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Leila Schneps (editor): Alexandre Grothendieck: a mathematical portrait, International Press, 2014. ISBN:978-1-57146-282-4, USD 85.00, 307+viii+6 pp.

REVIEWED BY BERND KREUSSLER

Alexandre Grothendieck, born on 28 March 1928 in Berlin, died on 13 November 2014 in the French Pyrenees. He was one of the greatest scientists of the twentieth century who has influenced significantly the development of a number of fields in pure mathematics: functional analysis, algebraic geometry, arithmetic geometry, category theory, logic, homological algebra and related areas. Grothendieck's contribution to the problem of space is considered to be of the same depth as Einstein's; his originality was to deepen the idea of a geometric point, see [4].

He was the greatest master of extracting the most essential features of a particular topic in a very general and abstract way so that solving an old problem often became a simple exercise. His method to solve problems was described by himself with the following allegory: instead of cracking a big nut by brute force one could immerse the nut in a softening fluid until the nut opens just by itself.

His personal life was as exceptional as his mathematical achievements. In 1966 he was awarded the Fields Medal for his fundamental contributions to algebraic geometry. In 1970, at the age of 42, he suddenly resigned from his prestigious position at the IHES, withdrawing himself from the mathematical community and later, in the 1990s, from social life completely.

The cubist portrait by Pablo Picasso, that was chosen as the front cover image of this book, reflects the style of this mathematical portrait of Grothendieck: his life and work is described on different levels and from many different angles by people who knew him personally, some of them very well, and who have included personal memories, anecdotes and explanations about Grothendieck's personality, his ideas and visions.

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The plan for this book took shape in August 2008 at a conference in the French Alps where former friends, colleagues, students, and collaborators of Alexandre Grothendieck met and discussed how one could explain to future generations the extraordinary and special nature of the contributions of this mathematical genius.

This book contains thirteen articles, each devoted to a particular aspect of Grothendieck's work. The authors did not concentrate on specific mathematical content of his monumental work but instead on the style how he did mathematics and on the impact he had on mathematics. To facilitate a better appreciation of Grothendieck's achievements, the authors also outlined the situation before the grand master started to work on a particular subject. This way the fundamental simplicity and the extraordinary power of his ideas become apparent.

A more detailed outline of the contents of the thirteen articles follows. At the end, the interested reader of this review is pointed to further literature about Alexandre Grothendieck.

Joe Diestel: **Grothendieck and Banach space theory**

The subject of this article is Grothendieck's work between 1953 and 1956 in Functional Analysis, especially on topological tensor products, nuclear spaces and the local theory of Banach spaces. It becomes obvious that he was a theory builder par excellence already at this early stage in his mathematical career just after receiving his Ph.D. in Nancy in 1953. The article concludes with a short account of the status of the problems posed by Grothendieck in this area.

Max Karoubi: **L'influence d'Alexandre Grothendieck en K-théorie**

This note deals with one of Grothendieck's revolutionary ideas: the introduction of the K-group of algebraic vector bundles, originally introduced for the proof of the Riemann-Roch Theorem. In Grothendieck's formulation, the Riemann-Roch formula for a morphism expresses the deviation from commutativity of a certain natural diagram that involves the Chern character map which links the K-group to ordinary (purely topological) cohomology. His ideas initiated the development of higher K-theory and of topological K-theory which has connections to Functional Analysis. From the personal memories the author has included in this article one learns how generous Grothendieck was with his ideas and advice.

Michel Raynaud: **Grothendieck et la théorie des schémas**

The notion of an algebraic scheme is at the core of Grothendieck's revolutionary re-foundation of algebraic geometry. An essential and influential novelty is the functorial point of view, which allows the description of a scheme by its 'functor of points'. As a consequence, commutative rings instead of fields are now in a unifying manner at the heart of algebraic geometry. It also suggests the shift of focus from objects to morphisms which played an essential role in Grothendieck's proof of the Riemann-Roch Theorem, his first outstanding masterpiece in algebraic geometry. This note is a kind of condensed survey of the essentials of EGA and (part of) SGA.

Steven L. Kleiman: **The Picard scheme**

Grothendieck's theory of the relative Picard scheme was an early impressive success of his new approach to algebraic geometry: the functorial point of view, focus on morphisms instead on objects (known as the relative point of view), the importance of considering nilpotent elements etc. In this article, the author describes the substance and spirit of Grothendieck's theory in an informal way. After an illuminating exposition of the historical context, he highlights the simplicity, natural generality and originality of Grothendieck's approach.

David Mumford: **My introduction to schemes and functors**

In this short note, Fields medallist David Mumford gives a very personal description of how Grothendieck influenced his own view of algebraic geometry in 1958 at Harvard University. Essential for Mumford, who was interested at that time in moduli spaces of curves and vector bundles, was that the moduli space idea of Riemann and Picard could be made precise using functors. For him the most convincing aspect of Grothendieck's theory of schemes was that it allows to consider infinitesimal deformations, used in an intuitive way by Enriques, as actual families of schemes over one-point bases that are spectra of Artin rings. At the end, the author gives a very brief summary how Grothendieck transformed algebraic geometry.

Carlos T. Simpson: **Descent**

The notion of descent, putting together a global object out of local pieces and gluing data, is ubiquitous in Grothendieck's work. It is crucially present in the theory of sheaves, leads to Grothendieck topologies and is essential in the theory of stacks. In topos theory

the idea of descent is pushed to a new level: the abstract collection of gluing data is the only true reality, no need for the existence of glued objects. After a very brief introduction to gluing, the author puts his main emphasis on modern developments and possible future directions of descent theory. This includes higher categories, higher stacks, higher non-abelian cohomology and derived stacks. These developments were strongly influenced by Grothendieck's 595-page manuscript *À la Poursuite des Champs* (Pursuing Stacks) which he wrote in 1983.

Jacob P. Murre: **On Grothendieck's work on the fundamental group**

This lecture gives the reader an impression of the power and beauty of Grothendieck's method. He primarily looked for naturalness, not generality, he aimed at simplifying situations by extracting the key features so that eventually the solution to the problem falls out easily. His theory of the algebraic fundamental group unified classical Galois Theory of fields and the topological theory of the fundamental group. It led to a deep understanding of the algebraic fundamental group of an algebraic curve in positive characteristic, which was out of reach before the introduction of schemes. The lecture concludes with some remarks on further developments, including Grothendieck's famous manuscript *Esquisse d'un Programme* from 1984.

Robin Hartshorne: **An apprenticeship**

In this essay, the author describes vividly his experience with Grothendieck when he wrote *Residues and Duality*, published in 1966. The story starts in 1963, when Grothendieck was busy with fundamental works (EGA, SGA) but nevertheless prepared a 250-page manuscript for Hartshorne to conduct a seminar about his theory of duality. After many rounds of corrections by Grothendieck to the drafts written by Hartshorne, the now published version was accepted by Grothendieck as the best possible at the time, but both agreed that the theory had not yet reached a satisfactory state.

Luc Illusie: **Grothendieck et la cohomologie étale**

This article deals with Grothendieck's concept of étale cohomology and the impact it had on arithmetic geometry. What inspired these ideas, how the Weil conjectures motivated this development and which obstacles had to be overcome is thoroughly explained.

Leila Schneps: **The Grothendieck-Serre correspondence**

The exchange of letters between Serre and Grothendieck started at the beginning of 1955 and continued until 1969. A bilingual version of the Grothendieck-Serre correspondence was published in [5]. The aim of the present article is to give a short explanation of the main results and notions discussed in these letters, whereby giving a first impression of the nature of [5] as a 'living maths book'. The explanations are enriched with information about the personalities and the lives of these two outstanding mathematicians. In the closing chapter of this article, letters are discussed that were exchanged after Grothendieck started in 1986 the distribution of his monumental autobiographical work *Récoltes et Semailles* (Reaping and Sowing).

Frans Oort: **Did earlier thoughts inspire Grothendieck?**

The author raises the question if Grothendieck's brilliant ideas had simply occurred to him out of the blue or whether they have their roots in earlier works. He analyses this question in the context of three topics: the fundamental group, Grothendieck topologies and anabelian geometry. In each case he describes the situation before Grothendieck entered the scene and then carves out the extraordinary contribution of Grothendieck. The author of this article advocates for writing a scientific biography of Alexandre Grothendieck in a similar style. For each aspect of his work, he suggests to include a discussion of possible roots, then describe the leap Grothendieck made from those roots to general ideas and finally investigate the impact his ideas had on the development of this branch of mathematics.

Pierre Cartier: **A country of which nothing is known but the name: Grothendieck and 'motives'**

In this note, the interactions between Grothendieck's outstanding scientific work and his extraordinary personality are discussed in a way in which it is accessible to a broad audience. The author, who was a very close friend of Grothendieck, tries to stay as rational as possible in his analysis of the work and biography of Grothendieck before he lets *Récoltes et Semailles* illuminate the situation 'from within'. He included a discussion of Grothendieck's sufferings, spirituality and obsession in his later years. When Grothendieck's story is compared with Botzmann's and Cantor's, a striking difference is that his scientific work was immediately and enthusiastically accepted by the scientific community. His work is unique in that his

fantasies and obsessions are not erased from them; he also delivered to us what he believed to be the meaning of his mathematical work. Included in this note is a very readable and brief overview of the scientific work of Grothendieck which covers, in a less detailed way, most of the content of the other articles in this book.

YURI I. MANIN: **Forgotten motives: the varieties of scientific experience**

The author of this short essay describes his experience when he visited Grothendieck in 1967 for five or six weeks. The central topic of this private tutoring was Grothendieck's newly emerging project of motives including the so-called standard conjectures. The content of this essay spans from the early history of motives to recent developments of this active field of research – Grothendieck's standard conjectures are still unproven. In addition to personal anecdotes, the author gives an intuitive description of Grothendieck's idea of motives and highlights the close relationship the theory of motives has to homological algebra and to mathematical physics.

The book ends with six pages of photographs of the contributing authors, some of them taken before 1970.

The algebraic geometer will find in each article interesting aspects of Grothendieck's work and life. Readers who are less familiar with algebraic geometry may enjoy most the articles of R. Hartshorne, L. Schneps, F. Oort, P. Cartier and Y. Manin as well as the less technical and more anecdotal parts of the other articles. Even without a detailed understanding of the mathematical ideas, through these articles it is possible to get a feeling of the mathematical atmosphere around Grothendieck and to appreciate the personality of this outstanding mathematician.

The book under review is not the only text that deals with the work and life of A. Grothendieck, but currently the best available mathematical portrait. Interesting other texts include the English translation [13] of Part 1 of Winfried Scharlau's four volume biographical project, the articles [6], [11], [10], [3] and from the Notices of the AMS: [8], [9], [12], [7], [1], [2]. The curious reader is referred to the Grothendieck Circle website `www.grothendieckcircle.org` where one finds some of Grothendieck's later texts, links to many other fine articles about the work and life of Grothendieck as well as an interesting collection of photographs.

References

[1] M. Artin, A. Jackson, D. Mumford, J. Tate (eds.): *Alexandre Grothendieck 1928–2014. I.* Notices Am. Math. Soc. 63, No. 3 (2016), 242–255.

[2] M. Artin, A. Jackson, D. Mumford, J. Tate (eds.): *Alexandre Grothendieck 1928–2014. II.* Notices Am. Math. Soc. 63, No. 4 (2016), 401–413.

[3] W. Bietenholz and T. Peixoto: *To the Memory of Alexander Grothendieck: a Great and Mysterious Genius of Mathematics*, `http://arxiv.org/abs/1605.08112`.

[4] P. Cartier: *A mad day's work: from Grothendieck to Connes and Kontsevich The evolution of concepts of space and symmetry*, Bull. Amer. Math. Soc. 38 (2001), 389–408.

[5] P. Colmez (ed.) and J.-P. Serre (ed.): *Grothendieck-Serre Correspondence: Bilingual Edition*, Société Mathématique de France and American Mathematical Society, 2004.

[6] A. Grothendieck: *Mathematical Life in the Democratic Republic of Vietnam*, 1967. `https://www.math.washington.edu/~koblitz/groth.pdf`

[7] L. Illusie, A. Beilinson, S. Bloch, V. Drinfeld: *Reminiscences of Grothendieck and his school*. Notices Am. Math. Soc. 57, No. 9 (2010), 1106–1115.

[8] A. Jackson: *Comme appelé du néant – as if summoned from the void: the life of Alexandre Grothendieck. I.* Notices Am. Math. Soc. 51, No. 9 (2004), 1038–1056.

[9] A. Jackson: *Comme appelé du néant – as if summoned from the void: the life of Alexandre Grothendieck. II.* Notices Am. Math. Soc. 51, No. 10 (2004), 1196–1212.

[10] D. Mumford: *Can one explain schemes to biologists*, 2014. `http://www.dam.brown.edu/people/mumford/blog/2014/Grothendieck.html`

[11] P. Pragacz: *Notes on the Life and Work of Alexander Grothendieck*. In: Topics in cohomological studies of algebraic varieties. Impanga lecture notes. Trends in Mathematics. Basel: Birkhäuser, 2005.

[12] W. Scharlau: *Who is Alexander Grothendieck?* Notices Am. Math. Soc. 55, No. 8 (2008), 930–941.

[13] W. Scharlau: *Who is Alexander Grothendieck? Anarchy, mathematics, spirituality, solitude. Part 1: Anarchy*. Transl. from the German. Revised and expanded edition. Norderstedt: Books on Demand, 2011.

Bernd Kreussler is Lecturer in Mathematics at Mary Immaculate College Limerick since 2002. His main research interests lie in the area of complex algebraic geometry and homological algebra.

Mary Immaculate College, Limerick
E-mail address: `bernd.kreussler@mic.ul.ie`