

**Alan Baker: A Comprehensive Course in Number  
Theory, Cambridge University Press, 2012,  
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REVIEWED BY TOM CARROLL

Baker's superb book is a much-expanded, substantial reworking of his *A Concise Introduction to the Theory of Numbers* [1], retaining the style and character of the original 'concise' edition in this 'comprehensive' account. The choice of topics follows lines which have been well-established by many excellent texts over the years. Indeed, the first four of seventeen chapters are entitled *divisibility, arithmetical functions, congruences, quadratic residues*. The feel is that of classical analysis where the goal is to solve problems and establish general results rather than to formulate general theories. This is not an 'elementary' book on number theory including, as it does, aspects of both algebraic and analytic number theory as required. At the same time, Baker does not skimp in his treatment of the more popular aspects of the subject such as perfect numbers, Mersenne and Fermat primes, Carmichael numbers, and has a nice, short chapter on *factorisation and primality testing*, which includes the RSA public key cryptosystem.

Each chapter is divided into short readable sections, the last two in each chapter being, without exception, *further reading* and *exercises*. Theorems are not often formally stated. Rather than being 'theorem-proof', the expository style is primarily discursive with many paragraphs beginning with the phrase 'for the proof'. The glue that holds the exposition together is the authority of the author which is evident throughout. Since Baker is clearly as much at ease with the law of quadratic reciprocity, with the prime number theorem, and with primes in arithmetical progressions as he is with the Euclidean algorithm and the Euler  $\phi$ -function, the exposition is never strained or forced. That said, despite the single conspicuous variation in its title compared to its predecessor, the mathematical

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narrative continues to be quite concise. This allows Baker to advance rapidly at the sole expense of requiring readers to pause often for thought and to convince themselves of the validity of his arguments. By Page 42, Baker has already established both Gauss's law of quadratic reciprocity and that every natural number can be written as the sum of four squares, so that these forty-odd pages alone would make a fine 24-lecture first undergraduate course in number theory. In fact, since the chapters are short and broken into bite-sized sections which are seldom longer than two or three pages (and are often much shorter), most chapters can reasonably be read in a single sitting. The book has a substantial feel to it so that, having worked through a chapter, one is rewarded with the sense that one now knows not only the key concepts but something significant concerning that chapter's topic. The final chapters, on *analytic number theory*, on *the zeros of the zeta-function*, on *the distribution of the primes*, on *the sieve and circle methods*, on *elliptic curves*, occupy the last one hundred pages, treating topics which are more advanced than those in the previous twelve chapters, always in the same assured manner.

Baker's rewarding book is a pleasure to read including, as it does, so much beautiful mathematics, beautifully expounded.

#### REFERENCES

- [1] A. Baker, *A Concise Introduction to the Theory of Numbers*, Cambridge University Press, 1984.

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