IRISH MATHEMATICAL SOCIETY

NEWSLETTER

No. 3

1980

CONTENTS

REPORT ON ACTIVITIES OF THE IRISH MATHEMATICAL SOCIETY

P.J. Boland

The Irish Mathematical Society has continued to be active in support of various mathematical activities since the issue of the last newsletter in April, 1979.

In May 1979, the second annual Group Theory conference was held under the auspices of the Society at University College Galway. Professor Martin Newell organised this successful International Conference which was highlighted by lectures from Dr. R. Dark (UCG), Dr. R. Gow (UCD), Dr. A. Camina (East Anglia) and Professor W. Deskins (Pittsburg). In May 1980, the third annual Group Theory conference was held, and it was again very successful. Speakers included Dr. M.R. Vaughan-Lee (Oxford), Dr. T.C. Hurley (UCD), Dr. D. MacHale (UCC) and the organiser, Professor Martin Newell.

Professor J.J.H. Miller (TCD) and Dr. B. Brown (Kevin Street) were the organisers of the conference *Numerical Analysis of Semi-Conductor Devices* held in Dublin on June 27-29, 1979. This conference, which was modestly supported by the Society, was also very successful and attracted a large participation.

Professor Walter Rudin (University of Wisconsin) delivered two lectures entitled *Non-Isotropy in Several Complex Variables* and *How Smooth must Infinitely Differentiable Functions Really be?* at UCD On September 25 and 26, 1979, respectively. In his first lecture, Professor Rudin showed very elegantly how nice the theory of several complex variables can be. In the second, given under the auspices of the Society, he delighted the audience with his clever construction of various differentiable functions.

Professor Gustave Choquet, well-known mathematician and member of the French Academy of Science, postponed his visit to Ireland in 1979 and we hope he will be visiting us later in 1980. It is hoped that Professor Choquet will give several lectures while here on both mathematical education and his mathematics.

In spite of the postal difficulties during the first half of 1979, the first Irish National Mathematics Contest, held on March 6, 1979, was a great success. Altogether, a total of 957 students registered to do the examination, and results were received on behalf of 674 pupils from 43 of the 58 schools that competed. The results compared favourably with other countries taking the exam, and this year a wider participation is expected. The Educational Company of Ireland generously sponsored a dinner for the five highest-ranking candidates of the 1979 contest on December 14, 1979, at the Green Isle Hotel in Dublin. The five winners and their mathematics teachers were honoured, and each of the students received a prize of a hand calculator. This year's contest was held on March 4, 1980, and it was even more successful. Almost a thousand students participated and even though the examination was somewhat more difficult than the previous one, the candidates scored well. The contest was won by Jonathan Griffin from Ard-Scoil Ris, Limerick. He also won the contest in 1979, so he completed a remarkable double, and so deserves our warmest congratulations.

Dr. Alan Bell (Nottingham) gave a lecture under the auspices of the Society in U.C.D. in May, 1980. Dr. Bell, who works in the area of mathematical education spoke on *Symbolism in Mathematics - Problems of Understanding Graphs and Algebra.*

The second conference on Matrix Theory and its Applications, run under the auspices of the society was held in U.C.D. on October 3,4, 1980. The conference was organised by F.J. Gaines and T.J. Laffey and it was sponsored by the Royal Irish Academy. It was well attended. The speakers included Professor Charles Johnson (Maryland and National Bureau of Standards), Dr. Stephen Barnett (Bradford), Professor Robert Grone (Auburn, Alabama and London), Professor Frank Uhlig (Aachen) and Professor Harald Wimmer (Wurzburg) who all travelled to Ireland specially for the conference, as well as Dr. Robin Harte (U.C.C.) and Professor D. Judge and Drs. F.J. Gaines, R. Gow, T.J. Laffey, D. Lewis and D. O'Connor, of U.C.D.

At the Annual General Meeting of the Society held on December 20, 1979, Ray Ryan was elected treasurer of the Society. The dues of the Society are still a modest £2.00 and Ray Ryan (UCG) would greatly appreciate receiving more (1979-1980) dues from members. The other officers of the society continue to hold their offices for another eyar, and a new committee was elected. A list of the officers and committee members appears on the last page of this newsletter.

Contributions to forthcoming issues of the newsletter on various mathematical topics are both needed and encouraged. Please send any to: The Editor, I.M.S. Newsletter, Mathematics Department, University College, Belfield, Dublin 4.

LOGO COMPETITION

A competition is now under way to choose a symbol for the Irish Mathematical Society. Entries to be sent to: T. Laffey, Mathematics Department, University College, Belfield, Dublin 4. Closing date for entries is December 31, 1980. The designer of the winning symbol will be given free membership of the Society for the years 1981 and 1982.

PERSONAL ITEMS

Professor J.R. Timoney retired as Professor of Mathematical Analysis at University College Dublin on November 30, 1979. Professor Timoney joined the staff of University College Dublin in 1932. Dr. Seán Dineen was appointed to the position vacated by Professor Timoney.

Professor Michael Hayes (U.C.D.) has been elected to membership of the Royal Irish Academy.

Dr. Michael Mortell has been appointed Professor of Applied Mathematics and Registrar at University College, Cork.

Dr. Richard Ward joined the staff of Trinity College in October, 1979. Dr. Ward works in differential geometry and wrote his thesis under the guidance of Roger Penrose at Oxford.

Dr. Richard Timoney joined the staff of Trinity College in January of 1980. Dr. Timoney works in Complex Analysis and obtained his Ph.D. from the University of Illinois at Champagne under the direction of Lee Rubel. Prior to joining Trinity College he held a position in the Mathematics Department of the University of Indiana at Bloomington.

Professor C. Van Rijksbergen (Cambridge and Australia) has recently been appointed Professor of Computer Science at University College Dublin.

Dr. Martin Stynes and his wife Jean have both joined the staff of Waterford Regional Technical College. They both work in Numerical Analysis.

Martin Newell (Mathematics), Bobby Curran (Computer Science), Iggy Muircheartaigh (Statistics) and Jim Flavin (Mathematical Physics) became members of the Governing Body of University College Galway in December 1979.

Professor Heini Halberstam, who was professor of Mathematics at T.C.D. during the middle sixties returned to Dublin in May to give the Donegall Lecture. Professor Halberstam has recently taken up the position of Chairman of the Department of Mathematics at the University of Illinois at Champagne-Urbana.

Dr. Phil Boland has been promoted to a Statutory Lectureship at U.C.D. Dr. Boland is on leave of absence from U.C.D., and is spending the session 1980-81 at Florida State University in Tallahassee.

Dr. Ted Hurley has been appointed to a Statutory Lectureship at U.C.G.

Professor Jorge Mujica (Campinas, Brazil) is visiting U.C.D. for the session 1980-81. Professor Mujica is a native of Chile. He works in infinite dimensional analysis.

Paul Barry (T.C.D. and Nancy) and Michael Clancy (U.C.G. and Notre Dame) have been appointed to temporary one-year positions at U.C.D. Dr. Barry works in infinite dimensional analysis and Dr. Clancy in differential geometry.

Dr. Tim Porter has left U.C.C. to take up a lectureship at the

University of Wales in Bangor.

Dr. Glan Thomas has left U.C.C. to take up a position with the B.B.C. Open University Production Unit.

Dr. J. Bruce (Liverpool) and Dr. P. Fitzpatrick have been appointed to lectureships at U.C.C. Dr. Bruce works in topology and Dr. Fitzpatrick in group theory.

Dr. J. Siemons, who spent the session 1979-80 in U.C.G., has been appointed to a temporary lectureship at U.C.C. Dr. Siemons works in group theory.

Dr. Alan Williamson has been appointed to a junior lectureship at U.C.G. Dr. Williamson works in group theory.

Dr. John Gibbon has left the Department of Mathematical Physics at U.C.D. to take up an appointment at Imperial College, London.

Dr. Ralph Saxton (Glasgow) has been appointed to a temporary position in the Department of Mathematical Physics, U.C.D. Dr. Saxton works in differential equations.

Dr. Ray Flood (Kevin Street) has been appointed to a position at N.I.H.E., Dublin.

Dr. John Hannah (Melbourne) has been awarded a Department of Education postdoctoral fellowship and will spend the session 1980-81 at U.C.D. Dr. Hannah works in ring theory.

It is hoped to include a regular Personal Items column in future editions of the Newsletter. Any items of interest, e.g. new appointments, vacancies, visitors, scandals, etc. should be forwarded to S. Dineen, Mathematics Department, U.C.D.

IRISH NATIONAL MATHEMATICS CONTEST

The contest was held on March 4, 1980. The examination paper is that of the U.S. National High School Contest and the examination is held simultaneously in participating countries. A total of 79 schools entered 1681 students for the contest, an increase of 724 over last year's entry. Results were received from 71 schools on behalf of 1344 students and a summary of their performance is given. The names of those who scored 80 or more (out of 150) is given also. Jonathan F. Griffin, who won last year's contest, repeated his success, and indeed, was the only one to score 100 marks or better.

It is interesting to note that 416,024 students from 6,887 schools in the United States, Canada, Puerto Rico, Jamaica, Italy, Virgin Islands and Guam did the examination. Of these, only 256 achieved marks greater than 100. There was, however, one perfect paper, written by

Anthony Y. Lee,
Laurel Senior High School,
8000 Cherry Lane,
Laurel, Maryland 20810.

The Irish Mathematical Society is very grateful to Abacus Systems Limited and the Educational Company of Ireland for sponsoring this year's contest.

Irish National Mathematics Contest

Distribution of Marks

Scores	Boys	Girls	Total
0 - 9	0	0	0
10 - 19	2	2	4
20 - 29	6	30	36
30 - 39	104	138	242
40 - 49	206	230	436
50 - 59	235	135	370
60 - 69	128	44	172
70 - 79	51	10	61
80 - 89	14	4	18
90 - 99	2	2	4
100 - 109	1	0	1
110 - 119			
120 - 129			
130 - 139			
140 - 149			
150			
Totals	749	595	1344
Median Score	52.40	45.54	48.94

Individual Role of Honour

Irish National Mathematics Contest 1980

Score	Student	School	County
109	Griffin, Jonathan F.	Ard Scoil Ris	Limerick
95	O'Sullivan, Oonagh	Holy Cross College	Kerry
95	Bowe, Frances	St. Dominic's College,Cabra	Dublin
92	Kavanagh, Richard C.	Presentation College	Cork
92	Cronin, Michael	lackrock College	Dublin
89	Quill, Adrian D.	St. Munchin's College	Limerick
87	Browne, Thomas	O'Connell School	Dublin
86	Moran, Seamus	O'Connell School	Dublin
85	Campbell, Aisling	St. Dominic's College, Cabra	Dublin
85	Moriarty, Derek	Templeogue College	Dublin
85	Sanfey, Peter	Belvedere College	Dublin
85	Palmer, John B.	Holy Cross College	Kerry
85	Dennehy, John A.	Presentation College	Cork
85	Nestor, Kieran D.	" "	"
84	O'Sullivan, Michael D.	Salesian College	Limerick
83	O'Dowd, John	St. Laurence's College, Shankill	Dublin
83	Horgan, Emer	Ashton Comprehensive School	Cork
82	Roche, Michael J.	Presentation College	Cork
81	Mattar, Joseph J.	Colaiste an Spioraid Naoimh	Cork
80	McCarthy, Patrick C.	Colaiste Chriost Ri	Cork
80	Moloney, Tom P.	St. Michael's College	Kerry
80	Deegan, Robert	St. Aidan's CBS, Whitehall	Dublin
80	Farrell, Paula	St. Dominic's College, Cabra	Dublin
80	Leonard, Verona	St. Dominic's College, Cabra	Dublin

CONFERENCE NEWS

Dublin Institute for Advanced Studies Mathematical Symposium

17,18 December, 1980

Principal Speakers:

Dr. R. Gow (*U.C.D.*) Partition functions and representation theory.

Dr. J. Kennedy (*U.C.D.*) The Kepler problem in R^4 co-ordinates.

Dr. P. Hogan (*U.C.D.*) The two-body problem in linearized gravity.

Dr. B. Goldsmith (*D.I.T.*) Model theory and algebra.

Dr. T. Laffey (*U.C.D.*) Some questions in matrix theory.

Professor D. Judge (*U.C.D.*) Boundary values and unbounded infinite matrices.

Reserve Speaker:

Professor D.L. Weaire (*U.C.D.*)

Conference details from the Registrar, D.I.A.S., 10 Burlington Road, Dublin 4.

The 1981 British Mathematical Colloquium will be held in Oxford on March 30 - April 3. The Secretary is Dr. W.B. Stewart, Exeter College, Oxford OX1 3DP.

The next International Congress of Mathematicians will be held in Warsaw, August 11-19, 1982. The Chairman of the Organising Committee is Professor Czesław Olech. The First Announcement containing more detailed information will be issued in Summer 1981.

The following conferences will be held in Dublin,
Ireland, under the auspices of the Numerical
Analysis Group

NASECODE II

the second international conference on the

NUMERICAL ANALYSIS OF SEMICONDUCTOR DEVICES AND INTEGRATED CIRCUITS

17th to 19th June, 1981.

This is sponsored by IEEE (Electron Devices Society), IEE (Irish Branch), Royal Irish Academy and Irish Mathematical Society.

Contributed papers are solicited on any topic relevant to the numerical simulation, optimization and computer aided design of semiconductor devices and integrated circuits. The preliminary version of such a paper should be submitted not later than Friday, 20th March, 1981, and it must be accompanied by a separate one-page abstract.

BAIL II

the second international conference on

BOUNDARY AND INTERIOR LAYERS – COMPUTATIONAL AND ASYMPTOTIC METHODS

15th to 18th June, 1982.

Contributed papers are solicited from biologists, chemists, engineers, mathematicians, physicists and other scientists on computational or asymptotic methods for problems involving boundary or interior layers. The preliminary version of such a paper should be submitted not later than Friday, 19th March, 1982, and it must be accompanied by a separate one-page abstract.

All communications concerning the above conferences should be addressed to NASECODE II or BAIL II, 39 Trinity College, Dublin 2, Ireland. Telephone (01) 772941 ext. 1889 or 1949. Telex 25442 or 31166 TCD EI. Cables "TRINITY DUBLIN."

MATHEMATICS IN U.C.D. 1854 TO 1974

J.R. Timoney

The school of mathematics in University College, Dublin, began in The Catholic University of which J.H. Newman, afterwards Cardinal was Rector. The first lecture in mathematics was given on Monday November 6th 1854 by the first professor of mathematics, Edward Butler.

To understand the educational scene, especially for catholics, about the middle of the last century it is necessary to go back beyond 1850. The only schools available to catholics before 1833 were the hedge schools. In these the teaching ranged over what we would call primary and secondary level so that they supplied students to second and third level institutions. In them "the pupil and his teacher met feloniously to learn".

After catholic emancipation in 1829 primary or national schools, open to all, began to operate in 1833. The primary school masters were government officials paid by the state and they soon displaced the hedge school teachers. As a result, there was no second level education outside the six royal schools, the three Erasmus Smith schools and some twenty one private Protestant foundations. The six royal schools were:

Armagh, Banagher, Cavan, Dungannon, Portora and Raphoe.

The Erasmus were:

Drogheda, Galway and Tipperary.

These schools were sufficient to prepare students for entry to Trinity College where the normal age of entry was then 16 years. However, the Government of Ireland decided to found in 1848 the Queen's University of Ireland with Colleges in Belfast, Cork and Galway. These Colleges, called the "Godless Colleges" by the catholic bishops had no secondary free schools and suffered from lack of students.

Newman's Catholic University was to suffer the same fate although Newman was aware of this and hoped his University would attract the sons of English catholic gentlemen. The removal of religious tests for entry to Oxford and Cambridge in the 1870's together with the foundation of University College, London, with no religious aspect, finished any hope of survival for the Catholic University, except its medical school. The medical school in Cecilia St. flourished because the College of Surgeons recognised its courses and enabled its graduates to get on the medical register. This liberal act of the Colleges of Physicians and Surgeons in 1854 was repaid in recent times when U.C.D. via N.U.I., recognised the courses of these Colleges for N.U.I. degrees.

An interesting fact concerning Jermy Bontham, an agnostic and supporter of the founding of University College, London, is that he made it a condition of his benefaction that he be present at every meeting. In the early days of U.C.L. he was present at every meeting, after his death, in a glass case dressed in top hat and morning suit.

I must acknowledge my debt to Fr. Fergal McGrath S.J., 35 Lower Leeson Street, for accurate information about the Catholic University. Fr. McGrath is archivist in 35 Lower Leeson St. where are preserved most of the Calendars of the Catholic University.

The appointment of Edward Butler as professor of mathematics was announced in the Catholic University Gazette, written by Newman, for October 19th 1854. Butler was chief inspector of the Board of National Education and had graduated from T.C.D., B.A. 1842, M.A. 1845. His tutor in T.C.D. is listed as Mr. Hamilton.

Butler is listed as giving lectures in advanced mathematics, Monday, Wednesday and Friday on books I to VI and XI of Euclid and elementary lectures on the same course on Tuesday, Thursday and Saturday. There is no evidence that he gave an inaugral or public lecture like all other professors. In the late 1850's, when Newman wished to return to England, there was a proposal to have a lay Vice-Rector and Butler was the chosen lay candidate. The proposal never came to anything, perhaps due to opposition from the bishops and Butler resigned when Newman left Ireland in 1858-9.

The chair of mathematics was filled in 1860 by William Goodenough Penny who was an ordained convert to catholicism in 1847 from being an Anglican Parson. He was first in mathematics in his year at Christ Church College, Oxford. Penny came to Ireland with Newman and was mathematical tutor in St. Mary's House, Number 6, Harcourt St. This was Newman's own house.

From 1859 on there was a professor of elementary mathematics, James W. Kavanagh, who was a Carlow man and an ex-inspector of the National Board, who resigned in 1858 as a protest against proselytism, which he alleged was practised in some shools.

Penny seems to have continued in the chair up to 1873 and Kavanagh continued on the staff to 1880.

Cardinal Cullen persuaded John Casey to accept the chair of mathematics in 1873. Casey, born in Mallow in 1821, was a monitor in a school in Kilkenny where he helped to look after a Trinity mathematician who seems to have been very ill with tuberculosis. Casey learned mathematics from the patient and after the patients death sent some original work to T.C.D. resulting in his being given a teaching job in Kingstown school and an opportunity to do a degree in T.C.D. He graduated from T.C.D. in 1862.

At one of his exams in T.C.D. Casey found on the paper a cut he had discovered himself earlier and was surprised when he got no marks for the answer. He was told that he gave no adequate explanation of his proof.

Casey was one of the pillars of the Catholic University and University College, Dublin, to his death in 1891. He became famous as a line and circle geometer and was elected to the Royal Society and given an honorary LL.D. by T.C.D.

Casey became a great tutor and taught in many places when the Catholic University had no money to pay salaries.

During the dwindling years of the Catholic University, one man mainly showed the way forward. This was Fr. William Delaney, S.J., who as master of novices at Tullabeg entered students for the examinations of London University. The high standards of these and others showed the necessity for second level education in Ireland and the intermediate board was set up in 1877.

The Royal University was established in 1879 as an Examining Body only. Casey and many staff members of Queen's Colleges became fellows of the Royal at salaries of £400 p.a. each. The Catholic University was reformed into colleges in 1883 with University College as the largest. Blackrock, Dominican, Loreto and some others also prepared students for the examinations of the Royal.

Fr. Delaney was president of University College and was remarkable for three things which were

1) spending money which he had not got on libraries and other essentials. (His College had no endowment);

2) high standards;

3) concentrating resources in 86 St. Stephen's Green rather than a thin scatter over many colleges.

He continued as president down to 1909 and ended the unendowed U.C.D. in a blaze of glory. In the last year of the Royal, U.C.D. won more prizes and awards than Belfast, Galway and Cork put together.

Two great pillars of the early U.C.D. were Casey and Fr. Tom Finlay, S.J. Fr. Finlay was in turn professor of classics, philosophy and economics. Women were not admitted to the lectures of the fellows in U.C.D. to 1901 on grounds of lack of classroom space. Fr. Finlay had erected a tin shed in the garden of 85-86 which was to become known as Fr. Finlay's tin university. This shed, now removed, made room for the women to attend lectures.

John Casey died in 1891 and was succeeded in the chair of mathematics by Henry Charles McWeeney who remained in office to his death in 1935. McWeeney had graduated from the Royal University, B.A. 1887, M.A. 1890, Studentship 1891. He was a magnificent teacher and a geometer of great elegance. A favourite expression of his was "if you attack it judiciously it will come out in a line". McWeeney held the chair to his death in 1935. He played a large role in the administration of the College. In 1901, Fr. Delaney decided to have a lay academic council and Mac was elected at the top of the Roll. He was the vice-president to Dr. Coffey from 1909 to his death.

In the 1890's there were on the mathematical staff two young men who had won all the prizes in the Royal, Gibney and O'Toole, but both died young and Michael F. Egan, S.J. replaced Gibney in 1900. He was fellow of the Royal and lecturer until he succeeded McWeeney. He was an analyst of the French-Belgian school and retired in 1947 to be succeeded by P.G. Gormley who died in 1973 to be succeeded by Don McQuillan.

Now a little about the school as I knew it since 1927. Our first lecture was given by McWeeney who started to do questions off the entrance scholarship paper, indeed the more outlandish bits of the toughest questions. Fr. Egan gave the second lecture. He described it later as his most unintelligible lecture on irrational numbers. He was not a good elementary lecturer and did not need to try to be difficult. This was the honours class clearance act, of course, and things moderated for the hard necks who stuck it out.

In 1928, I had the option to go on in Engineering or Mathematics, so

I looked up the results of the honours degree from 1910 to that time. The number of First Class awards was five. They were J.M. Fay 1910, an engineer and director of the E.S.B; F.D. Murnaghan 1913, who will be known to you in algebra, he became professor in John's Hopkins in Baltimore; R.C. Geary, 1916, who is very much with us and very well known abroad as a statistician and economist; Jeremiah O'Riordan 1924 who became a consulting engineer and is still with us; Liam Honohan 1928 afterwards government actuary and secretary of social welfare, still with us. These five became eight in 1930 with Gormley, myself and James Murphy in that order getting class one.

We were taught mathematical physics in first year honours by William McFadden Orr. He was a Belfast Presbyterian who had won the Royal Studentship in 1887 and was later a senior wrangler in the Cambridge Tripos. He was a great gentleman but very stern in every sense. He would sit with the pocket watch out waiting for 9 o'clock to start the lecture on the dot. If you came in after 9 a.m. you got a 'late' on the roll. He had come to U.C.D. from being professor in the College of Science. He was heard to say that research in the College of Science consisted of solving a quadratic equation which had not been solved before. We learned little from him for most of the time was spent criticizing the bad treatment of Newton's laws. We did not know very well what the laws were but we knew that Ernest Mach was the man to be respected.

The other large man in very sense in the school in our time was Arthur William Conway F.R.S., the professor of mathematical physics and registrar and president from 1940 to 1947. He won the Royal Studentship in 1898 and went to Corpus Christi College Oxford. He was an expert in special theory of relativity (at an early stage); the quantum jump idea and on quaternions. During our time as students he was editing the first volume of Hamilton's work with J.L. Synge and this produced some unusual and tasty questions on the exam papers.

To turn very briefly to the distinguished graduates of the school since the thirties there are many but could be more due to the rush to physics after the war. To name very few: Professor James McConnell, D.I.A.S., 1938, and Professor Loughlann O'Rafferty in the 50's, Professor Dennis Keeffe in Berkeley and Oliver McBryan in New York.

The treatment of honours students in the school has always been tough and standards very high. The result has been that our graduates can more than hold their own when they go as post-graduates to other schools. I hope these high standards will continue in this expanding school.

PROFESSOR J.R. TIMONEY

As noted elsewhere in the Newsletter, Professor J.R. Timoney retired from the chair of Mathematical Analysis at U.C.D. on November 30, 1979. In the above article, he, with characteristic modesty, does not refer to his own major role in building up the department of Mathematics at U.C.D., and indeed, in the development of U.C.D. itself.

As recorded in the article, P.G. Gormley held the chair of Mathematics at U.C.D. from 1947 to his untimely death in 1973. During the later years of his professorship, much of the day-to-day administration of the department was carried out by Professor Timoney. After Professor Gormley's death, Professor Timoney became head of the department, which position he held until 1976. So, for a period of almost twenty years, he played the principal role in the running and development of the department. He devoted himself unstintingly to this and more generally to the development of U.C.D. as a whole. He demonstrated great administrative skill, combining qualities of patience, quiet diplomacy, personal charm and determination most successfully. He inspired a spirit of loyalty, commitment and co-operation from his staff. He was, and still is, utterly dedicated to the improvement of U.C.D., and has worked assiduously to have his ideals achieved through his membership of numerous committees as well as the Governing Body and the Senate of the N.U.I. (of which he is still a member).

He has always retained his enthusiasm for Mathematics. His main interests lie in the "hard" parts of complex analysis - gap series, zeta function, Riemann hypothesis, etc. As a teacher he insisted on very high standards. He constantly sought elegant ways of presenting material to his students. He particularly liked to teach his students (and also inquisitive members of his staff) ingenious tricks by which, otherwise very difficult, problems could be solved quickly without much calculation. (Questions of this type set in examinations are often referred to as "Timoney specials" within the department.) He was very popular with students and was often the mandatory staff representative on various student committees. He has maintained contact with mathematics graduates and secondary school mathematics through his membership of the committee of the Irish Mathematics Teachers Association and the editorial board of its Newsletter. He has delivered many lectures to teachers under the auspices of the Association, or the Department of Education refresher courses. He has helped to improve the status of Mathematics and mathematicians in this country in countless ways. He has certainly earned the deepest gratitude of the whole Irish mathematical community.

On behalf of the Irish Mathematical Society, I wish him the enjoyable and fulfilling retirement which he so richly deserves.

Thomas J. Laffey

PROBLEM SECTION

(3.1) Find all numbers of the form $\frac{1}{2}n(n+1)$ (triangular numbers) which are perfect squares.

(3.2) Let $n > 1$ be a given natural number. Find a square matrix A whose entries are zeros and ones such that A^n is a matrix with all its entries equal to one. Show that 2^n is the least possible size for such a matrix A.

(3.3) Let f be a periodic real function. The least number $u > 0$ (if such exists) such that $f(x+u) = f(x)$ for all real x is called the period of f.

Suppose that f,g are periodic with periods $u \neq 0, v \neq 0$, respectively, and suppose that f is bounded. Prove that $f+g$ is periodic if and only if u/v is rational. Give an example of periodic functions f,g with periods $u>0, v>0$, respectively, such that (i) u/v is not rational and (ii) $f+g$ is periodic.

(3.4) Let A be a square matrix with entries in a field F. Prove that $A = D+N$ where N is nilpotent and D is diagonalizable over the field F.

[It is well known that if F is algebraically closed, $A = D+N$ as above with, in addition, $ND=DN$.]

(3.5) Let A be an $n \times n$ matrix with entries in an algebraically closed field F of characteristic $\neq 2$ and suppose that A has trace zero. Prove that if A has rank greater than one, then A is similar to a symmetric matrix with all its diagonal entries equal to zero.

(3.6) Let $\{a_n\}$ be a sequence of positive terms. Relate the convergence or divergence of $\sum a_n$ to that of $\sum \frac{a_n}{1+a_n^2}$.

(3.7) Let $t = \binom{k}{2}$ be an integer ≥ 10. Show that it is not possible to find an ascending sequence

$$0 = x_1 < x_2 \; . \; . \; . < x_k = t$$

of integers $x_1, \ldots, x_k$ so that the differences $x_j - x_i$ $(1 \leq i < j \leq k)$ are all distinct (and every one of $1,2,\ldots,t$ is such a difference).

(3.8) If A is a complex $n \times n$ matrix, set

$$||A|| = \sup_{||z||=1} ||Az||$$

where $||z|| = [|z_1|^2 + \ldots + |z_n|^2]^{\frac{1}{2}}$, $z = (z_1, \ldots, z_n) \in C^n$.

Let A^* denote the complex conjugate of A transposed. Suppose A,B are $n \times n$ complex matrices over C with

$$\text{trace}[(A^*A)^j A^*B] = 0 \quad \text{for } j = 0,1,\ldots,n-1.$$

Show that $||A|| \leq ||A+B||$.

(3.9) Evaluate

$$\int_0^1 \frac{\tan^{-1}\sqrt{\frac{v}{1+v}}}{\sqrt{1-v^2}}\, dv.$$

(3.10) A man carrying a horizontal 20ft. long pole runs at speed v such that+ $\gamma(v) = 2$ into a 10ft. long room and closes the door. Explain in the man's frame, in which the room is only 5ft. long, how this is possible. Show also that the minimum length of the room for the performance of this trick is $\frac{20}{\sqrt{3}+2}$ ft.

+ $\gamma(v) = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$, c = velocity of light.

(From W. Rindler, Special Relativity, 1960, Oliver & Boyd.)

I am grateful to Peter Hogan (U.C.D.), David Redmond (Maynooth), Wayne Sullivan (U.C.D.) and Richard Timoney (T.C.D.) for providing problems for this issue.

T.J. Laffey

SOLUTIONS TO PROBLEMS

We discuss some of the problems posed in earlier Newsletters. We begin with Problem 2 in Newsletter #1.

Let $f(x)$ *be a monic polynomial in* $\mathbb{Z}[x]$ *which divides* x^n-1 *and suppose that* a *is a natural number which divides all the coefficients of* $f'(x)$. *Prove that* $f(x) = g(x^a)$ *for some monic integral polynomial* $g(x)$.

Using induction on a, we reduce to the case where $a, = p$ say, is prime.

We first show that $p|n$. For let $x^n-1 = f(x)h(x)$. Differentiate with respect to x and then put $x=\omega$ where ω is a root of $f(x)=0$ to get $n\omega^{n-1} = f'(\omega)h(\omega)$ and thus n/p is an algebraic integer. So $p|n$.

Let $\Phi_d(x)$ be the cyclotomic polynomial of degree $\phi(d)$. Thus $\Phi_d(x) = (x-\omega_1) \dots (x-\omega_y)$ where $y=\phi(d)$ and $\omega_1,\dots,\omega_y$ are the primitive d^{th} roots of 1. We need

(1) if $p^2|d$, $\Phi_d(x) = \Phi_{d/p}(x^p)$.

[To see this, note that if ω is a primitive d^{th} root of 1, then ω^p

is a primitive $(d/p)^{th}$ root of 1. So $\Phi_d(x)$ divides $\Phi_{d/p}(x^p)$. On the other hand, since $p^2 | d$. $\phi(d) = p\phi(d/p)$.]

(2) if $p | d$, $\Phi_{pd}(x) = \Phi_d(x^p)/\Phi_d(x)$

[To see this, note that if ω is a primitive d^{th} root of 1, so is ω^p since $p | d$. So $\Phi_d(x)$ divides $\Phi_d(x^p)$. Also, as above, $\Phi_{pd}(x)$ divides $\Phi_d(x^p)$. But $\phi(pd) = (p-1)\phi(d) = p\phi(d) - \phi(d) = \deg \Phi_d(x^p) - \deg \Phi_d(x)$, so (2) follows.]

Now $f(x)$ divides $x^n - 1$, so we may write $f(x)$ as a product of $\Phi_d(x)$'s for various d's dividing n. We write

$$f(x) = \left[\prod_{p|d} \Phi_d(x)\right]\left[\prod_{p|e} \Phi_{pe}(x)\right]\left[\prod_{p^2|g} \Phi_g(x)\right]$$

(where some of the products may be empty).

Using (1),(2), we may write

$$f(x) = \left[\frac{\prod_{p\nmid d_o} \Phi_{d_o}(x)}{\prod_{p\nmid e_o} \Phi_{e_o}(x)}\right] \cdot \left[\prod_{p\nmid e} \Phi_e(x^p) \prod_{p|g} \Phi_{g/p}(x^p)\right]$$

$$= \left[\frac{u(x)}{v(x)}\right] \cdot k(x^p), \quad \text{say,}$$

where $(u(x),v(x)) = 1$.

Differentiating we thus find that p divides all the coefficients of $v(x)u'(x)-u(x)v'(x)$. Now since p does not divide any of the indices m for which $\Phi_m(x)$ occurs in $u(x)v(x)$ and $(u(x),v(x)) = 1$, $u(x)v(x)$ divides x^r-1 for some r with $p|r$. But then if $u(x)v(x) \neq 1$, writing $x^r-1 = u(x)v(x)w(x)$, differentiating and putting $x=\zeta$ where ζ is a root of $u(x)v(x) = 0$ we get a contradiction as in the first paragraph. Hence $u(x)v(x) = 1$ and $u(x) = 1 = v(x)$. This proves the result.

This result is due to Leonard Scott. His proof uses modular character theory (see Proc. of the Park City, Utah, Conference on Finite Groups).

We now discuss Problem 10 on Newsletter #1.

Let G *be a finite abelian group of order* n *and let* $g_1,\ldots,g_{2n-1}$ *be elements of* G. *Prove that there exists a subsequence* $g_{r_1},\ldots,g_{r_n}$ *of exactly* n *g's with* $g_{r_1}\cdots g_{r_n} = 1$.

Let A be a subset of a group X. We write $|A|$ to denote the number of elements in A. Given non-empty subsets A,B we write $AB = \{ab | a\varepsilon A, b\varepsilon B\}$.

We use the following result of Cauchy:

Let X be the cyclic group of prime order p and let A,B be non-empty subsets of X. Then either $AB=X$ or $|AB| \geqslant |A|+|B|-1$. (For a proof, see a paper of Davenport (Journal of the London Math. Soc. (1937).)

Suppose that G has prime order p and let $A = \{g_1,\dots,g_{2p-1}\}$. Let $A^2 = AA$, etc. Note that if $A^k \neq G$ for $1 \leq k \leq p$, then $|A^p| \geq p|A|-p+1 > |G|$ unless $|A| = 1$. If $A = 1$, the result is obvious. Assume A 1, then $A^k = G$ for some k with 1 k p and thus $A^{k+1} = AG = G$, etc. So $A^p = G$. So the result holds in this case.

In the general case we use induction on $|G|$. Let p be a prime divisor of $|G|$ and let L be a subgroup of order p. Applying the induction hypothesis we find $g_{r_1},\dots,g_{r_{n/p}}$ among the first $2(\frac{n}{p}) - 1$ of the g's with $w_1 = g_{r_1}+\dots+g_{r_{n/p}} \in L$. We apply the induction hypothesis to the sequence obtained by deleting $g_{r_1},\dots,g_{r_{n/p}}$ in the original sequence to get $_2 = g_{s_1}+\dots+g_{s_{n/p}} \in L$. Proceed thus. Note that since $2n-1 = \frac{n}{p}(2p-2)+2(\frac{n}{p})-1$ we can construct $w_1,\dots,w_{2p-1}$ in this way. Applying the theorem to L and the sequence $w_1,\dots,w_{2p-1}$ now gives a subsequence $z_1+\dots+z_p = 1$. But each z_i is a sum of $\frac{n}{p}$ elements of the original sequence and z_i, z_j $(i \neq j)$ involve g's with entirely different indices. The result follows. [This result is due to Erdos, Ginzburg and Ziv.]

Problem 6 of Newsletter #1 asked the following:

Let A, B *be* $n \times n$ *(complex) matrices such that* $AB-BA$ *has rank one. Prove that* A, B *have a common eigenvector (i.e. there exists a vector* $v \neq 0$ *such that* $Av = \lambda v$, $Bv = \mu v$ *for some* λ, μ*).*

The solution of this problem was discussed in Newsletter #2, where it was remarked that no short elementary proof was known. In a paper to appear in Linear & Multilinear Alg., M.-D. Choi, C. Laurie and H. Radjavi have provided such a proof. The key observation is

<u>Theorem</u> If A, B are any linear operators on a vector space V and if $AB-BA$ has rank one, then either the null-space or range of A is invariant under B.

<u>Proof</u> Assume that the null-space N of A is not invariant under B. (Note that this implies in particular that N is nontrivial, i.e. $0 \neq N \neq V$.) Then there exists a non-zero vector x in V with $Ax = 0$ and $ABx \neq 0$. Then $(AB-BA)x = ABx$ spans the (one-dimensional) range of $AB-BA$ and, for every $y \in V$, there exists a scalar λ_y such that

$$(AB-BA)y = \lambda_y ABx.$$

It follows that $BAy = AB(y-\lambda_y x)$, yielding

$$BAV \subseteq ABV \subseteq AV$$

as desired.

Let λ be an eigenvalue of A. Then

$$AB-BA = (A-\lambda I)B-B(A-\lambda I)$$

and the kernel and null-space of $A-\lambda I$ are proper subspaces of V. So V has a proper (A,B) - invariant subspace. The result of Q.6 now follows by induction.

I am grateful to Choi, Laurie and Radjavi for providing me with a preprint of their paper entitled "Commutators and invariant subspaces". Their paper contains extensions of the result to infinite dimensional spaces.

We now discuss two of the problems posed in Newsletter #2.

(2.2) *Prove that if* c *is a real number such that* n^c *is a natural number for every natural number* n, *then* c *is a non-negative integer.*

Let k be a natural number $>c$. Let $f(x) = x^c$. Note that $\sum_{r=0}^{k}(-1)^r\binom{k}{r}f(x+rh) = (-h)^n f^{(k)}(\xi)$ for some ξ between x and $x+kh$ [cf. Eggleston Elementary Real Analysis CUP (1962), page 119]. Let $h=1$. Then

$$\sum_{r=0}^{k}(-1)^r\binom{k}{r}(x+r)^c = (-1)^n c(c-1)\ \dots\ (c-k+1)\xi^{c-k}$$

where

$$x<\xi<x+k.$$

Let $x\to\infty$ through the set of natural numbers. By hypothesis, the left-hand side of (*) is an integer while the right-hand side tends to zero (since $k>c$). Hence the right-hand side is zero for some large x and thus c is one of the numbers $0,1,2,\dots,k-1$.

[This problem was posed in the Putnam Examination in 1971.]

(2.8) *Let* $f(x), g(x)$ *be monic integral polynomials and let* α, β *be roots of* $f(x), g(x)$, *respectively (in the complex field). Suppose* α, β *can both be expressed as integral linear combinations of square roots of integers. Prove that there exist integral polynomials* $h(x), k(x)$ *such that* h.c.f.$(f(h(x)), g(k(x)))$ *has degree greater than one. Generalize.*

Let $f_o(x), g_o(x)$ be irreducible factors of $f(x), g(x)$, respectively such that α is a root of $f_o(x)$, β a root of $g_o(x)$. Note that the conclusion for $f(x), g(x)$ will follow from that for $f_o(x), g_o(x)$. Thus we may assume that $f(x), g(x)$ are irreducible. Note that the forms of α, β imply that $Q(\alpha, \beta)$ is a normal extension of Q with abelian Galois group. By a theorem of Kronecker (Lang Algebraic Number Theory, p.210), $Q(\alpha, \beta) \subseteq Q(\omega)$ where $\omega^n = 1$ for some n. The set of algebraic integers in $Q(\omega)$ is just $Z[\omega]$ (Lang, ibid., p.75), so there exist integral polynomials $h(x), k(x)$ such that $\alpha = h(\omega)$, $\beta = k(\omega)$. But then ω is a root of $f(h(x))$ and also a root of $g(k(x))$. The result follows.

This problem was posed by Robert Gilmer (Tallahassee). He asks whether the result holds without any restriction on the form of α, β. This more general question appears to be open at present.

PERFECT NUMBERS

(Addendum to the article in Newsletter #2)

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Two new "Mersenne primes" have been discovered since the 25th (which was $(2^{21701}-1)$). They are $(2^{23209}-1)$ and $(2^{44497}-1)$. The first of these was found by Kurt Noll, who with Laura Nickel, had discovered the 25th. The second was the joint work of two other computer workers, Harry Nelson and David Slowinski. Incidentally, the last prime above has 13,395 digits (when expressed in the base 10). Because of the connection between Mersenne primes and perfect numbers, there are now twenty seven even perfect numbers known.

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