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RANGES OF BIMODULE PROJECTIONS AND CONDITIONAL EXPECTATIONS

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This is an abstract of the PhD thesis *Ranges of Bimodule Projections and Conditional Expectations* written by R. Pluta under the supervision of Prof. Richard M. Timoney at the School of Mathematics, Trinity College Dublin and submitted in September 2011.

The algebraic theory of corner rings introduced by Lam [1] (as an abstraction of the properties of Peirce corners eRe of a ring R associated with an idempotent $e \in R$) is investigated in the context of C^*-algebras and operator algebras. The main result is as follows.

Theorem. *Let H be a Hilbert space with an orthonormal basis $(e_i)_{i\in I}$ (which may be countable or uncountable), and $\mathcal{B}(H)$ the algebra of bounded operators on H. Let $\mathcal{E}\colon \mathcal{B}(H) \to \mathcal{B}(H)$ be a linear map with range S a subalgebra such that $\mathcal{E}\circ\mathcal{E} = \mathcal{E}$, $\mathcal{E}$ is an S-bimodule map, and $\mathcal{E}(x^*) = \mathcal{E}(x)^*$ for $x \in \mathcal{B}(H)$ ($\mathcal{E}$ is called a Lam conditional expectation). Then, if $e_i \otimes e_i^* \in S$ for $i \in I$, there is an equivalence relation on I such that $\mathcal{E}(x) = \sum_{j\in J} p_j x p_j$ for $x \in \mathcal{B}(H)$, where J is the set of equivalence classes, $p_j = \sum_{i\in j} e_i \otimes e_i^*$ for $j \in J$, and $e_i \otimes e_i^*$ is the operator that sends an element $h \in H$ to $\langle h, e_i\rangle e_i \in H$.*

This is generalized to purely atomic von Neumann algebras.

References

[1] T. Y. Lam: *Corner ring theory: a generalization of Peirce decompositions. I.* Algebras, rings and their representations, 153–182, World Sci. Publ., Hackensack, NJ, 2006.

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