is as serious a problem as illiteracy; those lacking such skills today will inevitably find themselves exploited by society. Many of the students with the problems discussed above would also be unable to determine whether a packet of 6, 10, 16 or 36 fish fingers is the best value. Consider also the level of competence in simple mathematics that is required in order to follow, let alone participate in, a debate on the safety of asbestos or of nuclear power. Similar skills are required if one is to make any judgement about modern economic theories expounded by politicians. The teachers of mathematics at all levels carry a grave responsibility and deserve every support. All those in a position to influence educational policy in mathematics should urgently consider whether present practices are achieving the desired objectives. On the evidence available to us we are forced to conclude that, for a significant number of school leavers, this is not the case.

Physics Department,
University College, Cork.

BOOK REVIEWS

RIESZ AND FREDHOLM THEORY IN BANACH ALGEBRAS

B.A. Barnes, G.J. Murphy, M.R.F. Smyth and T.T. West
Research notes in Mathematics, PITMAN (82) 117 pp.

A linear operator T on a Banach space X is continuous if and only if it is bounded, in the sense of mapping bounded sets into bounded sets, and if it actually maps bounded sets into totally bounded sets then it is called compact. If K is a compact operator and $0 \neq \lambda \in \mathbb{C}$ then $T = K - \lambda I$ is Fredholm, in the sense that its null space $T^{-1}0$ is finite dimensional and its range TX is closed and of finite dodimension. An operator K for which $T = K - \lambda I$ is Fredholm for each $\lambda \neq 0$ is called Riesz. It turns out that if K is Riesz then it need not be compact, and for each $\lambda \neq 0$ the index of T is zero, in the sense that the two finite dimensions coincide, and indeed that T will be invertible except for at worst a sequence of values of λ tending to 0.

As is familiar this is a circle of ideas of crucial importance to those who would use bounded linear operators to study differential equations: differential operators can never be bounded, but the associated integral equations are built upon operators which are not only bounded but actually compact, and it was among integral equations that Fredholm theory was born. Thus Fredholm theory should be tightly bound to the core of the theory of bounded operators. One perhaps rather perverse way of doing this is to try and "algebraicise" the theory - a sort of "Wilson cloud chamber" approach in which the properties of being compact, Fredholm or Riesz are to be expressed not spatially but in terms of all the other bounded linear operators on the space. If one should succeed in doing that, then there is an overwhelming compulsion to go on and study corresponding elements of more general Banach algebras.

For better or worse, this is the subject of the volume under review. It falls into six chapters, indexed by descriptive letters rather than by arabic numerals. The preliminary Chapter O, OPERATOR THEORY, was intended to be a summary of the basic definitions of compact, Fredholm and Riesz operators and an account of the algebraicisation of the second two properties - compactness is rather more elusive. It has however been augmented by a discussion of a new "enlargement process" for spaces and operators which gives a very sharp description of semi-Fredholm and of Fredholm operators; this technique is applied to obtain a new characterization of Riesz operators and to derive range inclusion theorems for them.

Chapter F, FREDHOLM THEORY, is the core of the book, and sets out to characterize the Fredholm elements of a Banach algebra. If A is a semi-simple Banach algebra then the sum of its minimal left ideals is equal to the sum of its minimal right ideals and forms a two-sided ideal called the socle: then an element x ∈ A will be called Fredholm if and only if it is invertible modulo the socle. If A is not semi-simple then this does not work: in that case first remove the radical and make it semi-simple. To see that this Fredholm theory works we must first restrict attention to primitive algebras and then climb back to a general semisimple algebra. If A is primitive, in the sense of having at least one faithful irreducible representation as operators on a linear space, and if the socle is not zero, then it turns out that a Fredholm element x induces an actual Fredholm operator of left multiplication on the quotient A/J by each minimal left ideal J, and further that the index and the two finite dimensions associated with this operator are independent of the particular ideal J. Thus the usual decomposition and perturbation theory for operators transfers back to elements of primitive algebras. If more generally A is semisimple then the authors show that x ∈ A is Fredholm if and only if the coset x+P is Fredholm for each primitive ideal P. The index and the two finite dimensions are now computed by adding the corresponding quantities for each coset x+P, having first shown that they vanish for all but finitely many P; it is then still possible to transfer decomposition and perturbation theory, and the "punctured neighbourhood theorem" to algebra elements.

Chapter R, RIESZ THEORY, turns to the concept of a Riesz element of an algebra A relative to an ideal K, and shows that for the theory to work it is necessary that the ideal K be closed and have the same "hull-kernel" as the socle, or presocle if the algebra is not semisimple. The authors prove a theorem showing that a left or right ideal lies in the hull-kernel of the socle if and only if 0 is not an accumulation point of the spectrum of any of its elements; if the whole algebra satisfies this condition it is called a Riesz algebra. For example the LCC and the RCC algebras of Kaplansky, and the group algebra of a compact group, are Riesz algebras in this sense.

Chapter C*, C*-ALGEBRAS, starts by picking up a "wedge operator" construction used in Chapter O to characterise Riesz operators and uses it

to define the finite rank elements and the compact elements of a C*-algebra: it then turns out that these just form the socle and its closure. The main burden of this chapter is to reproduce the famous West decomposition, and express a Riesz element of C*-algebra as the sum of a normal compact element and a quasinilpotent. Stampfli's generalization of the West decomposition is obtained on the way; the authors then go on to an improved characterization of Riesz algebras, and a version of the classic Gelfand-Naimark-Siegel construction which also preserves compactness and Fredholmness.

Chapter A, APPLICATIONS, looks at operators leaving a fixed subspace of a Banach space invariant, at triangular and quasitriangular operators, and at measures on a compact group. Chapter BA, BANACH ALGEBRAS, is a summary of the background material on minimal ideals and the socle, on primitive ideals and the hull-kernel topology, and on C*-algebras. There is finally a comprehensive bibliography and a valuable index.

"Riesz and Fredholm theory ... " is an exciting book for specialists, and makes available to a wider audience work of Smyth which has hitherto only been in preprints. Roger Smyth also contributes a unique author address to the title page. It is fair to warn the general reader that this is officially a "research note", and that it bears the signs of having been written by four people. This is a pity, because hidden in an exciting research note there is an even more exciting book. In seeking properly to algebraicise Fredholm and Riesz theory, these authors have started to write a new chapter of the theory of Banach algebras. Perhaps some specialist will become sufficiently irritated as he turns back and forth between Chapter BA, Chapter F and Chapter R to follow the advice offered, by another of these four authors, at the bottom of page 264 of the May 1982 issue of the Bulletin of the London Mathematical Society.

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Mathematics Department,
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"COUNTDOWN TO MATHEMATICS" Vols. 1 and 2.

By *Lynne Graham* and *David Sargent* (Open University)

Published by *Addison-Wesley Publishers Limited in association with the Open University Press, 1981.*

ISBN 201.13730.5 (Vol. 1) 201.13731.3 (Vol.2)

My overall impression of these two volumes on pre-calculus Mathematics is that they are excellent texts. They are primarily aimed at *intending* Open University students. Volume 1 concentrates on "basic skills and techniques in arithmetic, algebra, graphs and statistics" for courses in "the Social Science, Science, Education and Technology disciplines". Volume 2 continues the development of skills and techniques in these areas and introduces some of the basic ideas of geometry and trigonometry. Both volumes are intended to inculcate confidence in elementary mathematical skills and techniques particularly in manipulative skills. They further help to develop basic skills for self-study.

Both volumes present a number of Modules, each dealing with an individual area of basic Mathematics. The Module is then subdivided into five sections each prefaced by some diagnostic questions which enable the student to discern which, if any, of the sections may be omitted with confidence. Each section ends with a plentiful supply of exercises which coupled with the material in the section will help develop mastery of the particular topic treated.

The presentation of the content is informal, with many of the ideas illustrated or at least light-heartedly introduced by a liberal use of cartoons which are entertaining if not always informative. Many of the illustrative examples used in the text are both topical and relevant to real-life situations and/or to some of the basic disciplines of Science and Economics e.g. household budgets, level contours on maps, temperature conversion formulae, speed calculations (here however the authors could more consistently have used the SI unit system) etc. In relation particularly to the arithmetic sections I liked the authors' presentation of how to first obtain rough estimates in any calculation before proceeding to the detailed calculation, and also their encouragement to the student to do so. In addition, use of the pocket calculator is both encouraged and taught through examples accompanied by cartoon illustrations and step-by-step details of keys to be used.

Pictorial representations of algebraic manipulations and graphical, histogram, bar-chart and pie-chart representations of data are clear, informative and liberally used throughout the text. In the particular section on "Data in Tables" I like the way in which the authors present data in tabular form and clearly teach the student how to read information from Tables of Figures such as reciprocals, square roots etc.

In a number of areas, which were otherwise well presented, the authors could, I think, have included some further useful material without having to overly extend either the student or the size of the volumes. The section on simultaneous equations could easily have included something on 3 x 3 systems rather than just 2 x 2 systems; the Logarithm section which deals extensively with base 10 logarithms and includes some other-base logarithms could beneficially have included mention of natural logarithms; expansion formulae for sin(A $\pm$ B) and the like are omitted from the trigonometry section; and there is nothing involving elementary complex numbers which would have extended the otherwise excellent treatment of roots of quadratics to other than real roots. These however are minor shortcomings in what are in my opinion two excellent volumes.

In the light of our experiences here in University College, Cork in recent years, many of our First Year Science students would benefit enormously both in their Mathematics courses and particularly, in their Experimental Physics and Chemistry courses from using these texts to review and develop confidence in using material which they should have already mastered before entering University.

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University College, Cork.

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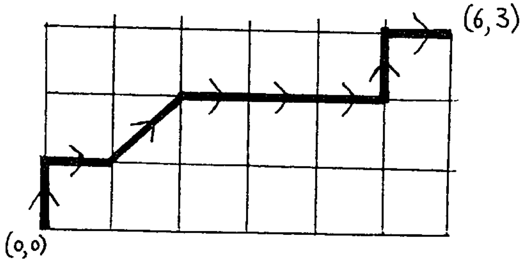
Problems Page

Not much response to the earlier problems yet. Gordon Lessells points out that Problem 1 is of a type considered by Professor Sir Alexander Oppenheim, with whom I have been in correspondence. He recommends the book 'Geometric Inequalities' by Bottema, Djordjevic, Janic, Mitrinovic and Vasic (Groningen 1969) which is entirely devoted to triangle inequalities. It seems that there is still some interest in finding a simple proof of Barrow's inequality.

Problem 3 contains a misprint, I'm afraid. The terms in the right-hand sum should have a d! in their denominators. The question, to which both these sums are the answer, is:-

> how many distinct paths on the positive integer lattice $\{(p,q):p,\ q \in \mathbb{Z}^+\}$ can be found which join (0,0) to (m,n)? The paths are to consist of horizontal, vertical or diagonal line segments in any order, but both the x and y co-ordinates of a point on the path must be monotonic.

For example:



Just two more problems this time.

1. (A Putnam Prize problem) Compute

$$\int_0^{\pi/2} \frac{dt}{1+\tan^{\sqrt{2}} t}$$

2. If a,b are points of a set A in $\mathbb{R}^n$ we write aSb whenever the line segment joining a to b lies in A.

If A is the union of two <u>convex</u> sets then:

(*) If $a,b,c \in A$ then at least one of the following is true:

aSb, bSc, cSa.

Problem: if (*) holds is A necessarily the union of two convex sets?

P. J. Rippon

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25.	J.H. RAWNSLEY, *The differential geometry of instantons*, 1976	£1.50
26.	W. ISRAEL, *Differential forms in general relativity*, 2nd ed., 1979	£1.50

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CONFERENCE REPORTS

BAIL II

The second international conference on Boundary and Interior Layers-Computational and Asymptotic Methods, BAIL II, was held in Trinity College, Dublin, Ireland, from 16th to 18th June, 1982, under the auspices of the Numerical Analysis Group. The eightytwo participants came from eighteen countries and included for the first time strong delegations from the USSR and China.

The conference was cosponsored by the American Institute of Aeronautics and Astronautics, the American Meteorological Society, the Institute for Numerical Computation and Analysis and the Irish Mathematical Society.

The aim of this series of conferences is to bring together biologists, chemists, engineers, mathematicians, physicists and other scientists who encounter problems having solutions which exhibit boundary or interior layer behaviour. Both computational and asymptotic methods were discussed extensively at BAIL II, and the degree of difficulty of the problems to which these were applied showed a marked increase over that of the first conference in the series.

The eleven keynote speakers presented papers covering a wide variety of applications and several new computational and asymptotic methods. The areas of application included plasmas, hydrodynamic shocks, transonic airfoils, free surface problems, viscous flows and a variety of phenomena in meteorology. The new methods were concerned with turning point and parabolic problems having a singular perturbation and stiff and other special initial value problems.

In addition there were forty-four contributed papers. Many of these were concerned with boundary and interior layer problems arising in biology, chemistry, elasticity, fluid flow, heat transfer, meteorology and petroleum reservoic modelling. Others discussed various computational and asymptotic topics including uniform numerical methods for problems with a singular perturbation, multigrid methods, defect correction techniques, sparse matrices and eigenvalue problems.

When boundary or interior layers are encountered in practical problems it is often found that standard numerical techniques are inaccurate, too expensive or even divergent. This underlines the importance of devising robust numerical algorithms which take account of such layers. In other cases the occurrence of layers may not have been recognised even though they may in fact be present. It is wise therefore to consider their presence as one possible cause for the degradation in performance of an otherwise well established numerical algorithm. That such is the case in a wide variety of situations is attested to by the many fascinating papers delivered at the first two conferences in this series.

In association with the BAIL II Conference an introductory short course was held on the same topic as the conference. This consisted of sixteen tutorial lectures on the various conference themes.

An exhibition of books and journals was also arranged. Eleven scientific and technical publishers exhibited fifty four books and sixteen journals.

For a representative collection of papers on the subject the reader may consult the three publications (4), (5) and (6) associated with the BAIL Conferences. The Proceedings of two earlier conferences on a similar topic are contained in (1) and (3). A comprehensive monograph on uniform numerical methods for problems with layers is (2), and an earlier monograph on complementary topics is (7).

It is proposed to hold the third conference in the series, BAIL III, from 20th to 22nd June, 1984.

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(2) E.P. Doolan, J.J.H. Miller, W.H.A. Schilders "Uniform Numerical Methods for Problems with Initial and Boundary Layers". Boole Press, Dublin (1980).

(3) P.W. Hemker, J.J.H. Miller (eds.) "Numerical Analysis of Singular Perturbation Problems". Academic Press, London (1979).

(4) J.J.H. Miller (ed.) "Boundary and Interior Layers - Computational and Asymptotic Methods. Proceedings of the BAIL II Conference". Boole Press, Dublin (1980).

(5) J.J.H. Miller (ed.) "Computational and Asymptotic Methods for Boundary and Interior Layers. Proceedings of the BAIL II Conference". Boole Press, Dublin (1982).

(6) J.J.H. Miller (ed.) "An Introduction to Computational and Asymptotic Methods for Boundary and Interior Layers. Lecture Notes of a Short

Course held in association with the BAIL II Conference". Boole Press, Dublin (1982).

(7) W.L. Miranker "Numerical Methods for Stiff Equations and Singular Perturbation Problems". Reidel, Dordrecht (1980).

J.J. H. Miller

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APPLIED STATISTICS IN IRELAND

A Conference on the above theme was held in Galway on May 13-14,1982, attended by a most representative group of statisticians from all over Ireland. The Conference was an ideal opportunity for statisticians to get together and discover what other statisticians were doing.

Contributors to the Conference included statisticians working in third level educational institutions, the Civil Service and private industry. Their brief was to present papers involving the application of statistical methods to real problems, and this resulted in a widely varied range of topics, from the treatment of wind power as a spatial time process to the identification of the authorship of a certain tract by means of multivariate techniques.

Two invited speakers gave special review presentations. Professor David Cox of Imperial College, London, gave a highly interesting and wide-ranging presentation on the role of Asymptotic Theory in statistical practice. Conference participants were privileged to hear a lecture by a man so clearly a master of all he surveyed. Colm O'Muircheartaigh, of the London School of Economics, gave a very stimulating and highly amusing talk on the impact of sample survey theory on sample survey practice; the insights provided into the historical development of sample survey theory were most illuminating.

The rest of the conference consisted of a series of presentations describing the application of statistical techniques to particular problems. Professor R.E. Blackith (TCD) described a computer model of the possible relations between radiation and cancer. He examined in particular the form of the dosage-response curve relating ionising radiation and cancer by treating the problem as one in biological assay. Professor M.A. Moran (UCC) and Dr. B. Murphy (RTC,Cork) compared the parametric and product kernel approaches to discriminant analysis. They showed that the latter approach was superior when the variables are independent or moderately correlated but that its behaviour otherwise is less predictable.

Dr. S.T.C. Weatherup (N.I. Dept. of Agriculture) described various statistical tests which are used to determine if newly bred varieties of crops can be distinguished from existing varieties. He described both univariate and multivariate methods of discriminating between varieties, and the determination of characteristics of new variates likely to be accepted by the present system.

Drs. J. Haslett, E. McColl and A. Raftery (TCD) presented the initial analyses of a project concerned with mapping the statistical characteristics of the Irish wind power resource. Their analysis involved the removal of non-stationarities in time and space, and an attempt to model the remaining space-time correlations. Subsequently, such a model would be used to develop improved estimates of long-term average wind power at sites where small amounts of data are available and by interpolation between sites to estimate wind power for sites at which no data is avialable.

Professor S.F. McNamara, Dr. K.F. McNamara, J. Conroy and T. Dooley (U.C.G.) spoke on a related topic. They described the analysis of the data collected from the international energy agency's prototype wave energy absorber KAIMEI during its sea trials at Yura in the Sea of Japan over the period September to March 1979. In order to determine the effectiveness of the KAIMEI in converting wave energy into useable electric power, the theory of multiple frequency response functions was applied to the data. Statistical measures were used to indicate the reasons why the KAIMEI failed to capture a great proportion of the available wave energy.

And now for something completely different! A.R. Unwin, D. Berman and M. Sloan (TCD) discussed a statistical approach to a long-running dispute concerning the authorship of "The Memoirs of Gaudentio Di Lucca". They used multivariate statistics to analyse authors' styles and the claims of two of the most serious candidates were assessed. They also discussed the relationships between the statistical evidence available. In answer to a question, they had to admit that they could not reject the null hypothesis that the memoirs were written by Myles Na Gopaleen!

Professor M.A. Moran (UCC) and J. Langan (IIRS) gave a very practical illustration of the application of discriminant analysis to the identification of edible 'fats' on the basis of their fatty acid composition. A variety of models appropriate to the compositional nature of the data were

described and discriminant functions based on these models were applied to some real data. M.R. Stevenson (Gallagher Ltd.) spoke about the truncation of data and its relevance to a particular problem: animals (experimental units) are allocated at random to treatment-dose groups and the time to a predetermined response (i.e. death!) measured for each experimental unit. In general, most animals indicate response before termination of the experiment, but in a few cases, particularly in the low dose treatment groups, some animals fail to respond before termination.

Dr. A.E. Raftery (TCD) described a non-parametric approach to measureing social mobility. This measurement is usually based on a sample survey of employed males in which they are asked to give their occupation and that of their father. Dr. Raftery developed a non-parametric approach which facilitated the construction of social mobility indices; these indices could then be used to enable comparisons to be drawn between surveys carried out in different countries or at different times, or between birth cohorts within the same survey.

Dr. G. Kelly (UCC) discussed an application of the errors in variables model to the comparison of two different methods for measuring cardiac output. This model is particularly appropriate when comparing two measurement techniques where both are subject to error. An analysis of real data was presented.

Dr. D.J. Kilpatrick (N.I. Dept. of Agriculture) spoke about sampling methods using information on a concomitant variable. The problem considered was the estimation of the mean of a variable y when information was available on a related variable x. Alternative estimation methods used included stratification according to x value, the adjustment of the mean of y through a ratio or regression procedure and sampling with probability proportional to x. Applications to recent forest survey data were described.

Dr. D. McSherry (QUB) presented a paper on optimal efficiency, within the constraints of cultivar incompatibility, in trials with mixtures of different cultivars of a single crop. Trials were identified which, within such constraints, were optimally efficient.

Last, but (to coin a phrase) by no means least, Dr. M. Stuart (TCD) gave a very stimulating talk on the increasing use of statistics and probabilities in law cases in Ireland. In particular, he reviewed some technical issues arising in the probabilistic assessment of forensic evidence.

The relevance of the substance of this paper to the breathalyser laws was discussed in depth.

The Conference, which was organised by Dr. S.T.C. Weatherup (N.I. Dept. of Agriculture) and Dr. I.G. O Muircheartaigh (UCG), was the second in a series of what is hoped will be annual conferences run by a (so far) ad hoc grouping of statisticians working in Ireland.

I. Ó Muircheartaigh

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CONFERENCE ANNOUNCEMENTS

THIRD CONFERENCE ON APPLIED STATISTICS IN IRELAND

This conference will be held in the Slieve Donard Hotel, Newcastle, Co. Down, on 28th and 29th March, 1983. As was the case with previous conferences, the preference is for papers concerned with the application of statistical methods but other papers will be considered. There will be a a special session devoted to statistical computing and another to the teaching of statistics. While these sessions will have some invited speakers, submitted papers concerned with these topics are as welcome as any others. The criteria for acceptance of a paper are that it can be delivered in a 20 minute presentation and that the organisers consider the subject matter appropriate to the conference. The organisers will make their decision on the basis of an abstract or brief synopsis of the proposed paper. Please send abstracts to, either:

Dr. S.T.C. Weatherup, Department of Agricultural Biometrics, Agricultural and Food Science Centre, Queen's University, Newforge Lane, Belfast BT9 5PX,
Telephone Belfast: 661166, Extension 209

Professor D. Conniffe, The Economic and Social Research Institute,
4 Burlington Road, Dublin 4:Telephone Dublin: 760115

The final date for receipt of abstracts is 10 January 1983 but the organisers urge potential contributors to write or telephone as soon as possible.

Even if you will not be submitting a paper please consider attending the conference. The conference is intended to be of interest to anyone who utilises statistical methods and not only to the professional statistician. The conference fee, which covers all meals and accommodation, is £48 sterling.

BRITISH MATHEMATICAL COLLOQUIUM

The thirty-fifth B.M.C. will be held at the University of Aberdeen from 5th to 9th April, 1983. The programme is:

Wednesday, 6th April

9.30 - 12.30 E.H. Brown, D.F. Holt, J. Toland;
G.J.O. Jameson, W.S. Kendall, J.C. Wood

4.45 - 5.45 D.G. Quillen (M.I.T.)

INFINITE DETERMINANTS OVER ALGEBRAIC CURVES ARISING FROM PROBLEMS IN GEOMETRY, DIFFERENTIAL EQUATIONS AND NUMBER THEORY

Thursday, 7th April

9.30 - 12.30 W.A. Hodges, C.J.K. Batty, J.T. Stafford; A.A. Ranicki, W.J. Harvey, J.P. Bourguignon

5.45 - 5.45 L. Nirenberg (Courant Institute)

**COMMENTS ON NONLINEAR PROBLEMS*

Friday, 8th April

9.30 - 12.30 E.J.N. Looijenga, C. Kosniowski, N.M.J. Woodhouse; N.J. Young, R.J. Cook

4.45 - 5.45 D.P. Sullivan (I.H.E.S. and C.U N.Y)

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The membership fee for the Colloquium is £10 (this includes the cost of refreshments). This fee will be increased to £15 for applications received after 31st January 1983.

The full charge for meals and accommodation in a hall of residence (beginning with the evening meal on 5th April and ending with breakfast on 9th April) has been specially reduced to £50. The charge for a shorter stay should be calculated on the basis:

Bed and Breakfast £8.50, Lunch £2.40, Evening Meal £2.90.

Further information from:

R.J. Archbold,
Colloquium Secretary,
Department of Mathematics,
University of Aberdeen,
The Edward Wright Building,
Dunbar Street,
Aberdeen AB9 2TY.

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THE NEXT INTERNATIONAL CONGRESS

OF MATHEMATICIANS

will take place in

WARSAW, POLAND

during

AUGUST 16 to 24, 1983

TRAVEL GRANTS FOR YOUNG MATHEMATICIANS

The International Mathematical Union offers financial support to well-qualified mathematicians who are under 35 years of age and wish to go to Warsaw for the 1983 Congress.

If you are interested, eligible, and wish to apply for a grant to enable you to go to the Congress next year, make out an application - which should include a brief curriculum vitae, the names of two referees, and any other supporting evidence you wish to submit - and send it to:

The Secretary,
National Committee for Mathematics,
Royal Irish Academy,
19 Dawson Street,
DUBLIN 2,

before January 19, 1983.