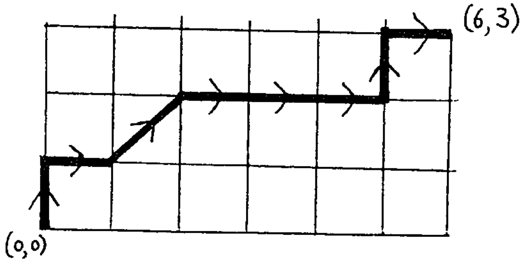
Problems Page

Not much response to the earlier problems yet. Gordon Lessells points out that Problem 1 is of a type considered by Professor Sir Alexander Oppenheim, with whom I have been in correspondence. He recommends the book 'Geometric Inequalities' by Bottema, Djordjevic, Janic, Mitrinovic and Vasic (Groningen 1969) which is entirely devoted to triangle inequalities. It seems that there is still some interest in finding a simple proof of Barrow's inequality.

Problem 3 contains a misprint, I'm afraid. The terms in the right-hand sum should have a d! in their denominators. The question, to which both these sums are the answer, is:-

> how many distinct paths on the positive integer lattice $\{(p,q):p,\ q \in \mathbb{Z}^+\}$ can be found which join (0,0) to (m,n)? The paths are to consist of horizontal, vertical or diagonal line segments in any order, but both the x and y co-ordinates of a point on the path must be monotonic.

For example:



Just two more problems this time.

1. (A Putnam Prize problem) Compute

$$\int_0^{\pi/2} \frac{dt}{1+\tan^{\sqrt{2}} t}$$

2. If a,b are points of a set A in $\mathbb{R}^n$ we write aSb whenever the line segment joining a to b lies in A.

If A is the union of two convex sets then:

(*) If a,b,c $\in$ A then at least one of the following is true:

aSb, bSc, cSa.

Problem: if (*) holds is A necessarily the union of two convex sets?

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