

**26th Annual Meeting of the  
Irish Mathematical Society  
National University of Ireland  
Maynooth**  
AUGUST 26–27, 2013

The 26th annual scientific meeting of the Irish Mathematical Society was held at the National University of Ireland Maynooth. The meeting was organised by Stephen Buckley, Ann O’Shea, David Wraith (all of the Mathematics and Statistics Department, NUI Maynooth), and Steve Kirkland (of the Hamilton Institute, NUI Maynooth). The meeting brought together researchers from a wide range of disciplines, with 64 registered participants from Ireland, Britain, and the United States. Below we give the meeting timetable, and abstracts of the talks. We also refer the reader to the meeting website at <http://archive.maths.nuim.ie/staff/sbuckley/meetings/13IMS/>

TIMETABLE

**Monday 26 August**

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|-------------|---|
| 10:30–10:45 | Registration  |
| 10:45–11:00 | Opening remarks   |
| 11:00–11:50 | <b>Robert Devaney</b> , <i>Boston University</i><br>The Fractal Geometry of the Mandelbrot Set  |
| 12:00–12:30 | <b>James Gleeson</b> , <i>University of Limerick</i><br>Dynamics on complex networks  |
| 12:30–13:30 | Lunch   |
| 13:30–14:20 | <b>Tim Rowland</b> , <i>University of Cambridge</i><br>University mathematics teaching: is this the real thing?                                   |
| 14:30–15:00 | <b>Helena Šmigoc</b> , <i>University College Dublin</i><br>Patterns of symmetric matrices that allow the characteristic polynomial to be a square |
| 15:05–15:30 | Coffee/Tea  |
| 15:30–15:45 | <b>Victor Bible</b> , <i>University College Dublin</i><br>Boundaries and geometric renorming theory   |

- 15:50–16:05 **Sonia Balagopalan**, *National University of Ireland Maynooth*  
An elementary construction of the Witt design on 22 points
- 16:10–16:25 **Philipp Hoffmann**, *University College Dublin*  
An overview of  $(m, p)$ -isometric operators and operator tuples
- 16:40–17:10 **Joost Slingerland**, *National University of Ireland Maynooth*  
Topological Quantum Computation
- 17:15–18:15 IMS Committee Meeting
- 19:00– Dinner

### Tuesday 27 August

- 09:30–10:00 **Anca Mustata**, *University College Cork*  
Stable map spaces for varieties with  $\mathbb{C}^*$ -actions
- 10:10–11:00 **Mladen Bestvina**, *University of Utah*  
Cube complexes and 3-manifolds
- 11:05–11:30 Coffee/Tea
- 11:30–11:45 **Eoin Lynch**, *Trinity College Dublin*  
Neural encoding of auditory stimuli
- 11:50–12:05 **Dave McConnell**, *Trinity College Dublin*  
The Glimm space of the minimal tensor product of  $C^*$ -algebras
- 12:10–12:25 **James McTigue**, *National University of Ireland Galway*  
Partial matrices whose completions all have the same rank
- 12:30–12:45 **Richard Ellard**, *University College Dublin*  
Generating New Realisable Lists from Old in the NIEP
- 12:50–13:50 Lunch
- 13:50–14:50 IMS Annual General Meeting
- 15:00–15:30 **Paul Barry**, *Waterford Institute of Technology*  
Exponential Riordan arrays and the Toda Chain equations

- 15:40–16:10 **Rachel Quinlan**, *National University of Ireland Galway*  
Special spaces of matrices
- 16:10–16:40 Prize awards\* and Coffee/Tea

\* After the last talk, the Fergus Gaines Cup was presented to the winner of 2013 Irish Mathematical Olympiad, Oisín Faust, and the award for the best postgraduate talk of the meeting was presented to Philipp Hoffmann.

#### ABSTRACTS

**Sonia Balagopalan**, *National University of Ireland Maynooth*  
***An elementary construction of the Witt design on 22 points***

The 22-point Witt design is the unique 3-(22,6,1) block design, invariant under an action of the sporadic group  $M_{22}$ . We present a new construction of this exceptional combinatorial object, based on well-known properties of the complete graph on 6 vertices. We also sketch a connection with the only known vertex-minimal triangulation of real projective 4-dimensional space.

**Paul Barry**, *Waterford Institute of Technology*  
***Exponential Riordan arrays and the Toda Chain equations***

The Toda Chain equations are a model of dynamical systems that are integrable. They are intimately associated with time-dependent orthogonal polynomials and Hankel determinants. In this talk we will show how to construct systems satisfying the Toda equations using exponential Riordan arrays. The combinatorial origin of some of these arrays will be described.

**Mladen Bestvina**, *University of Utah*  
***Cube complexes and 3-manifolds***

I will outline the recent application of geometric group theory to the structure of 3-manifolds due to Agol and Wise, particularly the virtual Haken conjecture. This breakthrough essentially finishes the list of problems proposed by Thurston.

**Victor Bible**, *University College Dublin*  
***Boundaries and geometric renorming theory***

Given a geometric property and a Banach Space  $X$ , we are concerned

with obtaining sufficient conditions for when  $X$  has a renorming with the property. A boundary of  $X$  is a subset of the dual unit ball with the following: Given an element  $x$  of  $X$ , there exists an element of the boundary which agrees with the norm of  $x$  when evaluated at  $x$ . Sometimes the existence of a boundary with certain properties will be such a sufficient condition. I will present some of these results for the property of polyhedrality. Upcoming areas of focus will then be discussed.

**Robert Devaney**, *Boston University*

***The Fractal Geometry of the Mandelbrot Set***

In this lecture we describe several folk theorems concerning the Mandelbrot set. While this set is extremely complicated from a geometric point of view, we will show that, as long as you know how to add and how to count, you can understand this geometry completely. We will encounter many famous mathematical objects in the Mandelbrot set, like the Farey tree and the Fibonacci sequence. And we will find many soon-to-be-famous objects as well, like the “Devaney” sequence. There might even be a joke or two in the talk.

**Richard Ellard**, *University College Dublin*

***Generating New Realisable Lists from Old in the NIEP***

Given a list of complex numbers  $\sigma := (\lambda_1, \lambda_2, \dots, \lambda_m)$ , we say that  $\sigma$  is realisable if  $\sigma$  is the spectrum of some (entrywise) nonnegative matrix. The Nonnegative Inverse Eigenvalue Problem (or NIEP) is the problem of categorising all realisable lists.

Given a realisable list  $(c_1, \lambda_2, \lambda_3, \dots, \lambda_m)$ , where  $c_1$  is the so-called Perron eigenvalue of the list, Šmigoc (2004) shows that we may replace the eigenvalue  $c_1$  with a list  $(\mu_1, \mu_2, \dots, \mu_n)$ , where  $\mu_1, \dots, \mu_n$  satisfy certain conditions, i.e. we may produce a new realisable list of the form  $(\mu_1, \mu_2, \dots, \mu_n, \lambda_2, \lambda_3, \dots, \lambda_m)$ . Subsequently, Šmigoc (2005) gave a construction to replace both the Perron eigenvalue  $c_1$  and another real eigenvalue  $c_2$  of a realisable list.

In this talk, we first expand on the above work by presenting some new lists which may replace the eigenvalues  $c_1$  and  $c_2$ . We then go on to give a construction which allows us to replace three eigenvalues of a realisable list (the Perron eigenvalue and a complex conjugate pair), i.e. from a realisable list  $(\rho, \alpha + i\beta, \alpha - i\beta, \lambda_4, \dots, \lambda_m)$ , we generate a new realisable list  $(\mu_1, \mu_2, \dots, \mu_n, \lambda_4, \dots, \lambda_m)$ .

**James Gleeson**, *University of Limerick*  
***Dynamics on complex networks***

I will discuss the modelling of the structure of large-scale networks, and dynamics that occur on the nodes (vertices) of such networks. A wide class of binary-state dynamics—including, for example, the voter model, the Bass diffusion model, and threshold models—can be described in terms of transition rates (spin-flip probabilities) that depend on the number of nearest neighbours in each of the two possible states. High-accuracy approximations for the emergent dynamics of such models on uncorrelated, infinite networks are given by recently-developed compartmental models or approximate master equations (AME). Pair approximations (PA) and mean-field theories can be systematically derived from the AME; we show that PA and AME solutions can coincide in certain circumstances. This facilitates bifurcation analysis, yielding explicit expressions for the critical (ferromagnetic/paramagnetic transition) point of such dynamics, closely analogous to the critical temperature of the Ising spin model.

**Philipp Hoffmann**, *University College Dublin*  
***An overview of  $(m, p)$ -isometric operators and operator tuples***

So-called  $m$ -isometric operators were defined in the 1980s on complex Hilbert spaces, and since then have attracted the interest of many researchers. In recent years, a generalisation to operator tuples has been given, as well as a generalisation to operators on general normed spaces. We give an overview of the history of these objects and present some new results.

**Eoin Lynch**, *Trinity College Dublin*  
***Neural encoding of auditory stimuli***

The problem of neural encoding is a central issue in theoretical neuroscience. That is, how are external stimuli encoded by neurons into sequences of electrical impulses referred to as spike trains? In this talk an overview of current theory in this area will be presented. Recent results of modelling the encoding process in auditory response data using a novel cascade spiking neuron model will be presented and discussed. The model consists of a linear stimulus filter cascaded with a neuron model from the class of two dimensional models with adaptation. An approach to fitting this model to data using a

spike train metric, the van Rossum distance, is also discussed. One advantage of this approaches used here is the generation of precise spike times as opposed to a rate function. The results will thus be presented along side those obtained with standard rate models.

**Dave McConnell**, *Trinity College Dublin*

***The Glimm space of the minimal tensor product of  $C^*$ -algebras***

We consider a  $C^*$ -algebra  $A$  and its space of Primitive ideals  $\text{Prim}(A)$  in its usual hull-kernel topology. The Glimm space of  $A$ ,  $\text{Glimm}(A)$ , is defined as the complete regularisation of  $\text{Prim}(A)$ —the quotient of  $\text{Prim}(A)$  modulo the relation of inseparability by continuous functions, with topology induced by the continuous functions on  $\text{Prim}(A)$ . To each such equivalence class in  $\text{Prim}(A)$  we assign a (closed, two-sided) ideal of  $A$  given by the intersection of the primitive ideals contained in it.

For the minimal tensor product  $A \otimes B$  of  $C^*$ -algebras  $A$  and  $B$  we describe  $\text{Glimm}(A \otimes B)$  in terms of  $\text{Glimm}(A)$  and  $\text{Glimm}(B)$ , both topologically and as a set of ideals of  $A \otimes B$ , generalising a result of Kaniuth (1994). We apply this to the study of the centre of the multiplier algebra of  $A \otimes B$ .

**James McTigue**, *National University of Ireland Galway*

***Partial matrices whose completions all have the same rank***

A partial matrix over a field  $\mathbb{F}$  is a matrix whose entries are either elements of  $\mathbb{F}$  or independent indeterminates. A completion of such a partial matrix is obtained by specifying values from  $\mathbb{F}$  for the indeterminates. We determine the maximum possible number of indeterminates in an  $m \times n$  partial matrix ( $m \leq n$ ) whose completions all have a particular rank  $r$ , and we fully describe those examples in which this maximum is attained, without any restriction on the field  $\mathbb{F}$ .

We also describe sufficient conditions for an  $m \times n$  partial matrix whose completions all have rank  $r$  to contain an  $r \times r$  partial matrix whose every completion is non-singular.

This is joint work with Rachel Quinlan and it extends some recent results of R. Brualdi, Z. Huang and X. Zhan.

**Anca Mustata**, *University College Cork*

***Stable map spaces for varieties with  $C^*$ -actions***

This talk is based on joint work with Andrei Mustata. The moduli

spaces of stable maps are compactifications of the spaces of smooth curves of a specific class in a complex projective variety. They are important in counting curves with given properties in that variety; the numerical answers to these counts are known as Gromov-Witten invariants. To provide a count that is consistent over complex deformations of the target variety, these moduli spaces have been endowed with a suitable homology class called the virtual fundamental class. The oldest and one of the most useful tools for calculating Gromov-Witten invariants is localization. For target varieties endowed with a torus  $(\mathbb{C}^*)^n$  action, it reduces the Gromov-Witten calculations to a study of the fixed-point locus of the stable map space under the given action. This has been particularly successful in the case of toric varieties, i.e.  $n$  dimensional varieties with a  $(\mathbb{C}^*)^n$  action. The fixed-point locus of the stable map space parametrizes combinations of the following curves: (i) curves mapping to the fixed loci of the target variety and (ii) curves which are unions of orbits. In the examples studied so far, the set of curves of the second type is relatively simple, so that the computations are concentrated on to the fixed loci of the target variety. In the case of  $\mathbb{C}^*$  actions however, the space of curves which are union of orbits is quite large and complex. In our project we describe this space as an inverse limit of GIT quotients of the variety under the  $\mathbb{C}^*$  actions, and use this to give a relatively simple description of the virtual fundamental class.

**Rachel Quinlan**, *National University of Ireland Galway*  
*Special spaces of matrices*

A large body of literature considers linear and affine spaces of matrices in which rank behaves in a controlled way - for example, the ranks of the non-zero elements are all the same or satisfy specified bounds. This talk will mention some selected highlights of this subject, including the Radon-Hurwitz numbers which give the maximum possible dimensions of spaces of real square matrices whose non-zero elements are all invertible. A few of the author's own contributions, arising from (separate) joint works with R. Gow and J. McTigue, will also be mentioned.

**Tim Rowland**, *University of Cambridge*  
*University mathematics teaching: is this the real thing?*

Mathematics teaching at the school level can be thought to be making various concessions to the immaturity of school students in its

treatment of mathematical objects and concepts. In particular, the subject is often represented by reference to various analogies and models, and rigour in proof is significantly compromised. Mathematics at university can be viewed as the point when such concessions and compromises can, and perhaps should, be set aside. So when mathematics is taught in the university, is this the real thing? My approach to this question will be shaped, in part, by a theory of mathematical knowledge in teaching—the Knowledge Quartet—developed in the context of school mathematics.

**Joost Slingerland**, *National University of Ireland Maynooth*  
***Topological Quantum Computation***

Quantum computation is the use of quantum mechanical processes to manipulate information. This allows one to solve certain types of problem more efficiently than using classical mechanics (pen and paper, or current computers). Topological quantum computation is an approach to quantum computation which uses exchanges of particles to manipulate the information. The information in the computer and the computations themselves are then described in terms of the representation theory of certain groups governing the exchanges, most prominently Artin's braid groups. I will discuss the types of representations that one may expect to find in physical systems, as well as some questions about them which emerge from the potential application to computation.

**Helena Šmigoc**, *University College Dublin*  
***Patterns of symmetric matrices that allow the characteristic polynomial to be a square***

Let  $G$  be an undirected graph on  $n$  vertices and let  $S(G)$  be the set of all real symmetric  $n \times n$  matrices whose nonzero off-diagonal entries occur in exactly the positions corresponding to the edges of  $G$ . We will discuss the question of for which graphs  $G$  does there exist a matrix  $A$  in  $S(G)$  whose characteristic polynomial is a square, i.e., the multiplicities of all the eigenvalues of  $A$  are even. This is joint work with P. Oblak.

Report by Stephen M. Buckley, National University of Ireland Maynooth.

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