

**Richard P. Stanley: Enumerative Combinatorics,
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REVIEWED BY THOMAS HÜTTEMANN

Ever since the first edition of “Enumerative Combinatorics” (Volume 1) appeared in 1986 its presentation has shaped the subject, and introduced countless mathematicians and mathematics students to the beauty of combinatorial arguments. However, the subject has seen many new and important developments in the last two decades, prompting Stanley to update and expand the text. So now the second addition is available. The sheer size is impressive: it has 626 pages (compared to the 325 of the 2008 paperback edition) and comes with 537 exercises (up from 180). The numbers indicate already that a lot of new material is covered; among the additions are q -analogues of permutations, hyperplane arrangements, the cd -index, and differential posets.

Just like the first edition, the second is divided into four chapters: basics of enumerative combinatorics, sieve methods, partially ordered sets, and rational generating functions are the headlines. Each chapter is accompanied by notes containing further information (mathematical and historical), a bibliography, exercises, and solutions to exercises.

The introductory first chapter contains a thorough study of permutation statistics as well as their q -analogues, and hence comprises much more material than the chapter title might one lead to expect. There is also a detailed account of Rota’s “twelfefold way” which is a systematic approach to solving fundamental counting problems.

In contrast, the second chapter on sieve methods is surprisingly short: the main result is phrased in linear-algebraic terms, with several variations and applications subsequently developed. This includes classical formulations (inclusion-exclusion for families of sets) and weighted versions.

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Sieve methods are put into perspective in the third chapter, which deals with the algebraic theory of partially ordered sets; relevant keywords include incidence algebras, Möbius functions, and order polynomials. Apart from covering the theory and its combinatorial applications, Stanley points out important connections with topology, for example identifying certain Möbius functions as Euler characteristics of cell complexes.

Finally, the fourth chapter contains a careful discussion of (rational) formal power series and their applications in combinatorics, an area hugely influenced by Stanley. Here the book really shines. Great care is taken to point out the similarities and the differences between formal power series and analytic functions, dispensing with the hand-waving account often encountered in other texts. The theory is applied to a wide range of problems including such gems as Ehrhart theory of rational polytopes and enumeration of magic squares.

Going back to the beginning, the book starts off with a discussion of “how to count”. More precisely, Stanley describes under what circumstances one might consider an answer to a combinatorial question a “good” answer. The approach is practical: wherever possible the terms of the resulting formula should have intuitive meaning. He similarly describes what should be considered a “combinatorial” argument, and what shouldn’t. Such meta-mathematical classification depends of course on a number of factors, not least on taste, experience and background of the reader. It is a useful and enjoyable way of entering the world of combinatorics though, emphasising intuitive understanding over merely establishing facts by whatever means available.

This approach is in fact taken all over the book. Strikingly often statements are proved several times, giving for example algebraic, analytic and combinatorial proofs side by side. Each proof illuminates a different facet of the problem, and illustrates a different way of thinking about the original question. At the same time this certainly demands some maturity from the reader who is expected to delve entirely into Stanley’s world, aiming for depth of understanding rather than rapid accumulation of knowledge. It is not a book for the casual reader.

The huge number of exercises makes this book well-suited for self study, in particular as most exercises come with solutions. However,

for many of the newly added exercises solutions have been intentionally omitted. This makes it easier, if the book is used for teaching, to find suitable homework problems to assign to students.

Stanley has a refreshing and easy to read style. He manages to explain concepts clearly, not shying away from informal language but without sacrificing mathematical rigour. Concepts are illustrated by numerous examples, with many more applications and further developments delegated to the exercises. This is an excellent textbook and on combinatorics that will have a place on many a mathematician's bookshelf.

Thomas Hüttemann is a lecturer in pure mathematics at Queen's University Belfast. His research interests include combinatorial methods in topology, homotopy theory and algebra.

QUEEN'S UNIVERSITY BELFAST, SCHOOL OF MATHEMATICS AND PHYSICS,
PURE MATHEMATICS RESEARCH CENTRE, BELFAST BT7 1NN, UK

E-mail address: `t.huettemann@qub.ac.uk`