

How two hundred years ago William Rowan Hamilton turned into a mathematician

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ABSTRACT. In 1822, the year he turned seventeen, William Rowan Hamilton became aware of his enormous mathematical talents, and wrote his first original mathematical papers. As a celebration of the two hundredth anniversary of the transition from the orientalist, theologian or statesman he was expected to become into the famous mathematician he would become, this is an overview of that remarkable year.

1. INTRODUCTION

On 26 January 1823 a central eclipse of the moon occurred; such eclipses have long durations and the moon is very dark. Two days later William Rowan Hamilton wrote from Trim, where he lived with his uncle James Hamilton, to his sister Eliza¹ that in the summer of 1822 he had “made calculations of all the circumstances” of the eclipse. When the time of emersion approached he could not find the moon, but shortly thereafter he saw Jupiter’s moon Io through his telescope and knew that also our moon had started to emerge. “For it is a remarkable coincidence that Jupiter’s moon emerged from a total eclipse only three minutes and a-half before ours did. At the same time Saturn was on the meridian” [1, 126].

On 23 February 1823 William wrote to ‘Cousin Arthur’,² who lived in Dublin, “[What] struck me was the near coincidence in point of time between the eclipse of our moon and that of the first Satellite of Jupiter. By an investigation founded on the successive propagation of light, I ascertained that there were places (not in this earth) at which the emersion of Jupiter’s moon and the middle of the eclipse of ours would have appeared to synchronise, and also that these places are all contained in a hyperboloid of revolution, Jupiter being in one focus, the earth in the other, and the axis equal to the space that light traverses in the difference of the times of the phenomena: about ninety millions of miles. The result is remarkable” [1, 128-129].³

This observation obviously not being something most amateur astronomers would make, it is one of the indications that William had become a mathematician, a transition which had happened in 1822.

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¹Both their parents died young. In 1817 Sarah Hamilton née Hutton died at thirty-seven; thereafter all five Hamilton siblings lived with relatives. Their father Archibald died in 1819, he was forty-one.

²Arthur Hamilton was a cousin of James and Archibald Hamilton.

³To verify William’s results, data from the free planetarium program *Stellarium*, <http://stellarium.org>, and the website *Eclipsewise*, <http://eclipsewise.com/lunar/LEcatalog/LEcatalog.html>, were combined. Mentioning ninety million miles, William apparently had calculated that the light of Io’s emersion started eight minutes after the time of greatest eclipse of our moon and that, travelling for about 38 minutes, it arrived on Earth 3.5 minutes before the emersion of our moon. From this result, in combination with his remark about Saturn, it appears that his calculations for the emersions of our moon and of Io were indeed very accurate.

2. FROM THE CLASSICS TO MATHEMATICS

In 1808 young William had been brought to Trim to be educated, and uncle James, linguist, curate and schoolmaster, had immersed him in the Classics from the moment he arrived. In 1810, when William was four, his mother Sarah wrote in an astonished and very proud letter to her sister Mary Hutton that he read “Latin, Greek and Hebrew!!” [1, 36-37]. Thereafter European languages were added, and “with a view to India,” as was his father’s wish [1, 57], also Oriental ones; Graves notes that “when thirteen years old [he] was in different degrees acquainted with thirteen languages.”⁴

Early in 1821 the Classics still had been William’s most important subject; in May 1821 he had written to Cousin Arthur, “You are not to imagine that because astronomical calculations take up the greater part of my letters to you, they therefore occupy the principal portion of my time; it is employed in the study of the classics as my serious business, and only occasionally in the sciences by way of recreation, in which light I consider them, however closely I may pursue them for a time” [1, 90]. But in August 1821⁵ Uncle James gave him Lloyd’s 1819 *Analytic Geometry*,⁶ to which William reacted strongly, in September 1822 writing to Cousin Arthur, “Ill-omened gift! it was the commencement of my present course of mathematical reading, which has in so great a degree withdrawn my attention, I may say my affection, from the Classics” [1, 112]. Still, late in 1821 or early in 1822 he had written a long poem, according to Graves of “the Prize-poem order,” with as its subject “the Literature of Rome” [1, 105-108].

In March 1822 William made “a great many calculations about the next eclipse of the moon: part of it will fall on August 3, my birthday. I have also made a view of the progress for Dublin,” as he later wrote to Eliza. And on 31 March he wrote an Essay, ‘On the value of 0/0, with preliminary remarks on Division.’ Graves adds, “[which] by a subsequent annotation of his own is discredited as ‘unnecessary’,” yet he thought it worthwhile to give the calculations, to show William’s “early interest in the elementary notions of science” [1, 101].

In April William contracted whooping-cough, which for adults is usually not acutely severe but it can last for months, and the coughing fits can be exhaustive. But William also suffered from chronic bronchitis which may have aggravated his symptoms; early in May he was allowed to go to Dublin to stay with Cousin Arthur for a ‘required change of air’, because he “had been for some time forbidden to read, coughed much, and had to struggle with great difficulty of breathing” [1, 99, 100, 303].

On 31 May 1822, while still in Dublin and reading the first volume of Laplace’s *Mécanique Céleste*,⁷ William found a “flaw in the reasoning by which Laplace demonstrates the parallelogram of forces” and gave a more general proof [1, 661-662]. According to Graves the document was found by Henry Hennessy “inserted at the pages it refers to in the copy of the *Mécanique Céleste* which belonged to Dr. Brinkley, and which subsequently came into the possession of Mr. Hennessy” [1, 103]. Although it was not the direct cause, it did lead to the chain of events which would bring William, towards the end of the year, in contact with Brinkley, who then was Royal Astronomer of Ireland and therefore lived at Dunsink Observatory.

⁴R. P. Graves: *Our Portrait Gallery*, Dublin University Magazine (19) (1842), 94–110. <https://www.maths.tcd.ie/pub/HistMath/People/Hamilton/Gallery/Gallery.html>. See for the discussion about this claim https://annevanweerden.nl/docs/Sir_William_Rowan_Hamilton_-_hyperpolyglot.pdf.

⁵William wrote that he had received Lloyd’s book “in August, while the King was in Dublin.” George IV visited Ireland from 12 August until 3 September 1821, <https://georgianpapers.com/2021/10/18/erins-king-the-politics-and-pageantry-of-george-ivs-visit-to-ireland-in-1821>, he therefore must have received it in the second half of August.

⁶B. Lloyd: *Analytic Geometry*, s.n., Dublin, 1819. <https://doi.org/10.48495/qj72pf99m>.

⁷P. S. Laplace: *Traité de Mécanique Céleste*, Vol. 1, J. B. M. Duprat, Paris, 1798. <https://archive.org/details/traitemcaniquec01lapl>. The ‘flaw’ is on p. 6.

In June 1822 William was back in Trim, and in July again in Dublin. On 11 July he solved a mathematical problem in Analytic Geometry, posed as the Prize Question for 1822 in the *Gentleman's Mathematical Companion* of November 1821.⁸ It had been shown to him by his later college tutor, the mathematician Charles Boyton, who was a son of a family friend and had become a Fellow on 13 July 1821; he solved it before Boyton did [1, 81, 90, 108]. William then still expected to enter College in October, or perhaps November.⁹

On 4 September 1822 William gave an overview of 1821, in a letter to Cousin Arthur. “I was amused this morning, looking back on the eagerness with which I began different branches of the Mathematics, and how I always thought my present pursuit the most interesting. I believe it was seeing Zerah Colburn¹⁰ that first gave me an interest in those things. For a long time afterwards I liked to perform long operations in Arithmetic in my mind; extracting the square and cube root, and everything that related to the properties of numbers. It is now a good while since I began Euclid. Do you remember when I used to go to breakfast with you, and we read two or three propositions together every morning? I was then so fond of it, that when my uncle wished me to learn Algebra, he said he was afraid I would not like its uphill work after the smooth and easy path of Geometry. However, I became equally fond of Algebra” [1, 111].

This is also the letter in which William mentioned the “Ill-omened gift”, the book by Lloyd he had received in 1821, and that he would become a mathematician was now inevitable; on 26 August 1822 he wrote a letter to his aunt Mary for which she apparently reprimanded him.¹¹ Graves comments on this “remarkable letter,” “After having entered upon the study of Newton, Laplace, and Lagrange, he began to feel that he possessed powers akin to theirs; perhaps, too, he had floating notions of some of the discoveries which lay before him, for to this year he himself assigns the composition of an Essay which contains the germ of his investigations respecting Systems of Rays,¹² which were begun in the following year” [1, 110].

What William wrote to aunt Mary was, “I have been continuing my Classics, as usual, with my uncle. But I fear I shall never be so fond of them as of the Mathematics that I am now reading. I know that an intimate acquaintance with Classical literature is of the greatest importance both in College and in society: that nothing contributes more to form and refine one’s taste; but still, in human literature, I think there is nothing that so exalts the mind, or so raises one man above his fellow-creatures, as the researches of Science. Who would not rather have the fame of Archimedes than that of his conqueror Marcellus, or than any of those learned commentators on the Classics, whose highest ambition was to be familiar with the thoughts of other men? If indeed I could hope to become myself a Classic, or even to approach in any degree to those great masters of ancient poetry, I would ask no more; but since I have not the presumption

⁸J. Hampshire: *The Gentleman's mathematical companion*, vol. 5, Davis and Dickson, London, 1821-1826, xxv, 160 (question), xxvi, 447-452 (question and answers). Due to incoherent page numbering, page numbers refer to pages of the scanned volume. <https://babel.hathitrust.org/cgi/pt?id=mdp.39015065321062>. The Prize Question posed in 1822 as no 36 was won by P. P. and EPSILON (p. 319), one of William’s answers is given on [1, 109].

⁹On 1 July one of the main entrance exams had been held, the next ones were on 14 October and 4 November. Trinity College Dublin Admissions Records, 1769-1825, <https://doi.org/10.48495/6q182n74x>.

¹⁰In 1817 William and Zerah had “engaged in trials of arithmetical skill,” in which William’s “antagonist was generally the victor.” They met again in 1819, and Zerah “seems to have very freely imparted to Hamilton the methods used by him in calculation” [1, 77].

¹¹Having been praised from very early childhood, all his life Hamilton had to work hard not to become vain. The support and criticisms of his family seems to have laid the foundation for his perseverance.

¹²See also page 54. For the ‘Theory of Systems of Rays’, leading to Hamiltonian mechanics and to his knighthood, see his ‘Mathematical Papers’, <https://www.maths.tcd.ie/pub/HistMath/People/Hamilton/Papers.html>.

to think so,¹³ I must enter on that field which is open for me. Mighty minds in all ages have combined to rear upon a lofty eminence the vast and beautiful temple of Science, and inscribed their names upon it in imperishable characters; but the edifice is not completed: it is not yet too late to add another pillar or another ornament. I have yet scarcely arrived at its foot, but I may aspire one day to reach its summit” [1, 110-111].

3. A MATHEMATICIAN

On 23 September 1822 William wrote to Eliza who had just entered the school of the Misses Hincks in Dublin,¹⁴ “I have some curious discoveries - at least they are so to me - to show Charles Boyton when next we meet: he will be my Tutor soon. No lady reads a novel with more anxious interest than a mathematician investigates a problem, particularly if in any new or untried field of research. All the energies of his mind are called forth, all his faculties are on the stretch for the discovery. Sometimes an unexpected difficulty starts up, and he almost despairs of success. Often, if he be as inexperienced as I am, he will detect mistakes of his own, which throw him back. But when all have been rectified, when the happy clue has been found and followed up, when the difficulties, perhaps unusually great, have been completely overcome, what is his rapture! Such in kind, though not in degree, as Newton’s, when he found the one simple and pervading principle which governs the motions of the universe, from the fall of an apple to the orbits of the stars” [1, 114-115].

About the ‘curious discoveries’ Graves writes, “There exists a Paper of twenty-one folio pages entitled “Essay on Equations representing Systems of Right lines in a given Plane. Part I.: On the manner in which they arise from problems determining a right line, which admit of more than one solution. By William Hamilton.” To this title is appended a note which I transcribe. (“This curious old Paper, found by me to-day in settling my study, must have been written at least as early as 1822. It contains the germ of my investigations respecting Systems of Rays, begun in 1823. W.R.H., February 27, 1834.”) [1, 115].

Apparently early in October William’s entrance into College was postponed until the next year; William mentioned it to Eliza in a letter written on 9 October [1, 116]. Graves writes, “This decision was arrived at after much discussion between his uncle and his Cousin Arthur, the determining motive being the state of his health, which during the spring and the summer had caused much uneasiness” [1, 115]. It did not keep William away from his mathematical researches however.

On 31 October William wrote to Cousin Arthur, “When was Mr. Kiernan’s letter left at Cumberland-street?¹⁵ He tells me that “I forgot your ‘queries about Laplace’ for a long time” [...]; “but at last I laid them before Dr. Brinkley, who said he thought them ingenious, and he was so good as to say that he would write an explanation for you. He also desired me to bring you to him, and that he would be happy to know you, and to show you the Observatory. This of course, you know, is a great honour”” [1, 119]. Graves remarks that he could not “supply any information” about the ‘queries’, which William seems to have written when reading Laplace.

¹³While at college Hamilton twice won the Chancellor’s Prize; for ‘The Ionian Islands’ and ‘Eustace de St. Pierre’ [1, 154], like his 1821 potentially prize winning poem both history poems. It made no difference.

¹⁴Bithia and Frances Hincks were aunts of the Reverend Edward Hincks, the famous Egyptologist and Assyriologist, <https://www.dib.ie/biography/hincks-edward-a4021>. They were related by marriage to the Huttons.

¹⁵Cousin Arthur lived at South Cumberland-street. George Shirley Kiernan was a family friend, State Apothecary, and a member of the Royal Irish Academy, <https://archive.org/details/transactionsofro13iris/page/n133>.

Graves then writes the concluding remarks, “I find among the early mathematical manuscripts of Hamilton one entitled ‘Example of an Osculating Circle determined without any consideration repugnant to the utmost rigour of Analysis,’ and dated November 14, 1822; a second, [of the same date],¹⁶ entitled ‘Osculating Parabola to Curves of Double Curvature’; and a third, dated December, 1822, of which the title is, ‘On Contacts between Algebraic Curves and Surfaces.’ These papers mark the year 1822, when he attained the seventeenth year of his age, as that in which Hamilton entered upon the path of original mathematical discovery. With the second and third of them in his hand, availing himself of the kind permission of Dr. Brinkley, he paid his first visit to him at the Observatory.¹⁷ Dr. Brinkley was impressed by their value, and desired to see some of the investigations in a more developed form; with this request Hamilton complied, by forwarding to him in the following month a paper entitled ‘Developments’” [1, 124]. Unfortunately, this paper is most likely lost; Graves remarks, “It was returned by him to Hamilton, and was in possession of the latter in the year 1841, but I have not discovered it among the manuscripts entrusted to me, nor I believe is it to be found in the Hamilton collection deposited in the manuscript-room of the Library of Trinity College” [1, 124].

On 23 February 1823 William wrote to Cousin Arthur, in the same letter in which he had written about the hyperboloid formed by the places of synchronicity of the emersion of Io and the greatest eclipse of our moon with which this narrative started, “Perhaps you heard that Dr. Brinkley expressed his full approbation of my “Developments’” [1, 128]. He finally entered College on 7 July 1823.¹⁸

REFERENCES

- [1] R. P. Graves: *Life of Sir William Rowan Hamilton*, Vol. I, Hodges, Figgis, & Co., Dublin, 1882.
<https://archive.org/details/lifeofsirwilliam01gravuoft>

Anne van Weerden As an information specialist working in Utrecht University Library and just having started her master’s program in theoretical physics, while enrolled in a seminar on the History of Vector Analysis she came across the distorted descriptions of Hamilton’s private life as an unhappily married alcoholic. She decided to stop her studies and find out what had happened.

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¹⁶Graves did not give a date; it was taken from the overview of Trinity College Dublin Library’s manuscript collection IE TCD MSS 7773-6, which contains the latter two papers, but not the first.

¹⁷William had visited Dunsink Observatory three years before, on 8 July 1819, but Brinkley had not been at home [1, 62].

¹⁸TCD Admissions Records, 1769-1825, pp. 343 and 344, <https://doi.org/10.48495/6q182n74x>. The second page contains some errors; William was born in Dublin, and his father Archibald had died before he entered College. See also the *Atlas blog* of November 2019, http://www.mathsireland.ie/blog/2019_11_cm.