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Theodore Frankel: The Geometry of Physics, 3rd Edition, Cambridge University Press, 2012. ISBN:978-1-107-60260-1, USD 69, 686+lvi pp.

REVIEWED BY BRUNO HARRIS

This is a most interesting and valuable book by a distinguished Geometer, explaining clearly and carefully a large part of Geometry and Topology and its application to concepts in Physics. Differential forms are introduced right away (explaining their relation to the usual vector analysis in Calculus), and indeed the author acknowledges his indebtedness to "Differential Forms" by Harley Flanders (his Ph.D. advisor), a book much appreciated by Mathematicians and Physicists. Frankel's book is also very rich in applications but is much larger than the Flanders book and contains much more. After the introductory bridge from Calculus to differential forms, the first chapter discusses differentiable manifolds, vector fields (and their flows), tensor fields in general and differential forms. Applications include Configuration Space (just a manifold) , its Tangent bundle, and its Cotangent Bundle (which is Phase Space). The author points out that the tangent bundle and the cotangent bundle are not the same thing. In Chapter 2 the Lagrangian of Mechanics is a function on the tangent bundle, whereas the Hamiltonian is a function on Phase Space. Chapter 4 continues this discussion with the symplectic 2-form on Phase space, and Chapter 10 goes further with this topic. Another topic that is carefully explained is that of Orientation of a manifold (in Chapter 2), and "pseudoforms (like differential forms but changing sign if orientation is reversed). Chapter 3 discusses integration of forms, and also introduces the Electromagnetic field as a 2-form (essentially done by Minkowski) and Maxwell's equations (only two are needed now). Maxwells equations reappear in chapter 14 on the Hodge theory of harmonic forms; this chapter also discusses Hodge theory on manifolds with boundary. Chapters 7 to 10 are on Riemannian Geometry; chapters 11, 12 continue with Einstein and Hilbert on Relativity. Topology enters

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in chapter 12 (Curvature) and 13 (De Rham cohomology), and Lie groups appear in chapter 15 and are applied to Dirac's equation in Chapter 19. Chapter 16 introduces vector bundles, connections, and applications to gauge theory in Electromagnetic theory, and (chapter 20) in Yang-Mills theory. Chapter 20 also explains Noether's theorem (which states that a symmetry of the Lagrangian gives a conserved current) and has more on Weyl's Gauge Invariance and Yang-Mills. Further important Topology comes in Chapters 21 (fundamental group, compact Lie groups) and 22 (Chern forms, higher homotopy groups, even Chern-Simons forms). Finally, the many readers who enjoy this book may also like the Mathematics books "Differential Forms in Algebraic Topology" by Bott and Tu, and "Differential Forms" by H. Cartan.

References

[1] R. Bott and L. Tu:: *Differential forms in algebraic Topology*, Springer, 1982.
[2] H. Cartan: *Differential Forms*, Dover, 2006.
[3] H. Flanders: *Differential Forms*, Academic Press. 1963.

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