

**Mary Kelly and Charles Doherty editors: Music and
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REVIEWED BY ROD GOW

I have taught lecture courses in the History of Mathematics for several years. I like to discuss topics that are substantial, and preferably relate well to mathematics as we know it and practise it today. There is no shortage of material when following the history of Greek mathematics, from Thales until Diophantus and Pappus in the third/fourth century CE. Endless time can be devoted to the study of the work of Pythagoras, Euclid, Archimedes, Apollonius, and others, and this can be related to the philosophical ideas of Zeno, Plato and Aristotle.

It is generally accepted that the wonderful creativity of Greek mathematicians was gradually lost, especially from the first century BCE onwards. Nobody, for instance, was able to extend the techniques of Archimedes, which foresaw the infinitesimal arguments of integral calculus long before the revolution in mathematical thought brought about by Descartes, Fermat, Newton, and Leibniz, among others.

The Romans cared little for the achievements of Greek mathematicians, and seem to have had no idea of the full extent even of Euclid's *Elements*. The situation was different in the Eastern Roman Empire, centred on Constantinople, but nonetheless, while manuscripts were copied and circulated for use in the institutions of higher learning in the Eastern Empire, mathematicians and scholars tended to confine their creative skills to writing commentaries on the existing works, and added nothing much that was new or significant. Of course, in some cases, these commentaries have proved to be of basic importance, as, having miraculously survived for centuries, they provide us with the scarce information we have on ancient mathematics and

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its creators. A famous example is the commentary of Proclus (410-485 CE) on the first book of Euclid's *Elements*, which relates almost all of what we know about the history of Greek geometry.

Towards the end of the Western Roman Empire, a few elementary mathematical works were produced in Rome, and some of these continued to be used extensively in medieval Western Europe. An *Arithmetic* and a *Geometry*, written by the Roman aristocrat Boethius (c. 480-524 CE), are the most important examples of this kind, but they are considered to be poor synopses, without proofs, of more substantial work by Nicomachus and Euclid. Boethius was also the author of books on astronomy and music. (The book on astronomy is now lost, and it is not certain that the surviving fragments of a work on geometry attributed to Boethius are indeed written by him.) More importantly, he undertook to translate and explain as much as possible of the work of such Greek philosophers as Aristotle, and despite his shortcomings, his contribution to the continuity and history of scholarship has proved to be vital.

The four scientific subjects, arithmetic, geometry, astronomy and music, are said to form the *quadrivium*, a name possibly introduced by Boethius. Three less scientific subjects, grammar, rhetoric and logic, form the *trivium*. Taken together, the seven subjects of the trivium and quadrivium constituted the *Seven Liberal Arts* (*Artes Liberales*), which dominated much thinking on early medieval European education. The concept of the seven liberal arts was expounded by Martianus Capella, born in Africa in the early 5th century CE, in his work *De Nuptiis Philologiae et Mercurii* (On the Marriage of Philology and Mercury). It is said to be a convoluted allegorical encyclopedia of the liberal arts, of which numerous manuscript copies exist. To illustrate the remarkable longevity of this unusual work, it was even printed in Vicenza in 1499, and is still available today in reprints.

The Roman senator Cassiodorus (c. 485-585 CE) composed an influential work entitled *Institutiones Divinarum et Saecularium Litterarum*, between the 530's and the 550's, whose second section relates to the trivium and quadrivium. He considered the seven subjects useful for the study of divinity. Cassiodorus proved to be important for the transmission of learning during tempestuous times, as he systematized the copying of manuscripts in libraries.

We do not associate early medieval Western Europe with advanced mathematical thinking. It is always dangerous to categorize a given era in a few succinct words, but quoting from the paper *Seventh-century Ireland: the cradle of medieval science?*, p.46 in the book under review, by Immo Warntjes: “With the fall of the Roman empire, the secular institutions of education and learning lost their foundation. Learning shifted into the newly developing monasteries, and, with this, became decidedly Christian in character. The principal object of learning was to receive an understanding of God’s creation, which manifested itself in two ways—the Holy Scripture and the Cosmos.” Continuing on p.47, Warntjes writes: “Especially from the sixth to the eighth centuries, but also beyond the Carolingian age right up to the reception of Arabic science, computus was first the only, then the principal science within Christian learning.”

Computus means *computation*, and is the science concerning the date of Easter. Much intellectual effort seems to have been expended on what one is tempted to regard nowadays as a fairly routine matter, or at least one that has been solved definitively. Of course, there were foundational problems, such as determining the dates of Christ’s birth and death. There was also a problem about the lack of a number zero, so that there was no year zero for Christ’s birth. Dionysius Exiguus (c. 470-544 CE), born in Scythia Minor, was the inventor of the *Anno Domini* system, used to number the years of the Gregorian and Julian calendar. In 525, Dionysius prepared a table of future dates of Easter, and a set of *arguments*, explaining their calculation (the *computus*), at the request of Pope John. In 725, the Venerable Bede incorporated the algorithms for dating Easter into his textbook on computistics, *De Temporum Ratione*.

Moritz Cantor, the German historian of mathematics, wrote as follows (in English translation) in Vol. 1 of his *Vorlesungen über Geschichte der Mathematik* (1892): “The computation of Easter-time, the real central point of time computation, is founded by Bede as by Cassiodorus and others, upon the coincidence, once in every nineteen years, of solar and lunar time, and makes no immoderate demands upon the arithmetical knowledge of the pupil who aims to solve simply this problem.”

This brings me to focus on the book under review. As most mathematicians are probably not especially familiar with the intricacies

of medieval mathematics and its principal aims, I thought it necessary to present the preceding introductory material, albeit somewhat incomplete and undetailed. An interest in computistics and *computus* has arisen in NUI Galway, and five international conferences on the subject have been held there between 2006 and 2014. I should point out, however, that the work under review is a volume of papers presented at a conference, *Music and the stars: mathematics in medieval Ireland*, held in Dublin in July 2012.

To give some understanding of why the study of *computus* relates to Ireland, I quote from Warntjes again, p.52: “The mathematical relation between the beginning of Lent and Easter Sunday had to be fixed, calendrical algorithms for establishing the right date and lunar age had to be invented. This was first done by Irish scholars.

“What Irish scholars of the seventh century achieved, therefore, was a comprehensive understanding of Easter reckoning, which was to become the unanimously accepted system for the calculation of Easter, from the ninth century onwards, for the rest of the Middle Ages and in the Orthodox Church to the present day. Their knowledge they put into their writing, computing, for the first time, comprehensive textbooks on the reckoning of time.”

As might be anticipated from what I have written above, I found the article by Warntjes especially helpful for presenting me with the main ideas in *computus* and for explaining the role of Irish scholars in systematizing the subject. Other articles I found enjoyable were: *Music and the stars in Cashel, Bolton Library, MS 1*, by Charles Burnett; *Boethius in early Ireland: five centuries of study in the sciences*, by Pádraig Ó Néill; *Music and the stars in early Irish compositions*, by David Howlett. There is also *Saints, scholars and science in early medieval Ireland*, by Dáibhí Ó Cróinín. He has become a leading light in the recent study of *computus*, and has identified a number of Irish *computus* manuscripts, dating from the 7th century, in various European libraries. *Music and the Stars* also contains coloured photographic reproductions of various manuscripts described in the text, and these give a good idea of how surviving documents actually appear, and what it is that scholars actually have to study.

It is difficult for an outsider and non-specialist to do justice to topics that have only marginal connection to his own interests and

knowledge. At first, I thought that the subject matter was too remote from me to attempt more than a feeble appraisal. In fact, I found several papers fascinating, especially that of Warntjes, which certainly served to give a very helpful overview of medieval mathematical objectives.

To conclude, I would like to mention a more recent example of how the problem of Easter calculation can inspire mathematical creativity. In his book *Choice and Chance* (first edition, 1867), the Reverend William Allen Whitworth (1840-1905), Vicar of All Saints, Margaret Street, London, proposed the following Question 115 (I quote from the 5th edition, 1901): “What is the chance that in a year named at random Easter should fall on April 25 of the Gregorian Kalendar?”

Now Easter may fall between March 22 at the earliest and April 25 at the latest. The answer given by Whitworth is

$$\frac{1}{19 \times 7} = \frac{1}{133}.$$

The 19 term arises from the 19 year cycle mentioned earlier, and the $1/7$ is just the probability that a certain phenomenon occurs on a Sunday, when it is equally likely on any day of the week. Whitworth notes that, on average, Easter will occur on April 25 about three times in four centuries. The last such occurrence was in 1943, and the next is in 2038. Since finding out this, I have always hoped that I would live long enough to appreciate the next late Easter (nobody currently alive is likely to see the next occurrence after that, which is in 2190).

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