wait for later times.

Bibliography

1. T. Bradwardine, Tractatus de proportionibus, translated by H.L. Crosby, Jr. (Madison, 1955).

2. E. Grant (Ed.), A Source Book in Medieval Science (Harvard University Press, 1974).

3. N. Oresme, De proportionibus proportionum and Ad pauca respicientes, translated by E. Grant (Madison, 1966).

PROBLEM SECTION

It is hoped to include in each issue of the Newsletter a set of problems of general mathematical interest. Readers are invited to submit problems for inclusion in this section. It is envisaged that the problems posed should be intelligible to (though not necessary soluble by) people with a degree in Mathematics. It is hoped that most of the problems posed should be soluble and it is intended to publish solutions to those in subsequent issues of the Newsletter. Readers are invited to submit solutions to the problems posed for consideration for inclusion in the Newsletter. Correct solutions will be acknowledged in the Newsletter.

PROBLEM SET # 1

1. Given a sequence of n^2+1 distinct integers, show that it is possible to find a subsequence of $n+1$ integers which is either increasing or decreasing.

2. Let $f(x)$ be a monic polynomial in $\mathbb{Z}[x]$ which divides x^n-1 and suppose that a is a natural number which divides all the coefficients of $f'(x)$. Prove that $f(x) = g(x^a)$ for some monic integral polynomial $g(x)$.

3. Let $A = (a_{ij})$ be the $n \times n$ matrix with $a_{ij} = (1+|i-j|)^{-1}$ where $|\cdot|$ denotes absolute value. Prove that $\det A > 0$.

4. Necklaces of n beads are to be made out of an infinite supply of beads in k different colours. How many distinctly different necklaces can be made.

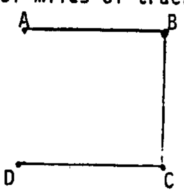
5. Let Q be the point $(2,0)$ in the plane and O the origin. Let P be a point in the first quadrant which lies on the curve $x^4+xy^2+y^2 = 3x^2$. Let θ be the angle in the interval $0 < \theta < \pi$ which the line PQ makes with the positive x-axis and let ϕ be the acute angle which the line OP makes with the positive x-axis. Prove that $\theta = 3\phi$ (i.e. OP trisects θ).

6. Let A,B be $n \times n$ (complex) matrices such that $AB-BA$ has rank one. Prove that A,B have a common eigenvector (i.e. there exists $v \neq 0$ with $Av = \lambda v$, $Bv = \mu v$ for some λ,μ).

7. S is an infinite set of points in the plane such that the distance between any pair of points in S is an integer. Prove that all the points in S are collinear (i.e. lie on one straight line).

8. Four towns A,B,C,D lie at the vertices of a square of side 100 miles. An engineer wishes to construct a railway line so that it is possible to travel from any of the towns to any other by rail. What is the minimum number of miles of track required.

Note



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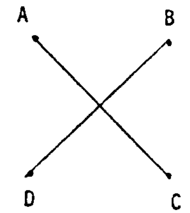


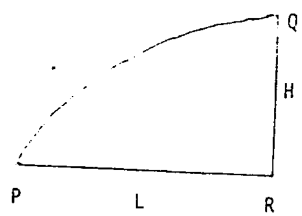
are good solutions, but not the best.

9. Prove that

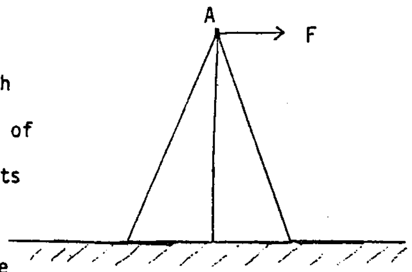
$$\frac{1}{2^2} + \frac{1^2.3}{2^2.4^2} + \frac{1^2.3^2.5}{2^2.4^2.6^2} + \frac{1^2.3^2.5^2.7}{2^2.4^2.6^2.8^2} + \ldots = 1 - \frac{2}{\pi}.$$

10. Let G be a finite abelian group of order n and let $g_1,\ldots,g_{2n-1}$ be elements of G. Prove that there exists a subsequence $g_{r_1}, g_{r_2}, \ldots, g_{r_n}$ of exactly n g's with $g_{r_1} g_{r_2} \cdots g_{r_n} = 1$.

11. Neglecting air resistence a particle projected under gravity travels on a parabola which has the property that at any point A on the path, the directrix is horizontal and is at a height $v^2/2g$ above A where v is the velocity of the particle at A. Q is at a height H above the horizontal plane containing P and PR has length L. What is the minimum speed of projection at P so that the particle reaches Q.



12. A right circular cone of weight W is placed on a rough horizontal table with coefficient of friction $\frac{1}{2}$. The radius of the base of the cone is one unit and its height is three units. A horizontal force is applied at the apex A. Prove that the cone begins to topple before its base begins to move. A heavy particle is attached to the cone, so that it begins to move before it begins to topple. What is the minimum weight of the particle required and where should it be attached to the cone.



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