Comments on LC Mathematics Paper 1 2023 (HL)

ICME working group (Julie Crowley, Kirsten Pfeiffer, Rachel Quinlan)

September 29, 2024

It is our view that the paper provides an appropriate mixture of basic/technical questions, and more explorative questions leading to challenging abstract tasks. The breadth of content and combination of abstract and applied topics is impressive, and the inclusion of substantial question on financial mathematics is appropriate. There are several good questions that really require students to demonstrate robust understanding of fundamental concepts, and present students with opportunities to demonstrate insightful and creative mathematical thinking. This is very positive. It is good to see that there are many questions that cannot be answered purely on the basis of memorized information or methods. That some questions are difficult does not mean that they are unfair. Indeed it is appropriate for an assessment of learning to include a few elements that challenge even the highest performing students, so that it can detect and recognize different levels of attainment. Overall it is our view that this paper presents a move away from reliance on rote learning and towards recognition of deeper engagement with the subject. Such a move, if sustained, could encourage better study habits and lead to a more satisfying experience for students.

We do have a concern that many questions that are posed in ways that are unclear, confusing or incorrect, and in language that is unconventional in mathematics. There are some places where we had genuine difficulty in figuring out what students are expected to do. These issues are serious and concerning in terms of their likely impact on students on exam day, and consequently on their overall experience in the LC exam period. They are listed in Section 1 below.

Also of concern are some instances where mathematical language and mathematical notation are used in an inaccurate or inconsistent ways (examples below in Section 2). This is arguably an issue of less importance, since it does not necessarily obscure the intended meaning. However, it is our view that internal inconsistencies in how the language of the subject is used, and usage that is not standard in mathematical communication, are problematic in contributing to an impression that precision and clarity are not important in written mathematics. Post-primary students should be able to look to the LC Mathematics papers for reliable exemplars of the use of mathematical terminology, notation and formalism. Unfortunately this paper falls short of such a standard.

A matter of possible concern is whether issues of unclear or nonstandard use of language have a disproportionate impact for students whose first language is not English or Irish, or who have not completed all of their mathematical education in Ireland, or who experience dyslexia.

1 Points of concern about clarity

Here we note items where the intended meaning is unclear to an extent that compromises the reliability of the process, with potential for serious adverse consequences for individual students.

1. Q2 (c) presents students with a graph of a function whose domain is a specified closed inteval, and asks for drawings of the graphs of some related functions, "on as large a domain as possible". The meaning and purpose of the phrase "as large a domain as possible" are very unclear. The domains of the functions in parts (i) and (ii) of the question can be deduced from the information provided about the given g, and can be fully included in the template that is provided for the sketch. The phrase "as large a domain as possible" adds nothing

to the question (except obfuscation). Students who understand the concepts and technical language of functions can only wonder why this phrase is included in the question, and whether it means that they are somehow expected to extend the domain beyond the definition that is provided. Deletion of the words "on as large a domain as possible" would leave a correctly posed and intelligible question.

2. Q4 (c) (i) presents a complex number $u = a + bi$ and asks students to write iu and $\overline{iu}$ "in their simplest form, in terms of a and b". The term "in their simplest form" has no precise mathematical meaning here, and it is reasonable to assert that (in the context) the simplest way to write iu in terms of a and b is $i(a + bi)$. We don't think this is what is intended. Mathematics does have standard, uncomplicated and unambiguous vocabulary for what we think *is* intended - the real and imaginary parts of the complex numbers in question. Precise terminology in technical subjects exists because it is necessary for conveying precise meaning, not because of a habit of pedantry.

3. Q4 (c) (iii). The definition of the term "transformation" was unclear to us here. An obvious answer to the question is "multiply by i and then conjugate", but from the reference in the question to a translation, and from looking at the marking scheme, it seems that what is expected is an explicitly geometric description of some action on the Argand plane. We acknowledge that this is consistent with the use of term "transformation" in in the Leaving Certificate syllabus documentation. However any student who is familiar with wider uses of the term, that are standard in mathematical language, might misinterpret this question. We suggest that the term "geometric transformation" might have been clearer.

4. Q7 (f). "the time that a driver could get from A to B" is an unusual and grammaticallly questionable construction whose intended meaning might not be clear to all readers.

5. Q8 (b)(ii) The intended meaning of "different" was not clear to us here. Different from what?

6. Q8 (c). The meaning of the acronym APR should be stated, to provide clarity for students and to distinguish from "quarterly interest rate" which is written out in full. We note that "APR" does not appear among the banking-related terms specified in the syllabus.

7. Q8 (e). The term "expected value" is used inappropriately here, in the absence of any formulation in terms of random variables or probability distribution. In the context, this should be called the mean. The meaning of the term "percentage of tickets sold" in line 2 is ambiguous and is arguably contradicted by the clarification on the next line.

8. Q9. Throughout this question, it appears that "factor" means positive integer factor. This is not implicit in the definition of the mathematical term "factor" and it should be stated for the sake of clarity.

9. Q10 (e). The description of the square in Diagram (B) as "a horizontal square that lies within the pyramid" is not adequate to capture what is intended, especially for part (ii), where the candidate has to interpret what happens to this square when x increases. There are multiple interpretations that are consistent with the stated information. Why does the question not refer to a horizontal cross section of the pyramid?

2 Mathematical consistency and accuracy

1. Notation for functions and graphs. Throughout the paper, the written language of functions is used inconsistently, and in many places inaccurately. Many of these points are unlikely to obscure the intended meaning, but collectively they create an impression of arbitrariness and inattention to detail. Here are some examples.

   - In Q2 (c), we are told that $g(x)$ is a function. The function should probably be called g. The curve that is labelled "$g(x)$" in Q2 (c)(i) and (ii) should be labelled "$y = g(x)$". There is no sense in which $g(x)$ is a curve in $\mathbb{R}^2$.

- In Q5 (a) we are told that f is a function, defined by $f(x) = \ldots$. This is correct, but the usage is not consistent with Q2 (c). Q5 (a) goes on to say that the derivative of f is $f'(x)$. This is not correct. The derivative of the function f is the function f'. The derivative of the expression $f(x)$ is the expression $f'(x)$.
  Q5 (b) defines a function which it calls $g(x)$, in the formulation $g(x) =$(expression involving x). This is inconsistent with the presentation of the function f (*not* $f(x)$) immediately above in part (a). It is incorrect to refer to $g(x)$ as a *function* here (the function is g), but the recurrent inconsistency is arguably more of a problem than the correctness.
- Similar problems occur in Q7, where v is asserted to be "speed t minutes after passing the point A". The v here should be $v(t)$, the information does not make sense as it is presented. The inconsistency noted above in Q5 (a) is repeated in Q7 (e), which says that $v'(t)$ is the derivative of v and $v''(t)$ is the second derivative of v. All of these should include (t), or none of them should (either would be fine). What is written is inaccurate, even if its only plausible interpretation is the correct one.

2. Q7 (b). The explanation of acceleration that is provided is not precise, but it is somewhat clarified by the unit ("km/hour per minute"). However, using hours to measure speed and minutes to measure acceleration, while not actually wrong, leads to the cumbersome unit "km per hour per minute" which conflates two different units of time and is not suited to physical analysis. Students should not be forced to engage in poor scientific practice of this nature.

3. Q7 (c). This is maybe a minor point (and probably not an issue for the examination process), but since this question has a "real world" context, and the application of mathematical meaning in everday contexts is a focus of the curriculum, we note that it is not standard in everyday communication to report time in minutes "correct to two decimal places". Writing 1.42 minutes (to mean one minute and 25.2 seconds), as students are asked to do here, would be likely to lead to misinterpretations in everyday contexts. Moreover, the unit in which the answer was expected should have been specified in the question.

4. Q8 (d) (i) "A geometric series in €A" is not a sensible thing to ask for. The square of €A is €$^2A^2$. What is intended is a geometric series in A. The question is confusing (and incorrect) as it is posed, and the confusion could be avoided by slightly changing the form of words in the presentation to "Take A to be the amount, in €, that ...", and asking for a geometric series in A. Moreover, this would be consistent with usage elsewhere in the paper, for example in Question 7, where a numerical quantity is defined as "the speed, in km/h".

3 Concluding Remarks

In general, the frequency of errors and inaccuracies of the kinds that we have noted can be much reduced by robust proofreading and checking processes. In conclusion, we repeat our positive impression of the scope and ambition of this paper, and the admirable effort to recognize and reward all forms of mathematical activity, including imaginative thinking as well as accurate and skillful implementation of methods. We hope that this scope and vision will persist in future years, and that it will have a positive influence on the study habits of students of mathematics.