Dark ages indeed. The IMTA has campaigned publicly (well before the present hubbub) about the need for syllabus reform; it has set up its commitees, held meeting upon meeting, poured out its sweat, fleshed out its ideas and after four years work submitted its Proposals for new Maths. Syllabi.

Here outside the Oracle the weather is as bleak as ever. We could do with good news to cheer us up.

References:

[1] Over 5 years ago John Kelly of Wicklow Vocational School listed the main faults as follows: (Irish Maths Teachers Association, Newsletter, No. 31, January 1977).

(a) Inadequate teacher training; (b) Demotion of traditional skills (mental arithmetic, graphs, estimation, logs(!), algebra);
(c) Distancing from Science curricula; (d) Over-abstraction (irrelevance to life of laws of associativity, etc., Axiomatics and Symbolism too soon); (e) Superficiality, e.g. treatment of groups;
(f) Unsuitability, general orientation towards University Mathematics.

[2] Article by Gerard Coogan, IMTA Newsletter, No. 30, October 1976. See also "Computer Mathematics 1". G. Coogan. Folens 1982.

[3] IMTA Newsletter, No. 31, January 1977.

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MATHEMATICS HORSES FOR ELEMENTARY PHYSICS COURSES

Niall Ó Murchú and Colm O'Sullivan

Recent publicity about the 1982 summer examinations at U.C.C. has served to draw attention to the fact that for a number of years significant numbers of first year students have been having difficulties in their first university Physics course. While this problem is not confined to U.C.C., nor indeed to Irish Universities, it does seem to exist in first science classes in U.C.C. in a particularly severe form. Given that the first year Physics course in U.C.C. is taught at a low level (significantly lower than U.K. A-levels and certainly not at any higher standard than prevails in most North American universities) it is important that we attempt to ident-

ify the reasons for this problem.

We reject totally the argument that the course is intrinsically too hard for the students. We feel that it is a counsel of despair to suggest that Irish university students are less able than their counterparts abroad. Further, those students who attempt either the exam or the course a second time have a high success rate. Thus one must conclude that the primary reasons for the student's failures must lie in their background and preparation.

One contributory factor is that more than 40% of the first science Physics class in U.C.C. have not done Physics at Leaving Certificate level. We accommodate this by teaching a course which assumes no previous knowledge of the subject. The course is very compact (only about half the Leaving course is covered) and nothing is included that is not on the Leaving Certificate syllabus. We believe, however, that an absence of a background in Physics is not the primary source of the difficulties. The failure rate among students who did Physics but obtained a poor grade (e.g. D,E or F on the higher paper) in the Leaving Certificate is significantly worse than the failure rate among students starting the subject from scratch.

We have become convinced that a major contribution to the difficulties that students are having comes from their grasp, or rather lack, of the fundamentals of Mathematics. We recognize that Physics is Physics and Mathematics is Mathematics in that each discipline has its own ethos and understanding. On the other hand it is clear that Mathematics provides the language of Physics. In all Physics courses at this level the insights and ideas involved are conveyed by mathematical relationships which both codify and illuminate the physical processes. In turn, we expect students to manipulate these formulae and be able to extract numbers from them.

Both the Science Faculty at U.C.C. as a whole and the Department of Physics have carried out a number of detailed surveys into the background and skills of incoming students during the past five or six years. Dianostic tests in basic mathematics have been a regular part of such investigations. We have discovered an appalling lack of the most elementary mathematical preparation among the first year science students (the problem is much less severe among the pre-medical, pre-dental and first engineering classes). The problem is not an absence of knowledge but rather a total lack of facility with even the simplest operations. It is not possible here to list all skills found lacking, but a few examples might help to illustrate the problem to some extent.

(i) Reassembling of terms done routinely in Physics classes such as

$$(a-b) - (c-d) = (a-c) - (b-d)$$

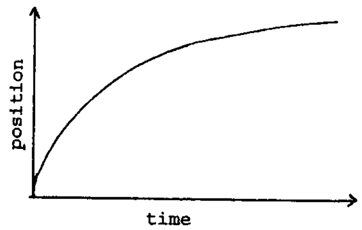
is seen by a significant number of students as a black art, designed to confuse.

(ii) While students are not expected to know any values of Sinθ except for θ=0 and θ= 90°, if sin0 = 0 or Sin90° = 1 are used to simplify an expression as much as 20% of the class may be lost.

(iii) The following question was asked on one of the diagnotic tests; over two-thirds of the class gave a wrong answer

This a rough position versus time graph for a moving car. Is the car

(1) increasing speed
(2) moving with constant speed
(3) slowing down
(4) going backwards



The sort of mathematics required for an elementary course in Physics is far from sophisticated as can be seen from the examples, but rather the material is that which should be covered in the junior cycle in secondary school. We can get by if students do not know how to differentiate sinθ, but we do expect them to know the general shape of the trigonometric functions. In these days of calculators, we do not expect students to be able to multiply, however we do expect them to be at least able to estimate an answer. A significant fraction of the class is not convinced that the inverse of a number less than one is greater than one and, when dealing with exponents, up to 30% will get division wrong.

There seems to be continuing tension in the second-level schools where some teachers feel that the courses are 'too academic'. This is often understood to be a result of having the courses 'geared to the needs of third-level'. The basis for the latter view is unclear. We, as third-level teachers, feel that many of the criticisms of the 'anti-academic' camp are valid and ought to be considered seriously. We see too many students with 'good' Leaving results coming out lacking basic survival mathematics. They are unable to interpret graphs, they do not understand scaling (double something in the numerator and the quantity doubles,

double the denominator and the quantity halves). They are unable to perform the simplest of manipulations (A=BC → B=A/C); all variables are either x or y, everything else is a constant; they do not understand sine or cosine. They are unable to transfer knowledge; they seem to have been drilled to solve a particular sort of problem when it is posed in a set way, but are unable to recognise an equivalent problem in a different context.

We feel sure that none of our students have passed through secondary school without meeting the ideas mentioned above. We expect that all students knew at one time that sin 90° = 1. What seems to be missing from the learning experience is any attempt to reinforce the elementary concepts. One of the most undesirable, and we presume unintended, side effects of the course reforms in second level education in the last decade or so has been an increase in the strategy of leaving out large sections of the syllabus for the purposes of examinations. This seems to be coupled with an increased reliance on rote memory as an alternative to understanding. Of course, these problems are by no means unique to Mathematics, they pervade the entire system. One obvious response to this problem is to reduce the quantity of new material introduced at each stage. This would enable the basic hard-core material to be covered in more depth and would allow more time for students to achieve mastery of the basic skills involved.

We hope that it will be understood by all that these criticisms are intended to be constructive and are not an attempt to distract attention from our own shortcomings. Physics in school, or in university is an immediate user of mathematics. Like any course which depends on simple mathematics, it will prove to be a difficult subject for those students who lack the basic skills. We readily accept that a large number of students leave second level well motivated and well equipped in basic mathematics. The quality of Mathematics teaching in some Irish schools compares favourably, in our view, with the very best in other countries. Unfortunately we have found that a large number of students also emerge from secondary school very badly equipped indeed. Given that we are talking here about minimal mathematics (the sort of mathematics that is used to justify the importance of the subject in the schools) this is deeply disturbing and calls for some response.

Much of the basic training in Mathematics required for success in an introductory course in Physics is the same as that required for survival in any situation. There seems to be no conflict here between the needs of third-level education and 'education for life'. Indeed we would argue that the simple mathematical skills we have isolated as being required for elementary physics are also precisely those required for survival in our modern technological society. A serious lack of these skills in an adult population is as serious a problem as illiteracy; those lacking such skills today will inevitably find themselves exploited by society. Many of the students with the problems discussed above would also be unable to determine whether a packet of 6, 10, 16 or 36 fish fingers is the best value. Consider also the level of competence in simple mathematics that is required in order to follow, let alone participate in, a debate on the safety of asbestos or of nuclear power. Similar skills are required if one is to make any judgement about modern economic theories expounded by politicians. The teachers of mathematics at all levels carry a grave responsibility and deserve every support. All those in a position to influence educational policy in mathematics should urgently consider whether present practices are achieving the desired objectives. On the evidence available to us we are forced to conclude that, for a significant number of school leavers, this is not the case.

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BOOK REVIEWS

RIESZ AND FREDHOLM THEORY IN BANACH ALGEBRAS

B.A. Barnes, G.J. Murphy, M.R.F. Smyth and T.T. West
Research notes in Mathematics, PITMAN (82) 117 pp.

A linear operator T on a Banach space X is continuous if and only if it is bounded, in the sense of mapping bounded sets into bounded sets, and if it actually maps bounded sets into totally bounded sets then it is called compact. If K is a compact operator and $0 \neq \lambda \in \mathbb{C}$ then $T = K - \lambda I$ is Fredholm, in the sense that its null space $T^{-1}0$ is finite dimensional and its range TX is closed and of finite dodimension. An operator K for which $T = K - \lambda I$ is Fredholm for each $\lambda \neq 0$ is called Riesz. It turns out that if K is Riesz then it need not be compact, and for each $\lambda \neq 0$ the index of T is zero, in the sense that the two finite dimensions coincide, and indeed that T will be invertible except for at worst a sequence of values of λ tending to 0.

As is familiar this is a circle of ideas of crucial importance to those who would use bounded linear operators to study differential equations: differential operators can never be bounded, but the associated integral equations are built upon operators which are not only bounded but actually compact, and it was among integral equations that Fredholm theory was born. Thus Fredholm theory should be tightly bound to the core of the theory of bounded operators. One perhaps rather perverse way of doing this is to try and "algebraicise" the theory - a sort of "Wilson cloud chamber" approach in which the properties of being compact, Fredholm or Riesz are to be expressed not spatially but in terms of all the other bounded linear operators on the space. If one should succeed in doing that, then there is an overwhelming compulsion to go on and study corresponding elements of more general Banach algebras.

For better or worse, this is the subject of the volume under review. It falls into six chapters, indexed by descriptive letters rather than by arabic numerals. The preliminary Chapter O, OPERATOR THEORY, was intended to be a summary of the basic definitions of compact, Fredholm and Riesz operators and an account of the algebraicisation of the second two properties - compactness is rather more elusive. It has however been augmented by a discussion of a new "enlargement process" for spaces and operators which gives a very sharp description of semi-Fredholm and of Fredholm