

Ian Roulstone and John Norbury: Invisible in the Storm: The Role of Mathematics in Understanding Weather, Princeton University Press, 2013, ISBN:978-0-691-15272-1, USD 35, 384 pp

REVIEWED BY PETER LYNCH

The development of mathematical models for weather prediction is one of the great scientific triumphs of the twentieth century. Accurate weather forecasts are now available routinely, and quality has improved to the point where occasional forecast failures evoke surprise and strong reaction amongst users. The story of how this came about is of great intrinsic interest.

General readers, having no specialised mathematical knowledge beyond school level, will warmly welcome an accessible description of how weather forecasting and climate prediction are done. There is huge interest in weather forecasting and in climate change, as well as a demand for a well-written account of these subjects. In this book, the central ideas behind modelling, and the basic procedures undertaken in simulating the atmosphere, are conveyed without resorting to any difficult mathematics.

The first chapter gives a good picture of the scientific background around 1900. It opens with an account of the circulation theorem derived by the Norwegian meteorologist Vilhelm Bjerknes. This theorem follows from work of Helmholtz and Kelvin, but makes allowance for a crucial property of the atmosphere, that pressure and density surfaces do not usually coincide. This is what is meant by the term *baroclinicity*. The theorem specifies how the circulation can change when baroclinicity is present. It enables us to calculate how vortices in the atmosphere and oceans behave, giving a holistic, but quantitative, description. Bjerknes' circulation theorem initiated the study of geophysical fluid dynamics.

Received on 17-4-2013.

This article originally appeared in the September 2013 issue of the Notices of the American Mathematical Society and is reprinted here with permission of the AMS. Copyright 2013 American Mathematical Society.

The basic mechanical and physical laws governing the atmosphere were in place by 1900. Bjerknes developed a vision of how weather forecasting could be put on solid theoretical foundations, and drew up what amounted to a manifesto for scientific prediction. He considered how precise, long-range predictions of astronomical events were possible, and aimed at employing a similarly systematic approach in meteorology. He recognised the intractability of the governing equations, and realized that analytical solution of them was impossible. But even an approximate solution, which would have required many months to carry out, might eventually lead to more practical methods.

The authors give an excellent account of developments over the first two decades of the twentieth century. The discovery by Henri Poincaré of sensitive dependence on initial conditions arose in the context of studying the three-body problem, but had much wider implications. It meant that unavoidable errors in the specification of the initial state of the atmosphere would grow over time, ultimately rendering the forecast useless. It imposed an inherent limit on prediction of future weather. A central aim of the book is to explain how rational prediction of weather is possible at all in the presence of chaos.

Predicting the weather is vastly more complex than predicting the return of a comet. The authors show how the problem can be reduced to the solution of a system of seven equations (coupled non-linear partial differential equations) in seven variables: pressure, temperature, density, humidity and three components of the wind. The mathematical details are very sketchy, even in the ‘Tech Boxes’ — boxes separate from the running text, with additional technical details — but the overall ideas are well conveyed.

Most of the key meteorologists are recognised in the narrative. In particular, William Ferrel’s work in formulating the equations on a rotating Earth is given due prominence. But a description of the important work of the scientists working in Vienna, specifically Max Margules and Felix Exner, is omitted. Margules anticipated the problems that would arise if the continuity equation were used for prediction, and Exner carried out several numerical forecasts using a highly simplified set of equations.

The method of solving a complicated system of equations by reducing them to a manageable, algebraic form is given good treatment.

The consequences of discretization are described by a nice analogy with pixilation of a painting, Constable's *Hay Wain*. Unfortunately, the indifferent quality of the illustration on page 34 detracts somewhat from the presentation. Overall, the diagrams in the book are good but some of the photographs are poorly reproduced.

Bjerknes' original idea was to use the mathematical equations to forecast the weather. However, the complexity of this task convinced him and his team to follow a more empirical line, which turned out to be enormously fruitful. The conceptual models of warm and cold fronts, and of the life cycles of frontal depressions, that emerged from the Bergen School dominated synoptic meteorology for most of the twentieth century and were of great practical benefit to humankind. More-quantitative methods had to await scientific and technical developments in mid-century.

During the First World War, an extraordinary numerical experiment was carried out by Lewis Fry Richardson who, using the best data set he could find, calculated changes in pressure and wind using the basic equations of motion. However, he was unaware that errors in the initial data could completely spoil the forecast, and his results were completely unrealistic. Richardson's attempt at practical forecasting by numbers was so unsuccessful and so impractical at the time, that it had a deterrent effect on other meteorologists. But of course, Richardson's approach was ultimately the right one, and the causes of the error in his forecast are now well understood and quite avoidable. The authors provide a clear description of what Richardson achieved and of the remarkable prescience of his work.

In a chapter entitled 'When the wind blows the wind', the authors attempt to convey the ideas of nonlinearity, a phenomenon that 'makes forecasting so difficult and weather so interesting'. This is a key idea, and I feel that the attempt can at best be described as a qualified success. Some of the discussion is lacking in clarity and may not provide readers with the desired level of understanding.

Part 2 opens with a chapter where the brilliant work of Carl Gustaf Rossby is described. Rossby had the capacity to reduce a problem to its essentials, and to devise conceptual models that elucidated the mechanism of atmospheric phenomena, unencumbered by extraneous details. In a landmark paper published in 1939, he explained the basic dynamics of the large wave-like disturbances in the atmosphere by using a simple model based on conservation of absolute

vorticity. Linearising this, he produced an expression for the phase speed of the waves, thereby explaining the mechanism of propagation and also providing a means of predicting the propagation of wave disturbances. Some mathematical details of Rossby's model are presented in a Tech Box.

Rossby's model assumed a wave disturbance of a fluid with uniform depth. When the fluid depth varies, the conserved quantity is the ratio of absolute vorticity to depth, the potential vorticity (PV) in its simplest form. This can be used to explain the effect of a mountain chain on the flow. The authors describe a flow over the Andes but do not mention that in the southern hemisphere the configuration of troughs and ridges is reversed. Thus, their account and their Figure 5.12 are likely to be a source of confusion to readers.

Rossby's formula was of limited value in practical forecasting. The atmosphere is complex and its behaviour cannot be reduced to a simple travelling wave on a uniform background flow. A much more complete understanding of how mid-latitude disturbances develop from small beginnings was provided by Jule Charney when he showed that they grow through baroclinic instability. Charney's work is rightly given prominence in the book. Having explained the mechanism of wave growth, he went on to produce a system of equations that could be used for practical numerical prediction whilst avoiding the problems encountered by Richardson. Charney then led the team that carried out the first successful prediction on the ENIAC computer in 1950. This was the beginning of real numerical weather prediction. The story is very well told in the book.

The limitations on prediction imposed by the chaotic nature of the atmosphere are then discussed. The work of Edward Lorenz was crucial to our understanding of what can and cannot be achieved. With our growing appreciation of the inherent limitations on weather forecasting, the emphasis has shifted from deterministic to probabilistic prediction, and the method of ensemble forecasting is now at the forefront of operational practise. All this is well described, including the application of probability forecasts to loss/cost models that can be used for rational decision-making with great economic benefits.

The authors have an interest in symplectic geometry, the mathematical framework underlying Hamiltonian mechanics. They include a description of the main ideas of symplectic geometry, but this is as likely to mystify as to inspire readers, especially as the link with PV is not clarified. More practical is the account of Lagrangian advection schemes, which have led to substantial increases in numerical efficiency of forecasting models. The components of climate models are also described. In general, more schematic diagrams showing, for example, the components of an earth system model and the principal physical processes parameterised in models, would have been welcome.

The book concludes by considering a number of outstanding issues. One of these is how best to represent moving surfaces of discontinuity. It is interesting that, in the Preface of his 1922 book, Richardson asked ‘how are we to deal with discontinuities by finite differences?’

Recognising that many readers are strongly discouraged by the appearance of even a few mathematical equations, the authors have endeavoured to elucidate the key ideas of modern weather prediction without explicit mathematical material. This is quite a difficult task. The attempt has been reasonably successful, and readers without advanced scientific knowledge but with an interest in scientific matters should get an accurate if incomplete impression of how modern weather forecasts are made.

In the main text, the authors strive to avoid mathematics and relegate details to ‘Tech Boxes’. I feel that more extensive use of mathematics in these boxes would have been appropriate and would have enabled mathematically inclined readers to get a more concrete understanding of the various technicalities discussed in the boxes (without frightening away more-general readers). Readers with more extensive mathematical knowledge may be frustrated by the absence of fuller quantitative detail.

In summary, this is a well-written book giving a generally clear and accessible account of how weather forecasts are prepared. The historical detail enlivens the narrative and makes for an enjoyable read. The authors have considerable knowledge and expertise, and the book is scientifically sound. It can be warmly recommended to anyone who wishes to understand, in broad terms, how modern

weather forecasts are made, and how we may use models of the atmosphere to anticipate changes in the Earth's climate.

Peter Lynch is Professor of Meteorology at UCD. His interests include dynamic meteorology, numerical weather prediction, Hamiltonian mechanics and the history of meteorology. He writes an occasional mathematics column *That's Maths* in the *Irish Times*; see his blog at <http://thatsmaths.com>.

SCHOOL OF MATHEMATICS, UNIVERSITY COLLEGE DUBLIN
E-mail address: `Peter.Lynch@ucd.ie`